Name ______________________________ Date ______________

PATTERNS AND RULES

A pattern can be found in a list of numbers or shapes that follow a rule to determine their order. There are different kinds of patterns.

Growing Pattern: 3, 5, 7, 9, 11, …

To find the rule, compare each pair of numbers in the list.

Notice: 3 + **2** = 5
5 + **2** = 7
7 + **2** = 9
9 + **2** = 11

To find the next number in the pattern, the rule is to add 2 to the last term.

Repeating Pattern: 1, 2, 2, 1, 2, 2, …

To find the rule, look for a pattern in the way the numbers are repeated.

The rule is to repeat 1, 2, 2.

Tell if each pattern is a growing pattern or a repeating pattern. Find the rule for the pattern.

1. 1, 4, 7, 10, 13, … ____________________

2. 5, 9, 13, 17, 21, … ____________________

3. 7, 4, 7, 4, 7, 4, … ____________________

4. 5, 15, 25, 35, 45, … ____________________

5. 3, 5, 1, 3, 5, 1, 3, 5, … ____________________

6. 8, 13, 18, 23, 28, … ____________________

7. 6, 16, 6, 16, 6, … ____________________

8. 30, 300, 30, 300, 30, … ____________________

CHALLENGE

Find the next number in each pattern in Exercises 1–5.

____

Name ______________________________ Date ______________

PATTERNS AND RULES

A pattern can be found in a list of numbers or shapes that follow a rule to determine their order. Use the rule to find the next number in the pattern.

Increasing Pattern:

2, 12, 22, 32, 42, …

Notice: 2 + **10** = 12
12 + **10** = 22 and so on.

The rule is to add 10.
The next number in the pattern is
42 + 10, or 52.

Decreasing Pattern:

39, 36, 33, 30, 27, …

Notice: 39 – **3** = 36
36 – **3** = 33 and so on.

The rule is to subtract 3.
The next number in the pattern is
27 – 3, or 24.

Repeating Pattern:

2, 3, 3, 2, 3, 3, …

To find the rule, look for a pattern in the way the numbers are repeated.

The rule is to repeat 2, 3, 3.
The next number in the pattern is 2.

Write the rule for each pattern. Then find the next number in the pattern.

1. 8, 12, 8, 12, 8, … ____________________

2. 4, 9, 14, 19, 24, … ____________________

3. 50, 48, 46, 44, 42, … ____________________

4. 100, 90, 80, 70, 60, … ____________________

5. 21, 41, 61, 81, … ____________________

CHALLENGE

Explain how to find the next two numbers in the following pattern:
2, 3, 4, 3, 6, 3, 8, 3, …
What are the next two numbers in the pattern?

____

____

Name ____________________ Date ____________

PATTERNS AND RULES

A pattern can be found in a list of numbers or shapes that follow a rule to determine their order. Use the rule to find the next number in the pattern.

Growing Pattern:

1.2, 2.4, 4.8, 9.6, …

Notice: $1.2 \times \mathbf{2} = 2.4$

$2.4 \times \mathbf{2} = 4.8$ and so on.

The rule is to multiply by 2.
The next number in the pattern is 9.6×2, or 19.2.

Decreasing Pattern:

95, 87, 79, 71, …

Notice: $95 - \mathbf{8} = 87$

$87 - \mathbf{8} = 79$ and so on.

The rule is to subtract 8.
The next number in the pattern is $71 - 8$, or 63.

Combined Pattern:

3, 2, 5, 2, 7, 2, …

The number 2 is repeated. The other numbers increase by 2.

A rule for the pattern is to add 2 to the numbers in the odd places in the pattern. Repeat the number 2 in the even places. The next number in the pattern is 9.

Write a rule for each pattern. Then find the next three numbers in the pattern.

1. 10.1, 9.6, 9.1, 8.6, … ____________________
2. 1, 3, 9, 27, … ____________________
3. 3, 6, 12, 24, … ____________________
4. 200, 5, 180, 5, 160, 5, … ____________________

____

5. 2.5, 3.6, 4.7, 5.8, … ____________________

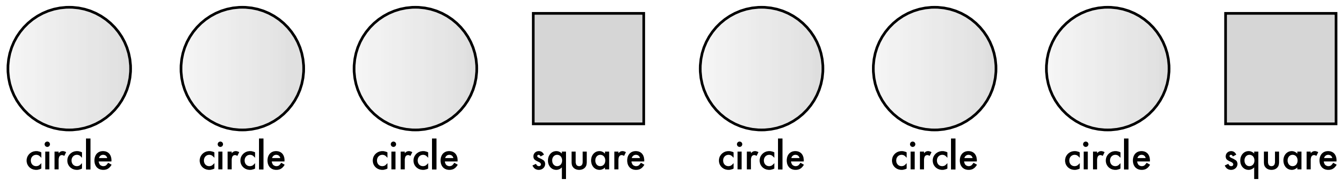
CHALLENGE

Write a rule for the following pattern. Then find the next two numbers in the pattern.
8, 15, 12, 19, 16, 23, … ____________________

Name ______________________________ Date ______________

REPRESENTING PATTERNS

You can continue a pattern to find what comes next.

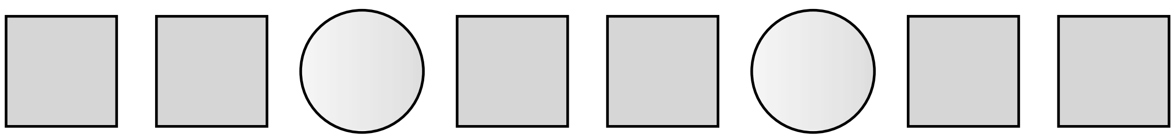


The pattern is a repeating pattern of shapes: circle, circle, circle, square.
You can also use numbers to describe the pattern: 1, 1, 1, 2, 1, 1, 1, 2.

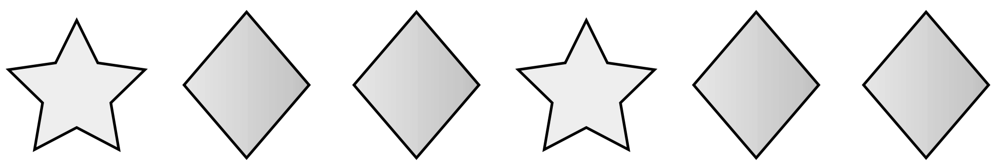
Continue the pattern to find the next shape in the pattern.
Since a circle follows the square, the next shape is a circle.

Draw the next shape in each pattern.
Then use numbers to describe the pattern.

1.



2.



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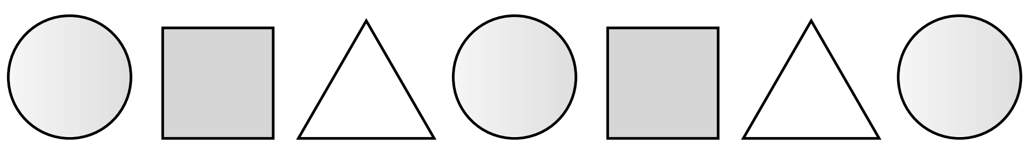

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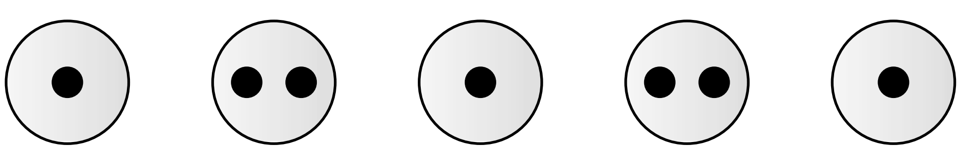
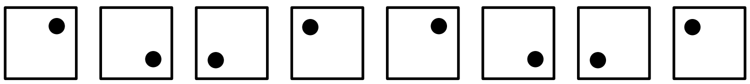


Name ______________________________ Date ______________

REPRESENTING PATTERNS

You can continue a pattern to find what comes next.

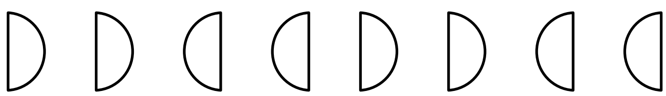


The pattern is a repeating pattern of squares.
The dot rotates $\frac{1}{4}$ turn clockwise each time.

You can also use numbers to describe the pattern: 1, 2, 3, 4, 1, 2, 3, 4.

