



## K-12 Mathematics

# Foreword

We are excited to announce that the State Board of Education took a key step in reforming Ohio's education system on December 11, 2001, when it unanimously adopted academic content standards in reading, writing and mathematics. Clear standards about what students should know and be able to do in reading, writing and mathematics is the first component of an aligned system that will ensure no child is left behind.

This enormous undertaking could not have occurred without the hard work and dedication of Ohio's educators and community members. Classroom teachers, parents, higher education faculty and business community leaders worked in teams across the state to develop these standards over the last several years. Fifty percent of the math writing team and 48 percent of the English Language Arts team were classroom teachers, so we especially want to extend our gratitude to the men and women in the teaching profession who gave their time, energy and expertise to create these standards.

The people of Ohio played a key role in the development of these standards. The Office of Curriculum and Instruction at the Ohio Department of Education, which facilitated the standards writing process, aggressively engaged the public in reviewing the standards in draft forms. Thousands of Ohioans gave suggestions that were evaluated and incorporated, as appropriate, by the writing teams into the final adopted standards.

The standards fulfill the requirement and timeline of Amended Substitute Senate Bill 1 for the State Board of Education to develop and adopt clear academic content standards in the areas of reading, writing and mathematics by the end of 2001. The bill gives the Department of Education 18 months from the standards adoption to design and produce model curricula for kindergarten through 12th grade, which Ohio school districts may, but are not obligated to, use.

The State Board of Education will use these standards as the basis for the development of achievement or diagnostic assessments for kindergarten through grade 10.

# ACADEMIC CONTENT STANDARDS

Achieve, Inc., an independent, bipartisan, nonprofit national organization that helps states raise academic standards, considers Ohio's standards in both areas to be among the best in the country. A review of the new standards to the State Board of Education reported that they are clear and comprehensive and that they set high expectations for student learning. "We commend the State on the high quality of the mathematics benchmarks and grade-level indicators. They represent a good balance of conceptual, procedural, and practical knowledge and skills," the review stated. "Ohio has standards, indicators and benchmarks that serve as the basis for a powerful language arts framework for literacy."



Jennifer L. Sheets  
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## K-12 Mathematics

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ACADEMIC CONTENT STANDARDS



## K-12 Mathematics

# Overview





## K-12 Mathematics

The mathematics academic content standards prepare all students for success in the workplace and post-secondary education. Competency in mathematics includes understanding of mathematical concepts, facility with mathematical skills, and application of concepts and skills to problem-solving situations. Students are able to communicate mathematical reasoning using mathematical and everyday language.

Whenever possible, students should have opportunities to learn mathematics through real-world contexts, including practical applications, real data, and numbers often associated with situations and problems encountered in the workplace and daily life. All students should be exposed to a mathematics program rich in technology, including calculators, computers, and technology applications.

The six standards that follow represent the mathematics content and processes all students should know and be able to use as they progress through school. These include:

Content Standards:    Number, Number Sense and Operations  
                                   Measurement  
                                   Geometry and Spatial Sense  
                                   Patterns, Functions and Algebra  
                                   Data Analysis and Probability

Process Standard:     Mathematical Processes

The rigorous, yet realistic standards provide a comprehensive foundation for all students to think and reason mathematically and use mathematics knowledge and skills effectively in post-secondary education, the workplace, and daily life. These standards represent a connected body of mathematical understandings and competencies, rather than a menu of discrete topics from which to choose.

Throughout the five content standards, students will use mathematical processes, including reasoning, communication and representation skills, and appropriate technology within problem-solving situations. Making connections within mathematics and between mathematics and other disciplines is critical for student success in using mathematics effectively in school, work and daily life.

Even though each of these six standards applies to all grades, emphases will vary both within and between the grade clusters. For example, the emphasis on num-

ber, number sense and operations is greatest in elementary grades and by grades 9-12, topics related to number, number sense and operations represent a smaller portion of the curriculum.

This set of six standards does not neatly separate the curriculum into separate or discrete topics. The content described by the standards and benchmarks are interrelated. For example, number pervades all areas of mathematics, and some topics in measurement are closely related to those within geometry. And mathematical processes are used in all content areas and are best learned when carefully integrated with content in instruction and assessment.

Technology, such as calculators and computers, help students learn mathematics and support effective mathematics teaching. Rather than replacing the learning of basic concepts and skills, technology can connect skills and procedures to deeper mathematical understanding. For example, geometry software allows experimentation with families of geometric objects, and graphing utilities facilitate learning about the characteristics of classes of functions.

The following terms and definitions are used in the document:

**Standard:** An overarching goal or theme in mathematics. The standard statement describes, in broad terms, what students should know and be able to do as a result of the K-12 program.

**Benchmark:** A specific statement of what a student should know and be able to do at a specific time in his/her schooling. Benchmarks are used to measure a student's progress towards meeting the standard. Benchmarks are defined for grades 2, 4, 7, 10 and 12.

