Example 5-3: Impulse Response of Cascade

To illustrate the utility of the results that we have obtained for cascaded LTI systems, consider the cascade of two systems defined by

$$h_1[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{otherwise} \end{cases} \qquad h_2[n] = \begin{cases} 1 & 1 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

The results of this section show that the overall cascade system has impulse response

$$h[n] = h_1[n] * h_2[n]$$

Therefore, to find the overall impulse response we must convolve $h_1[n]$ with $h_2[n]$. This can be done by using the polynomial multiplication algorithm of Section 5-3.3.1. In this case, the computation is as follows:

n	$n<0$	0	1	2	3	4	5	6	$n>6$
$h_1[n]$	0	1	1	1	1	0	0	0	0
$h_2[n]$	0	0	1	1	1				
$h_1[0]h_2[n]$	0	0	0	0	**0**	0	0	0	0
$h_1[1]h_2[n-1]$	0	0	1	1	**1**	1	0	0	0
$h_1[2]h_2[n-2]$	0	0	0	1	**1**	1	1	0	0
$h_1[3]h_2[n-3]$	0	0	0	0	**1**	1	1	1	0
$h[n]$	0	0	1	2	**3**	3	2	1	0

Therefore, the equivalent impulse response is

$$h[n] = \sum_{k=0}^{6} b_k \delta[n-k]$$

where $\{b_k\}$ is the sequence $\{0, 1, 2, 3, 3, 2, 1\}$. This result means that a system with this impulse response $h[n]$ can be implemented either by the single difference equation

$$y[n] = \sum_{k=0}^{6} b_k x[n-k] \tag{5.32}$$

where $\{b_k\}$ is the above sequence, or by the pair of difference equations

$$w[n] = \sum_{k=0}^{3} x[n-k] \qquad y[n] = \sum_{k=1}^{3} w[n-k] \tag{5.33}$$

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