Continue the pattern to find the next shape in the pattern.
The next number in the number pattern is 1.
The next shape in the shape pattern is the first figure.

Draw the next shape in each pattern.
Then use numbers to describe the pattern through two cycles.

1. 

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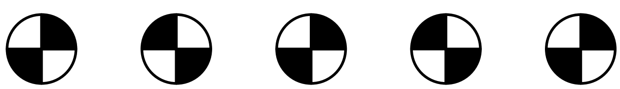
2. 

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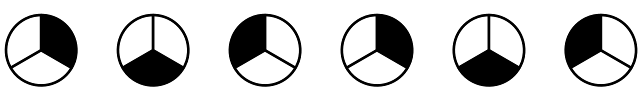
3. 

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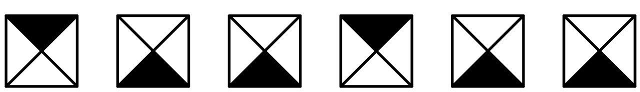
4. 

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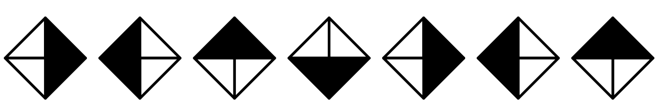
5. 

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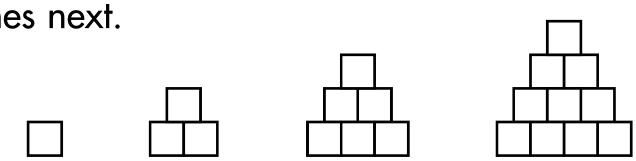
Name ______________________ Date ______________

REPRESENTING PATTERNS

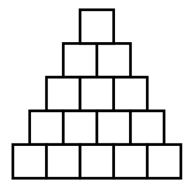
You can continue a pattern to find what comes next.

The pattern is a growing shape pattern.
Each time a row is added at the bottom
with 1 more square than in the previous shape.

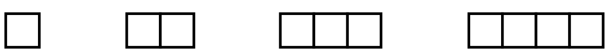


You can also use numbers to describe the pattern: 1, 3, 6, 10, …

To draw the next shape in the pattern,
add on a row with 5 squares at the bottom.
To continue the number pattern,
add 5 to the last number: 10 + 5 = 15.



Draw the next shape in each pattern.
Then use numbers to describe the pattern through four elements.

1. 

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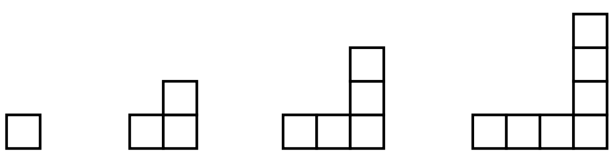
2. 

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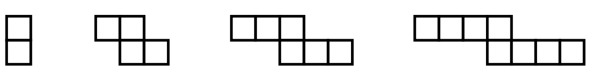
3. 

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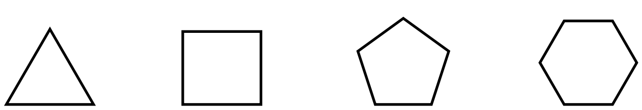
4. 

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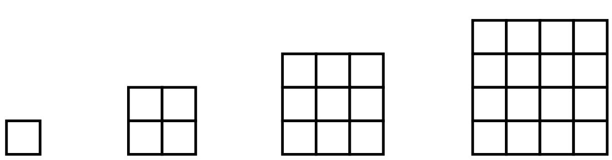
5. 

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Name ______________________________ Date ____________

Discovery Education

VARIABLES

A **variable** can be a symbol used to represent a not yet known or determined number.

You can use a variable to represent an answer.
You can choose any letter for the variable.

$4 + 7 = a$ → a represents the total of $4 + 7$.
$15 - 7 = n$ → n represents the difference of $15 - 7$.
$3 \times 6 = x$ → x represents the product of 3×6.
$18 \div 2 = w$ → w represents the quotient of $18 \div 2$.

Tell what each variable represents.

1. $7 \times 5 = m$ ______________________
2. $12 - 4 = b$ ______________________
3. $24 \div 8 = k$ ______________________
4. $9 + 3 = s$ ______________________
5. $17 - 9 = p$ ______________________
6. $8 + 6 = t$ ______________________
7. $36 \div 4 = q$ ______________________
8. $3 \times 9 = c$ ______________________

CHALLENGE

Find a value for each of the variables in Ex. 1–8.

____

Name ____________________ Date ____________

VARIABLES

A **variable** can be a symbol used to represent a not yet known or determined number.

John saw 6 bison.
Some more bison joined them.
How many bison are there?

Use a variable to represent the number of bison that joined them.
You can choose any letter for the variable.
Use *b* for the number of bison.

There were 6 bison and *b* more bison joined them.

$$6 + b$$

number John saw   number that joined them

There are $6 + b$ bison.

Choose a variable.
Tell what the variable represents.
Then use the variable to tell how many.

1. Rachel saw 10 caribou.
Then some of the caribou left.
How many caribou are there now?

____

2. A monkey eats 2 bananas for breakfast.
The monkey eats more bananas for dinner.
How many bananas does the monkey eat in all?

____

3. There are 8 cans of soup.
Some friends eat some of the soup for lunch.
How many cans of soup are left?

____

DISCOVERING MATH

LEVEL 2

Name ______________________________ Date ______________

VARIABLES

A **variable** can be a symbol used to represent a not yet known or determined number. An **expression** may combine numbers, operation signs, and variables.

The ranger counted 34 bison in one group.
There is another group of bison across the river.
How many bison are there in all?

You can write an expression to represent the total number of bison.
Choose a variable to represent the number of bison across the river.
You can choose any letter for the variable.

Let b = the number of bison across the river.

Expression: $34 + b$

number ranger counted — number across the river

There are $34 + b$ bison.

Write an expression with a variable.
Tell what the variable represents.

1. A large group of caribou is migrating.
500 of the caribou move ahead of the group.
How many caribou are behind?

____

2. The monkeys eat some bananas for breakfast.
They eat twice as many bananas for dinner as for breakfast.
How many bananas do they eat for dinner?

____

3. There are some cans of soup on the shelf.
Josh puts the cans into 5 equal groups.
How many cans of soup are in each group?

____

DISCOVERING MATH LEVEL 3

Name ______________________________ Date ____________

EQUALITY

The equals sign, =, is used to show that two quantities are equal.

You can show how to represent the number 4 in various ways using the equals sign.

Sums:	$0 + 4 = 4$	$1 + 3 = 4$	$2 + 2 = 4$
Differences:	$6 - 2 = 4$	$5 - 1 = 4$	$8 - 4 = 4$

Write three different sums and differences that are equal to each number, and use an equals sign to show it for each one.

1. 7 Sums: ______________________________

Differences: ______________________________

2. 5 Sums: ______________________________

Differences: ______________________________

3. 8 Sums: ______________________________

Differences: ______________________________

4. 6 Sums: ______________________________

Differences: ______________________________

5. 10 Sums: ______________________________

Differences: ______________________________

6. 12 Sums: ______________________________

Differences: ______________________________

7. 15 Sums: ______________________________

Differences: ______________________________

CHALLENGE

Write a sum, difference, and product equal to 20.

____

Name ______________________________ Date ______________

EQUALITY

In a number sentence, the equals sign, =, shows that two quantities are **equal**.

You can show how to represent numbers in various ways using the equals sign.

Each of these sums is equal to 12.

$0 + 12 = 12$ $3 + 9 = 12$ $4 + 8 = 12$ $6 + 6 = 12$

Each of these differences is equal to 12.

$13 - 1 = 12$ $15 - 3 = 12$ $17 - 5 = 12$ $20 - 8 = 12$

There are other sums and differences that are also equal to 12.

Write three different sums and differences that are equal to each number, and use an equals sign to show it for each one.

1. 15 Sums: ______________________

Differences: ______________________

2. 18 Sums: ______________________

Differences: ______________________

3. 20 Sums: ______________________

Differences: ______________________

4. 25 Sums: ______________________

Differences: ______________________

5. 32 Sums: ______________________

Differences: ______________________

CHALLENGE

Write a product equal to each number in Exercises 1–5.

____

Name ______________________________ Date ______________

EQUALITY

An **equation** is a number sentence that shows that two quantities are **equal**.
The equals sign, =, shows equality.
The following symbol shows that two quantities are not equal: ≠.

You can make statements about numbers in different ways using the equals sign.

Each of these sums is equal to 50.

$20 + 30 = 50 \quad 25 + 25 = 50 \quad 15 + 35 = 50$

None of these sums are equal to 50.

$30 + 30 \neq 50 \quad 25 + 35 \neq 50 \quad 30 + 35 \neq 50$

Write = or ≠ to make each number sentence true.

1. $9 + 12$ ◯ $10 + 11$ **2.** $15 + 3$ ◯ 6×3 **3.** $20 - 5$ ◯ $10 + 2$

4. $15 - 6$ ◯ $6 + 3$ **5.** 5×4 ◯ 3×6 **6.** $14 \div 2$ ◯ $3 + 4$

7. $4 + 16$ ◯ $16 \div 4$ **8.** 6×2 ◯ $5 + 7$ **9.** $6 + 7$ ◯ $16 - 7$

10. 3×2 ◯ $12 \div 4$ **11.** $10 + 6$ ◯ 4×4 **12.** $20 - 1$ ◯ $18 + 1$

Write equations showing that a sum, a difference, a product, and a quotient that are equal to each number.

13. 20 ______________________________

14. 25 ______________________________

15. 36 ______________________________

16. 60 ______________________________

CHALLENGE

Explain how to use equality to find the missing number in $37 + m = 45$.
What is the value of the missing number?

____

Name ______________________________ Date ______________

SOLVING OPEN SENTENCES

You can write an equation to solve a problem.

Danika has $8 in her wallet. She has some more money in her pocket. She has $13 in all. How much money does she have in her pocket?

Let m = the amount of money Danika has in her pocket.

8	$+$	m	$=$	13
↑		↑		↑
amount in wallet		amount in pocket		total amount

Think: $8 + 5 = 13$ So, $m = 5$.

Danika has $5 in her pocket.

Solve each equation.

1. $5 + k = 12$ ______

2. $w - 4 = 6$ ______

3. $p + 9 = 11$ ______

4. $s \div 3 = 4$ ______

5. $x + 7 = 13$ ______

6. $n \times 4 = 16$ ______

Write an equation to solve each problem.

7. Ross has saved $9. He needs $15 to buy a concert ticket. How much more money does Ross need to buy the ticket?

______

8. There are 5 tables for 20 people. The same number of people sit at each table. How many people are at each table?

______

Name ______________________________ Date ______________

DISCOVERY EDUCATION

SOLVING OPEN SENTENCES

An equation is like a balance. It shows that the expression to the left of the equal sign has the same value as the expression to the right of the equal sign.

$6 + n = 18$ → Both sides must equal the same value, 18.
$n = 12$ → Think: $6 + 12 = 18$, so $n = 12$.

Solve each equation.

1. $8 + k = 17$

2. $w - 5 = 6$

3. $p + 7 = 20$

4. $s \div 6 = 3$

5. $x + 16 = 20$

6. $n \times 8 = 24$

7. $9 \times b = 45$

8. $y \div 7 = 4$

9. $a - 15 = 15$

Write an equation to solve each problem.

10. Celine has saved \$24. She needs \$60 to buy a bike to ride to school. How much more money does Celine need to buy the bike?

11. Thomas is cooking wings for a party. He prepares 4 wings for each person who will be at the party. He cooks 36 wings in all. How many people will Thomas have at his party?

12. Wendy took some money out of her bank to go shopping. She spent \$20. She had \$18 left. How much money did she take shopping?

DISCOVERING MATH

LEVEL 2

Name ______________________ Date ______________

SOLVING OPEN SENTENCES

An equation is like a balance. It shows that the expression to the left of the equal sign has the same value as the expression to the right of the equal sign.

You can use inverse operations to solve an equation. Remember, if you change one side of an equation, you must change the other side in exactly the same way.

Solve:	$6x = 72$	$\rightarrow$	Think: The inverse of multiplication is division.
	$\frac{6x}{6} = \frac{72}{6}$	$\rightarrow$	Divide both sides by 6.
	$x = 12$	$\rightarrow$	Simplify. Think: $\frac{6}{6} = 1$ and $\frac{72}{6} = 12$.

Solve each equation.

1. $12 + k = 109$ ______

2. $w - 26 = 38$ ______

3. $p + 27 = 53$ ______

4. $s \div 6 = 9$ ______

5. $x + 78 = 206$ ______

6. $7n = 63$ ______

7. $12b = 132$ ______

8. $y \div 4 = 10$ ______

9. $a - 65 = 19$ ______

Write an equation to solve each problem.

10. Edward saved some money for a vacation. He spent \$225 to buy an airline ticket. Then he had \$620 left to spend on the vacation. How much money did Edward save for the vacation?

______

11. This month Kelsie rode her bike 3 times as many miles as she rode last month. She rode 72 miles this month. How many miles did Kelsie ride last month?

______

12. Frankie spent \$4 for breakfast and \$7 for lunch. Then he bought dinner. He spent \$23 in all. How much did he spend on dinner?

______

DISCOVERING MATH

LEVEL 3

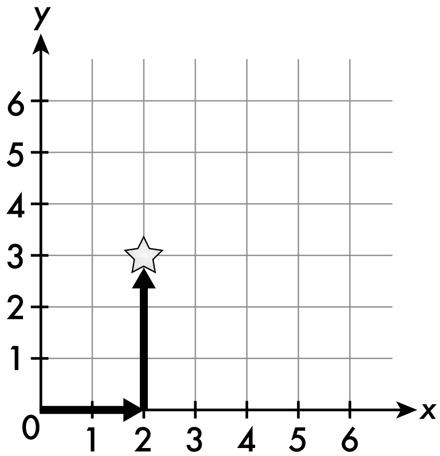
Name ______________________________ Date ______________

RECTANGULAR COORDINATE SYSTEM

An **ordered pair** gives the location of a point on a coordinate grid.

To locate a point on a coordinate grid, first start at 0. For positive numbers, count the number of units right and then the number of units up. Always move right first and up second.

The ordered pair for the star on the coordinate grid at the right is (2, 3). Start at 0. Move 2 units to the right. Then move 3 units up.



An archaeologist found pottery bowls with different pictures. The coordinate grid shows the location of each bowl.

Write the ordered pair for each picture.

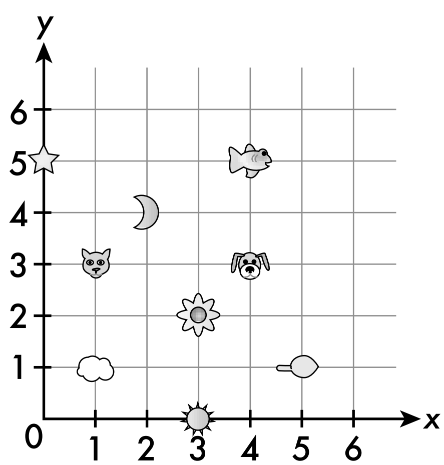
1. ☆ ____________

2. ☽ ____________

3. ☼ ____________

4. ☁ ____________

5. ____________



Name the picture located at each ordered pair.

6. (3, 2) ____________

7. (5, 1) ____________

8. (1, 3) ____________

9. (4, 5) ____________

10. (2, 4) ____________

11. (0, 5) ____________

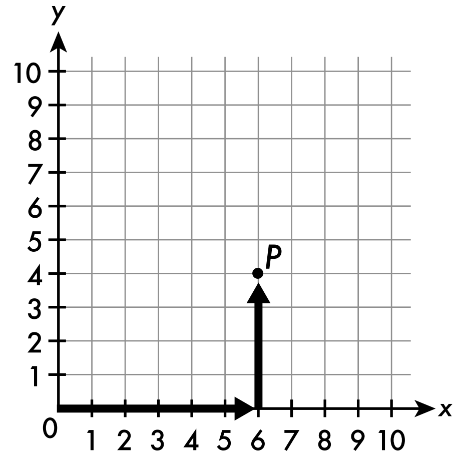
Name ______________________________ Date ______________

RECTANGULAR COORDINATE SYSTEM

An ordered pair gives the location of a point on a coordinate grid.

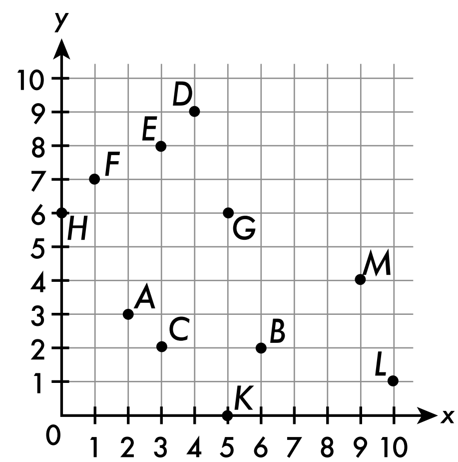
To locate a point on a coordinate grid, first start at 0. For positive numbers, count the number of units right and then the number of units up. Always move right first and up second.

The ordered pair for point *P* on the coordinate grid at the right is (6, 4). Start at 0. Move 6 units right. Then move 4 units up.



An archaeology team recorded the locations of items excavated on the coordinate grid.

Write the ordered pair for each item.



1. ring at point *H* ____________

2. cup at point *B* ____________

3. necklace at point *F* ____________

4. bowl at point *K* ____________

5. mirror at point *L* ____________

Name the point at which each item is located.

6. shield: (2, 3) ____________

7. fork: (9, 4) ____________

8. bracelet: (3, 8) ____________

9. pen: (5, 6) ____________

10. tablet: (3, 2) ____________

11. spoon: (4, 9) ____________

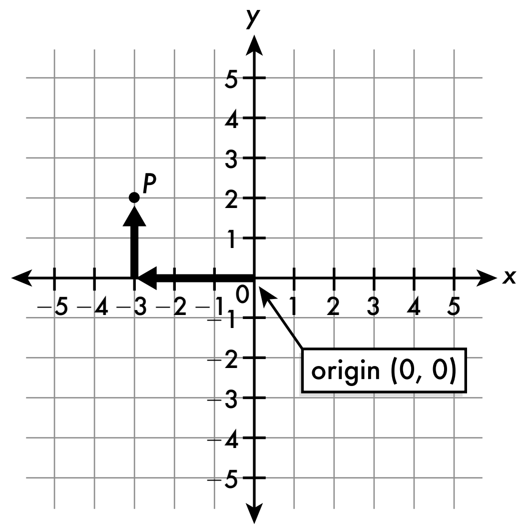
Name ____________________ Date ____________________

RECTANGULAR COORDINATE SYSTEM

A **coordinate plane** is formed by two perpendicular number lines. The horizontal number line is called the **x-axis**. The vertical number line is called the ***y*-axis**. They intersect at the **origin**.

To locate an **ordered pair** (*x*, *y*) on a coordinate grid:
(1) Start at the origin.
(2) Count *x* units right (+) or left (–).
(3) Then count y units up (+) or down (–).

The ordered pair for point *P* on the coordinate grid at the right is (–3, 2). Start at 0. Move 3 units left. Then move 2 units up.



Some ants left their anthill to look for food. The points show the locations where each ant found food. Write the ordered pair for each point.

1. *A* ____________________

2. *B* ____________________

3. *D* ____________________

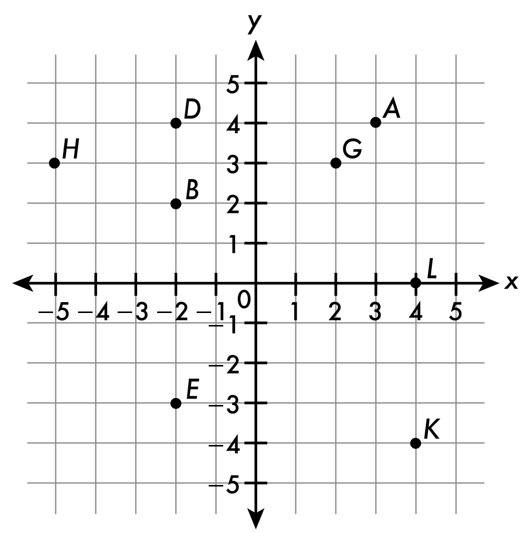
4. *E* ____________________

5. *G* ____________________

6. *H* ____________________

7. *K* ____________________

8. *L* ____________________



Graph and label the ordered pairs on the coordinate plane above.

9. *R* (5, 1) **10.** *S* (–3, 3) **11.** *T* (–1, –3)

12. *U* (–2, –5) **13.** *V* (–4, 0) **14.** *W* (3, –1)

Name ______________________________ Date ______________

PATTERNS AND RULES

A pattern can be found in a list of numbers or shapes that follow a rule to determine their order. There are different kinds of patterns.

Growing Pattern: 3, 5, 7, 9, 11, ...

To find the rule, compare each pair of numbers in the list.

Notice: 3 + **2** = 5
5 + **2** = 7
7 + **2** = 9
9 + **2** = 11

To find the next number in the pattern, the rule is to add 2 to the last term.

Repeating Pattern: 1, 2, 2, 1, 2, 2, ...

To find the rule, look for a pattern in the way the numbers are repeated.

The rule is to repeat 1, 2, 2.

Tell if each pattern is a growing pattern or a repeating pattern. Find the rule for the pattern.

1. 1, 4, 7, 10, 13, ... ____________ [growing pattern; add 3]

2. 5, 9, 13, 17, 21, ... ____________ [growing pattern; add 4]

3. 7, 4, 7, 4, 7, 4, ... ____________ [repeating pattern; repeat 7, 4]

4. 5, 15, 25, 35, 45, ... ____________ [growing pattern; add 10]

5. 3, 5, 1, 3, 5, 1, 3, 5, ... ____________ [repeating pattern; repeat 3, 5, 1]

6. 8, 13, 18, 23, 28, ... ____________ [growing pattern; add 5]

7. 6, 16, 6, 16, 6, ... ____________ [repeating pattern; repeat 6, 16]

8. 30, 300, 30, 300, 30, ... ____________ [repeating pattern; repeat 30, 300]

CHALLENGE

Find the next number in each pattern in Exercises 1–5.

[16; 25; 7; 55; 1]

Name ______________________________ Date ______________

Discovery EDUCATION

PATTERNS AND RULES

A pattern can be found in a list of numbers or shapes that follow a rule to determine their order. Use the rule to find the next number in the pattern.

Increasing Pattern:

2, 12, 22, 32, 42, …

Notice: 2 + **10** = 12
12 + **10** = 22 and so on.

The rule is to add 10.
The next number in the pattern is
42 + 10, or 52.

Decreasing Pattern:

39, 36, 33, 30, 27, …

Notice: 39 – **3** = 36
36 – **3** = 33 and so on.

The rule is to subtract 3.
The next number in the pattern is
27 – 3, or 24.

Repeating Pattern:

2, 3, 3, 2, 3, 3, …

To find the rule, look for a pattern in the way the numbers are repeated.

The rule is to repeat 2, 3, 3.
The next number in the pattern is 2.

Write the rule for each pattern. Then find the next number in the pattern.

1. 8, 12, 8, 12, 8, … ____________________ [repeat 8, 12; 12]

2. 4, 9, 14, 19, 24, … ____________________ [add 5; 29]

3. 50, 48, 46, 44, 42, … ____________________ [subtract 2; 40]

4. 100, 90, 80, 70, 60, … ____________________ [subtract 10; 50]

5. 21, 41, 61, 81, … ____________________ [add 20; 101]

CHALLENGE

Explain how to find the next two numbers in the following pattern:
2, 3, 4, 3, 6, 3, 8, 3, …
What are the next two numbers in the pattern?

[Part of the pattern is increasing: add 2 to get the next number. This alternates with a repeating 3 in the pattern. The next two numbers are 10 and 3.]

DISCOVERING MATH
LEVEL 2

Name ______________________ Date ______________

DISCOVERY EDUCATION

PATTERNS AND RULES

A pattern can be found in a list of numbers or shapes that follow a rule to determine their order. Use the rule to find the next number in the pattern.

Growing Pattern:

1.2, 2.4, 4.8, 9.6, …

Notice: $1.2 \times \mathbf{2} = 2.4$
$2.4 \times \mathbf{2} = 4.8$ and so on.

The rule is to multiply by 2.
The next number in the pattern is 9.6×2, or 19.2.

Decreasing Pattern:

95, 87, 79, 71, …

Notice: $95 - \mathbf{8} = 87$
$87 - \mathbf{8} = 79$ and so on.

The rule is to subtract 8.
The next number in the pattern is $71 - 8$, or 63.

Combined Pattern:

3, 2, 5, 2, 7, 2, …

The number 2 is repeated. The other numbers increase by 2.

A rule for the pattern is to add 2 to the numbers in the odd places in the pattern. Repeat the number 2 in the even places. The next number in the pattern is 9.

Write a rule for each pattern. Then find the next three numbers in the pattern.

1. 10.1, 9.6, 9.1, 8.6, … ______ [subtract 0.5; 8.1, 7.6, 7.1]
2. 1, 3, 9, 27, … ______ [multiply by 3; 81, 243, 729]
3. 3, 6, 12, 24, … ______ [multiply by 2; 48, 96, 192]
4. 200, 5, 180, 5, 160, 5, … [subtract 20 from numbers in the odd places in the pattern and repeat 5 in the even places; 140, 5, 120]
5. 2.5, 3.6, 4.7, 5.8, … ______ [add 1.1; 6.9, 8.0, 9.1]

CHALLENGE

Write a rule for the following pattern. Then find the next two numbers in the pattern.
8, 15, 12, 19, 16, 23, … [Possible rule: add 7 then subtract 3 or, add 4 to the number before the previous one in the list; 20, 27]

DISCOVERING MATH

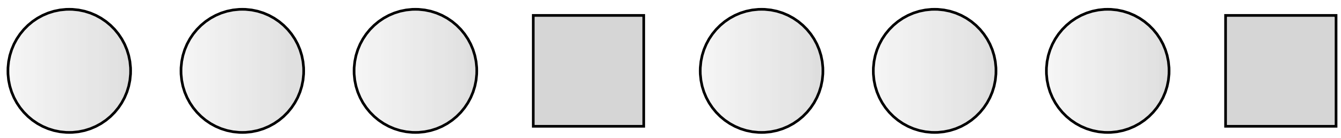
LEVEL 3

Name ______________________ Date ____________

DISCOVERY EDUCATION

REPRESENTING PATTERNS

You can continue a pattern to find what comes next.



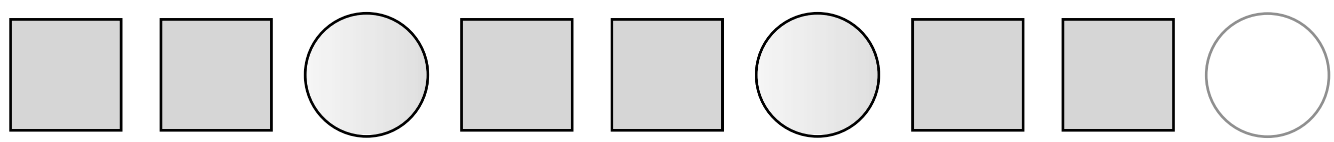
circle circle circle square circle circle circle square

The pattern is a repeating pattern of shapes: circle, circle, circle, square.
You can also use numbers to describe the pattern: 1, 1, 1, 2, 1, 1, 1, 2.

Continue the pattern to find the next shape in the pattern.
Since a circle follows the square, the next shape is a circle.

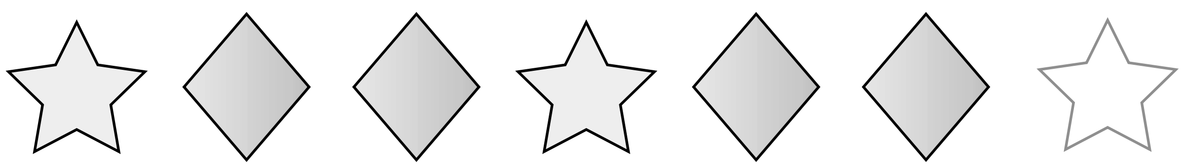
Draw the next shape in each pattern.
Then use numbers to describe the pattern.

1.



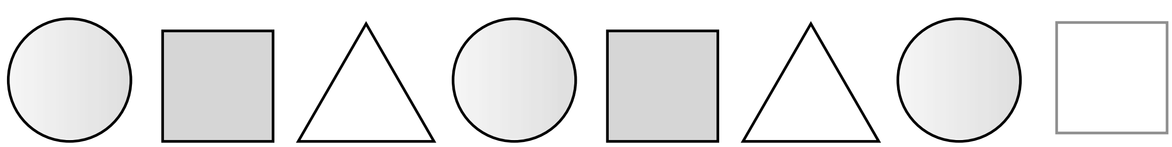
[1, 1, 2, 1, 1, 2]

2.



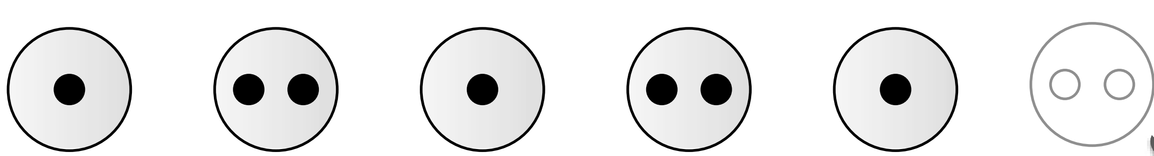
[1, 2, 2, 1, 2, 2]

3.



[1, 2, 3, 1, 2, 3]

4.



[1, 2, 1, 2, 1, 2]

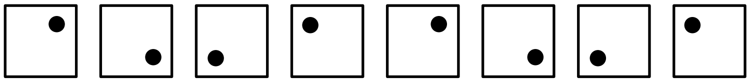
DISCOVERING MATH

LEVEL 1

Name ______________________________ Date ______________

REPRESENTING PATTERNS

You can continue a pattern to find what comes next.

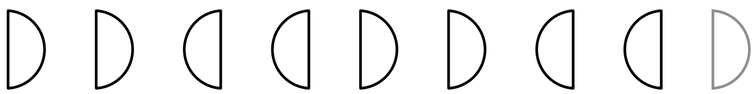


The pattern is a repeating pattern of squares.
The dot rotates $\frac{1}{4}$ turn clockwise each time.

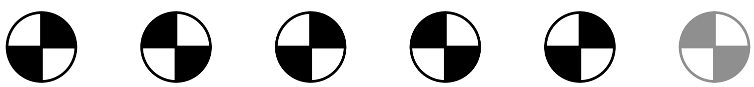
You can also use numbers to describe the pattern: 1, 2, 3, 4, 1, 2, 3, 4.

Continue the pattern to find the next shape in the pattern.
The next number in the number pattern is 1.
The next shape in the shape pattern is the first figure.

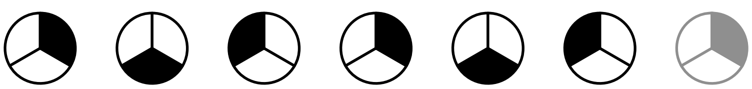
Draw the next shape in each pattern.
Then use numbers to describe the pattern through two cycles.

1. 

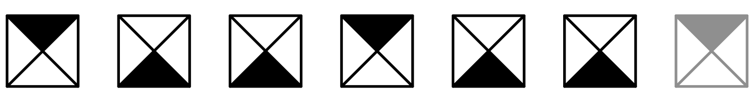
[1, 1, 2, 2, 1, 1, 2, 2]

2. 

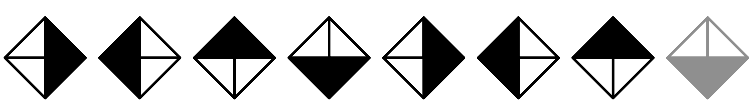
[1, 2, 1, 2]

3. 

[1, 2, 3, 1, 2, 3]

4. 

[1, 2, 2, 1, 2, 2]

5. 

[1, 2, 3, 4, 1, 2, 3, 4]

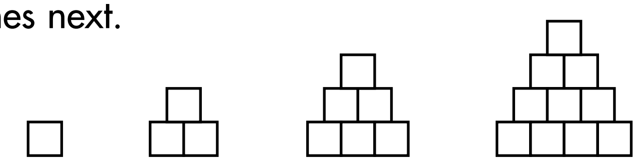
LEVEL 2

Name ______________________________ Date ____________

REPRESENTING PATTERNS

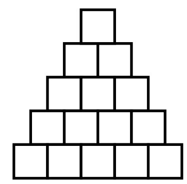
You can continue a pattern to find what comes next.

The pattern is a growing shape pattern.
Each time a row is added at the bottom
with 1 more square than in the previous shape.



You can also use numbers to describe the pattern: 1, 3, 6, 10, …

To draw the next shape in the pattern,
add on a row with 5 squares at the bottom.
To continue the number pattern,
add 5 to the last number: 10 + 5 = 15.

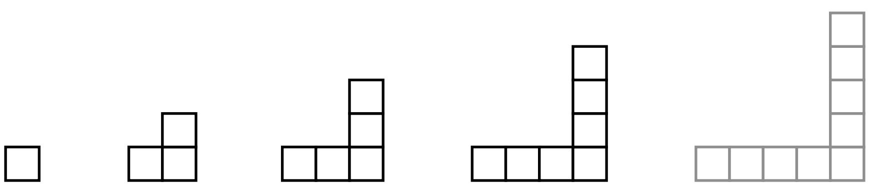


Draw the next shape in each pattern.
Then use numbers to describe the pattern through four elements.

1.

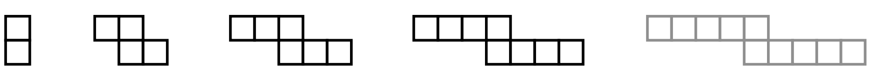
[1, 2, 3, 4, …]

2.



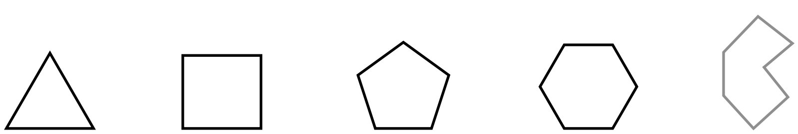
[1, 3, 5, 7, …]

3.



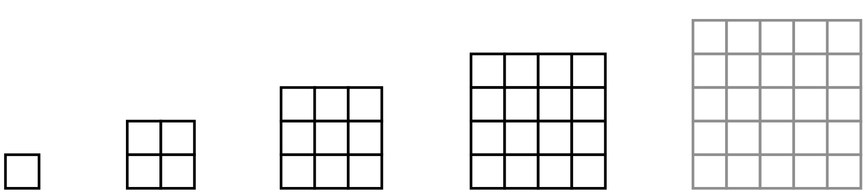
[2, 4, 6, 8, …]

4.



[3, 4, 5, 6, …]

5.



[1, 4, 9, 16, …]

Name ______________________________ Date ______________

VARIABLES

A **variable** can be a symbol used to represent a not yet known or determined number.

You can use a variable to represent an answer.
You can choose any letter for the variable.

$4 + 7 = a$	$\rightarrow$	a represents the total of $4 + 7$.
$15 - 7 = n$	$\rightarrow$	n represents the difference of $15 - 7$.
$3 \times 6 = x$	$\rightarrow$	x represents the product of 3×6.
$18 \div 2 = w$	$\rightarrow$	w represents the quotient of $18 \div 2$.

Tell what each variable represents.

1. $7 \times 5 = m$ [m represents the product of 7×5.]

2. $12 - 4 = b$ [b represents the difference of $12 - 4$.]

3. $24 \div 8 = k$ [k represents the quotient of $24 \div 8$.]

4. $9 + 3 = s$ [s represents the sum of $9 + 3$.]

5. $17 - 9 = p$ [p represents the difference of $17 - 9$.]

6. $8 + 6 = t$ [t represents the sum of $8 + 6$.]

7. $36 \div 4 = q$ [q represents the quotient of $36 \div 4$.]

8. $3 \times 9 = c$ [c represents the product of 3×9.]

CHALLENGE

Find a value for each of the variables in Ex. 1–8.

[$m = 35$; $b = 8$; $k = 3$; $s = 12$; $p = 8$; $t = 14$; $q = 9$; $c = 27$]

DISCOVERING MATH

LEVEL 1

Name ______________________ Date ______________

VARIABLES

A **variable** can be a symbol used to represent a not yet known or determined number.

John saw 6 bison.
Some more bison joined them.
How many bison are there?

Use a variable to represent the number of bison that joined them.
You can choose any letter for the variable.
Use b for the number of bison.

There were 6 bison and b more bison joined them.

$$6 + b$$

number John saw   number that joined them

There are $6 + b$ bison.

Choose a variable.
Tell what the variable represents.
Then use the variable to tell how many.

[Possible variables are shown. Any letter may be used in each case.]

1. Rachel saw 10 caribou.
Then some of the caribou left.
How many caribou are there now?

[Use c for the number of caribou that left; $10 - c$]

2. A monkey eats 2 bananas for breakfast.
The monkey eats more bananas for dinner.
How many bananas does the monkey eat in all?

[Use b for the number of bananas eaten for dinner; $2 + b$]

3. There are 8 cans of soup.
Some friends eat some of the soup for lunch.
How many cans of soup are left?

[Use n for the number of cans of soup used; $8 - n$]

Name ______________________________ Date ______________

VARIABLES

A **variable** can be a symbol used to represent a not yet known or determined number. An **expression** may combine numbers, operation signs, and variables.

The ranger counted 34 bison in one group.
There is another group of bison across the river.
How many bison are there in all?

You can write an expression to represent the total number of bison.
Choose a variable to represent the number of bison across the river.
You can choose any letter for the variable.

Let b = the number of bison across the river.

Expression: $34 + b$

number ranger counted — number across the river

There are $34 + b$ bison.

Write an expression with a variable.
Tell what the variable represents.

[Possible variables are shown. Any letter may be used in each case.]

1. A large group of caribou is migrating.
500 of the caribou move ahead of the group.
How many caribou are behind?

[Let c = the total number of caribou; $c - 500$]

2. The monkeys eat some bananas for breakfast.
They eat twice as many bananas for dinner as for breakfast.
How many bananas do they eat for dinner?

[Let b = the number of bananas the monkeys eat for breakfast; $2 \times b$]

3. There are some cans of soup on the shelf.
Josh puts the cans into 5 equal groups.
How many cans of soup are in each group?

[Let s = the number of cans of soup; $s \div 5$]

Name ______________________ Date ______________

DISCOVERY EDUCATION

EQUALITY

The equals sign, =, is used to show that two quantities are equal.

You can show how to represent the number 4 in various ways using the equals sign.

Sums: $0 + 4 = 4$ $1 + 3 = 4$ $2 + 2 = 4$

Differences: $6 - 2 = 4$ $5 - 1 = 4$ $8 - 4 = 4$

Write three different sums and differences that are equal to each number, and use an equals sign to show it for each one. [Possible answers are shown.]

1. 7 Sums: [$4 + 3 = 7$; $2 + 5 = 7$; $1 + 6 = 7$]

Differences: [$10 - 3 = 7$; $8 - 1 = 7$; $9 - 2 = 7$]

2. 5 Sums: [$0 + 5 = 5$; $1 + 4 = 5$; $2 + 3 = 5$]

Differences: [$5 - 0 = 5$; $6 - 1 = 5$; $10 - 5 = 5$]

3. 8 Sums: [$4 + 4 = 8$; $3 + 5 = 8$; $2 + 6 = 8$]

Differences: [$10 - 2 = 8$; $9 - 1 = 8$; $8 - 0 = 8$]

4. 6 Sums: [$3 + 3 = 6$; $2 + 4 = 6$; $1 + 5 = 6$]

Differences: [$7 - 1 = 6$; $8 - 2 = 6$; $9 - 3 = 6$]

5. 10 Sums: [$1 + 9 = 10$; $8 + 2 = 10$; $5 + 5 = 10$]

Differences: [$10 - 0 = 10$; $12 - 2 = 10$; $20 - 10 = 10$]

6. 12 Sums: [$10 + 2 = 12$; $9 + 3 = 12$; $8 + 4 = 12$]

Differences: [$13 - 1 = 12$; $15 - 3 = 12$; $16 - 4 = 12$]

7. 15 Sums: [$8 + 7 = 15$; $9 + 6 = 15$; $10 + 5 = 15$]

Differences: [$16 - 1 = 15$; $17 - 2 = 15$; $18 - 3 = 15$]

CHALLENGE

Write a sum, difference, and product equal to 20.

[Possible answer: $15 + 5 = 20$; $25 - 5 = 20$; $10 \times 2 = 20$]

DISCOVERING MATH

LEVEL 1

Name ______________________________ Date ______________

EQUALITY

In a number sentence, the equals sign, =, shows that two quantities are **equal**.

You can show how to represent numbers in various ways using the equals sign.

Each of these sums is equal to 12.

$0 + 12 = 12$ $3 + 9 = 12$ $4 + 8 = 12$ $6 + 6 = 12$

Each of these differences is equal to 12.

$13 - 1 = 12$ $15 - 3 = 12$ $17 - 5 = 12$ $20 - 8 = 12$

There are other sums and differences that are also equal to 12.

Write three different sums and differences that are equal to each number, and use an equals sign to show it for each one. [Possible answers are shown.]

1. 15 Sums: [8 + 7 = 15; 9 + 6 = 15; 10 + 5 = 15]

Differences: [16 − 1 = 15; 17 − 2 = 15; 18 − 3 = 15]

2. 18 Sums: [9 + 9 = 18; 10 + 8 = 18; 11 + 7 = 18]

Differences: [19 − 1 = 18; 20 − 2 = 18; 21 − 3 = 18]

3. 20 Sums: [10 + 10 = 20; 12 + 8 = 20; 15 + 5 = 20]

Differences: [30 − 10 = 20; 40 − 20 = 20; 25 − 5 = 20]

4. 25 Sums: [20 + 5 = 25; 21 + 4 = 25; 25 + 0 = 25]

Differences: [27 − 2 = 25; 26 − 1 = 25; 25 − 0 = 25]

5. 32 Sums: [32 + 0 = 32; 31 + 1 = 32; 30 + 2 = 32]

Differences: [35 − 3 = 32; 34 − 2 = 32; 33 − 1 = 32]

CHALLENGE

Write a product equal to each number in Exercises 1–5.

[Possible answer: 3 × 5 = 15; 2 × 9 = 18; 5 × 4 = 20; 5 × 5 = 25; 4 × 8 = 32

Name ______________________________ Date ______________

EQUALITY

An **equation** is a number sentence that shows that two quantities are **equal**.
The equals sign, =, shows equality.
The following symbol shows that two quantities are not equal: ≠.

You can make statements about numbers in different ways using the equals sign.

Each of these sums is equal to 50.

$20 + 30 = 50 \quad 25 + 25 = 50 \quad 15 + 35 = 50$

None of these sums are equal to 50.

$30 + 30 \neq 50 \quad 25 + 35 \neq 50 \quad 30 + 35 \neq 50$

Write = or ≠ to make each number sentence true.

1. $9 + 12 = 10 + 11$ **2.** $15 + 3 = 6 \times 3$ **3.** $20 - 5 \neq 10 + 2$

4. $15 - 6 = 6 + 3$ **5.** $5 \times 4 \neq 3 \times 6$ **6.** $14 \div 2 = 3 + 4$

7. $4 + 16 \neq 16 \div 4$ **8.** $6 \times 2 = 5 + 7$ **9.** $6 + 7 \neq 16 - 7$

10. $3 \times 2 \neq 12 \div 4$ **11.** $10 + 6 = 4 \times 4$ **12.** $20 - 1 = 18 + 1$

Write equations showing that a sum, a difference, a product, and a quotient that are equal to each number. [Possible answers are shown.]

13. 20 [10 + 10 = 20; 25 − 5 = 20; 4 × 5 = 20; 100 ÷ 5 = 20]

14. 25 [10 + 15 = 25; 30 − 5 = 25; 5 × 5 = 25; 100 ÷ 4 = 25]

15. 36 [30 + 6 = 36; 38 − 2 = 36; 9 × 4 = 36; 72 ÷ 2 = 36]

16. 60 [30 + 30 = 60; 75 − 15 = 60; 20 × 3 = 60; 120 ÷ 2 = 60]

CHALLENGE

Explain how to use equality to find the missing number in $37 + m = 45$.
What is the value of the missing number?

[Possible answer: Subtract 37 from each side; $m = 8$]

Name ______________________ Date ______________

SOLVING OPEN SENTENCES

You can write an equation to solve a problem.

Danika has \$8 in her wallet. She has some more money in her pocket. She has \$13 in all. How much money does she have in her pocket?

Let m = the amount of money Danika has in her pocket.

8	$+$	m	$=$	13
↑		↑		↑
amount in wallet		amount in pocket		total amount

Think: $8 + 5 = 13$ So, $m = 5$.

Danika has \$5 in her pocket.

Solve each equation.

1. $5 + k = 12$

[k = 7]

2. $w - 4 = 6$

[w = 10]

3. $p + 9 = 11$

[p = 2]

4. $s \div 3 = 4$

[s = 12]

5. $x + 7 = 13$

[x = 6]

6. $n \times 4 = 16$

[n = 4]

Write an equation to solve each problem.

7. Ross has saved \$9. He needs \$15 to buy a concert ticket. How much more money does Ross need to buy the ticket?

[Possible equation: 9 + s = 15; s = 6; He needs to save \$6 more.]

8. There are 5 tables for 20 people. The same number of people sit at each table. How many people are at each table?

[Possible equation: 5 × p = 20; p = 4; There are 4 people at each table.]

Name ______________________ Date ______________

SOLVING OPEN SENTENCES

An equation is like a balance. It shows that the expression to the left of the equal sign has the same value as the expression to the right of the equal sign.

$6 + n = 18 \rightarrow$ Both sides must equal the same value, 18.
$n = 12 \rightarrow$ Think: $6 + 12 = 18$, so $n = 12$.

Solve each equation.

1. $8 + k = 17$

[$k = 9$]

2. $w - 5 = 6$

[$w = 11$]

3. $p + 7 = 20$

[$p = 13$]

4. $s \div 6 = 3$

[$s = 18$]

5. $x + 16 = 20$

[$x = 4$]

6. $n \times 8 = 24$

[$n = 3$]

7. $9 \times b = 45$

[$b = 5$]

8. $y \div 7 = 4$

[$y = 28$]

9. $a - 15 = 15$

[$a = 30$]

Write an equation to solve each problem.

10. Celine has saved $24. She needs $60 to buy a bike to ride to school. How much more money does Celine need to buy the bike?

[Possible equation: $24 + m = 60$; $m = 36$; She needs to save $36 more.]

11. Thomas is cooking wings for a party. He prepares 4 wings for each person who will be at the party. He cooks 36 wings in all. How many people will Thomas have at his party?

[Possible equation: $4 \times g = 36$; $g = 9$; He will have 9 people at the party.]

12. Wendy took some money out of her bank to go shopping. She spent $20. She had $18 left. How much money did she take shopping?

[Possible equation: $m - 20 = 18$; $m = 38$; She took $38.]

DISCOVERING MATH
LEVEL 2

Name ____________________ Date ____________________

SOLVING OPEN SENTENCES

An equation is like a balance. It shows that the expression to the left of the equal sign has the same value as the expression to the right of the equal sign.

You can use inverse operations to solve an equation. Remember, if you change one side of an equation, you must change the other side in exactly the same way.

Solve: $6x = 72$ → Think: The inverse of multiplication is division.

$\frac{6x}{6} = \frac{72}{6}$ → Divide both sides by 6.

$x = 12$ → Simplify. Think: $\frac{6}{6} = 1$ and $\frac{72}{6} = 12$.

Solve each equation.

1. $12 + k = 109$

[$k = 97$]

2. $w - 26 = 38$

[$w = 64$]

3. $p + 27 = 53$

[$p = 26$]

4. $s \div 6 = 9$

[$s = 54$]

5. $x + 78 = 206$

[$x = 128$]

6. $7n = 63$

[$n = 9$]

7. $12b = 132$

[$b = 11$]

8. $y \div 4 = 10$

[$y = 40$]

9. $a - 65 = 19$

[$a = 84$]

Write an equation to solve each problem.

10. Edward saved some money for a vacation. He spent $225 to buy an airline ticket. Then he had $620 left to spend on the vacation. How much money did Edward save for the vacation?

[Possible equation: $s - 225 = 620$; $s = 845$; He saved $845.]

11. This month Kelsie rode her bike 3 times as many miles as she rode last month. She rode 72 miles this month. How many miles did Kelsie ride last month?

[Possible equation: $3 \times m = 72$; $m = 24$; She rode 24 miles last month.]

12. Frankie spent $4 for breakfast and $7 for lunch. Then he bought dinner. He spent $23 in all. How much did he spend on dinner?

[Possible equation: $4 + 7 + d = 23$; $d = 12$; He spent $12 on dinner.]

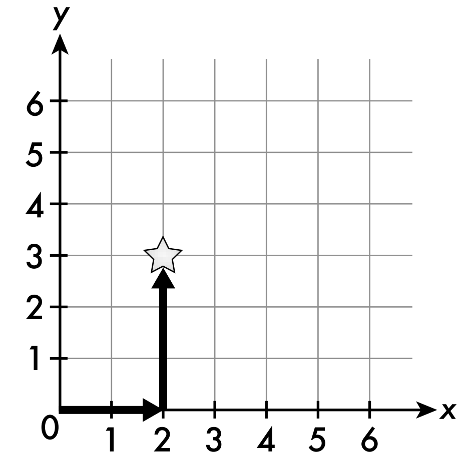
Name ______________________________ Date ______________

RECTANGULAR COORDINATE SYSTEM

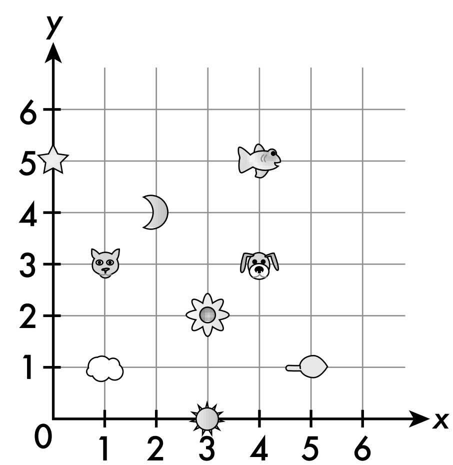
An **ordered pair** gives the location of a point on a coordinate grid.

To locate a point on a coordinate grid, first start at 0. For positive numbers, count the number of units right and then the number of units up. Always move right first and up second.

The ordered pair for the star on the coordinate grid at the right is (2, 3). Start at 0. Move 2 units to the right. Then move 3 units up.



An archaeologist found pottery bowls with different pictures. The coordinate grid shows the location of each bowl.



Write the ordered pair for each picture.

1. ☆ ____________ [(0, 5)]

2. ☽ ____________ [(2, 4)]

3. sun ____________ [(3, 0)]

4. cloud ____________ [(1, 1)]

5. dog ____________ [(4, 3)]

Name the picture located at each ordered pair.

6. (3, 2) [flower]

7. (5, 1) [leaf]

8. (1, 3) [cat]

9. (4, 5) [fish]

10. (2, 4) [moon]

11. (0, 5) [star]

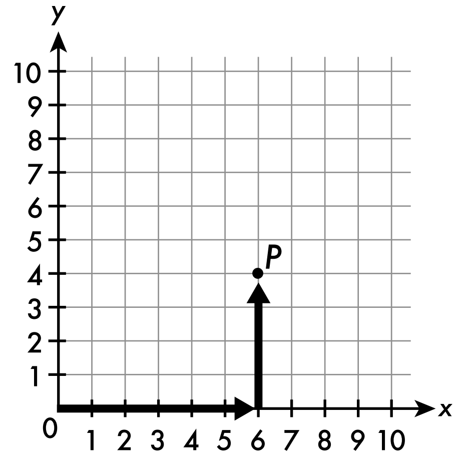
Name ______________________________ Date ______________

RECTANGULAR COORDINATE SYSTEM

An ordered pair gives the location of a point on a coordinate grid.

To locate a point on a coordinate grid, first start at 0. For positive numbers, count the number of units right and then the number of units up. Always move right first and up second.

The ordered pair for point *P* on the coordinate grid at the right is (6, 4). Start at 0. Move 6 units right. Then move 4 units up.



An archaeology team recorded the locations of items excavated on the coordinate grid.

Write the ordered pair for each item.

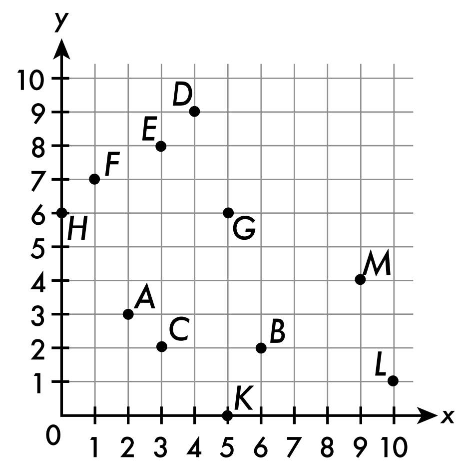
1. ring at point *H* ____________ [(0, 6)]

2. cup at point *B* ____________ [(6, 2)]

3. necklace at point *F* ____________ [(1, 7)]

4. bowl at point *K* ____________ [(5, 0)]

5. mirror at point *L* ____________ [(10, 1)]



Name the point at which each item is located.

6. shield: (2, 3)

[*A*] ____________

7. fork: (9, 4)

[*M*] ____________

8. bracelet: (3, 8)

[*E*] ____________

9. pen: (5, 6)

[*G*] ____________

10. tablet: (3, 2)

[*C*] ____________

11. spoon: (4, 9)

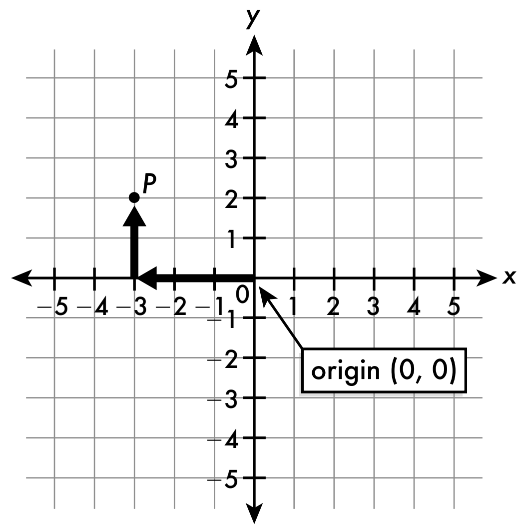
[*D*] ____________

Name ______________________ Date ______________

DISCOVERY EDUCATION

RECTANGULAR COORDINATE SYSTEM

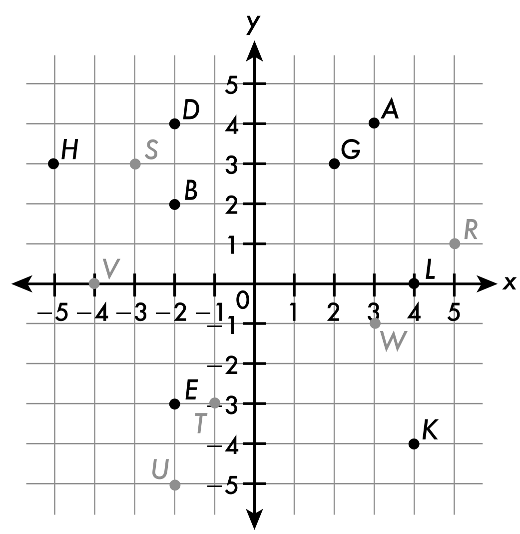
A **coordinate plane** is formed by two perpendicular number lines. The horizontal number line is called the **x-axis**. The vertical number line is called the ***y*-axis**. They intersect at the **origin**.



To locate an **ordered pair** (*x*, *y*) on a coordinate grid:
(1) Start at the origin.
(2) Count *x* units right (+) or left (–).
(3) Then count y units up (+) or down (–).

The ordered pair for point *P* on the coordinate grid at the right is (–3, 2). Start at 0. Move 3 units left. Then move 2 units up.

Some ants left their anthill to look for food. The points show the locations where each ant found food. Write the ordered pair for each point.



1. *A* [(3, 4)]

2. *B* [(–2, 2)]

3. *D* [(–2, 4)]

4. *E* [(–2, –3)]

5. *G* [(2, 3)]

6. *H* [(–5, 3)]

7. *K* [(4, –4)]

8. *L* [(4, 0)]

[See plane above]

Graph and label the ordered pairs on the coordinate plane above.

9. *R* (5, 1) **10.** *S* (–3, 3) **11.** *T* (–1, –3)

12. *U* (–2, –5) **13.** *V* (–4, 0) **14.** *W* (3, –1)

DISCOVERING MATH

LEVEL 3