**Grade-level Indicator:** A specific statement of the knowledge and/or skills that a student demonstrates at each grade level. These indicators serve as checkpoints that monitor progress toward the benchmarks.

# The Development of Academic Content Standards

Joint Council of the  
State Board of Education and  
the Ohio Board of Regents  
Academic Content Standards



The process for developing academic content standards began in 1997 when the State Board of Education and the Ohio Board of Regents created a Joint Council to oversee the implementation of recommendations made by the Secondary and Higher Education Remediation Advisory Commission. The boards began to build a common long-term agenda for pre-K through 16 education.

The Joint Council started its work by establishing a set of common expectations for what all students should know and be able to do upon completion of high school. The initial work established common expectations in six content areas: (1) the arts, (2) English language arts, (3) foreign languages, (4) mathematics, (5) science, and (6) social studies. These drafts were transformed into Ohio's Academic content standards.

The Joint Council assembled advisory groups to assist in completing preliminary planning for the process to draft Ohio's new academic content standards. This preliminary planning included review of exemplary world-class standards from the United States and other countries and the formulation of strategic policy recommendations. The recommendations assured that the drafting and refining of academic content standards would respect Ohio's history for sharing responsibility for curriculum decisions with Ohio's diverse learning communities.

Writing Teams were made up of representatives from all twelve regions served by the Ohio Department Regional Professional Development Centers and included educators from each grade level, K-12, as well as career-technical, special education, and gifted education. Ohio's diverse ethnicity, geography, types of school districts, and colleges and universities were represented on the writing teams. Parent and business and industry representatives also were represented on the writing teams. All original members of the teams who wrote the Common Expectations were invited back to join the writing teams.

When the writing teams completed the draft academic content standards documents, these documents were subjected to a period of extensive public engagement and rigorous review. Focus group meetings and electronic feedback via the web page allowed all stakeholders to express their opinions. The writing teams reviewed the public feedback and made revision recommendations to respond to the issues raised by feedback. The draft standards presented to the State Board of Education for adoption reflect the final recommendations of this writing process and include grade-level indicators of progress (K-12), benchmarks that will serve as checkpoints at key grade bands, philosophies and guide principles.

## Development and Implementation Timeline

Based on Amended Substitute Senate Bill 1

	English Language Arts	Mathematics	Science	Social Studies	Technology Foreign Languages The Arts
↑ Development ↓	(1) Assemble Advisory Committee	↓	↓	↓	↓
	(2) Identify Writing Team				
	(3) Develop Draft Standards and Benchmarks				
	(4) Convene Writing Team				
	(5) Seek Focused Input				
	(6) Engage the Public				
	(7) Revise Draft Standards and Benchmarks				
(8) Adoption of Academic Content Standards by the State Board of Education	December 2001	December 2001	December 2002	December 2002	
↓ Implementation ↓	(9) Develop products and services	↓	↓	↓	
	(10) Design Curriculum Models				
	(11) Present for Public Review				
	(12) State Board Review				
(13) Adoption of Curriculum Models by State Board of Education	June 2003	June 2003	June 2004	June 2004	
(14) Deliver Curriculum Models; Professional Development	September 2003	September 2003	September 2004	September 2004	



## Mathematics Writing Teams

The Ohio Department of Education wishes to express appreciation and gratitude to the writing teams who contributed expertise and time to the development of Ohio's Mathematics Academic Content Standards. Many hours were devoted to research and thoughtful consideration of issues to ensure the standards reflect wise and responsible thinking regarding mathematics teaching and learning. The writing team members represent the many caring and concerned individuals across the state dedicated to their profession and to high quality mathematics education for all Ohio students.

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# Ohio's K-12 Mathematics Standards

## Number, Number Sense and Operations Standard

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

## Measurement Standard

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

## Geometry and Spatial Sense Standard

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

## Patterns, Functions and Algebra Standard

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

## Data Analysis and Probability Standard

Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

## Mathematical Processes Standard

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.

*Note: Mathematical processes are used in all content areas and should be incorporated within instruction and assessment of the content-specific standards, benchmarks and grade-level indicators.*



# National Council of Teachers of Mathematics' Standards for School Mathematics: Pre-Kindergarten through Grade 12\*

Instructional programs from prekindergarten through grade 12 should enable all students to:

## Number and Operations Standard:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- Understand meanings of operations and how they relate to one another; and
- Compute fluently and make reasonable estimates.

## Measurement Standard:

- Understand measurable attributes of objects and the units, systems, and processes of measurement; and
- Apply appropriate techniques, tools, and formulas to determine measurements.

## Geometry Standard:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- Apply transformations and use symmetry to analyze mathematical situations; and
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

## Algebra Standard:

- Understand patterns, relations, and functions;

\* from *Principles and Standards for School Mathematics*

- Represent and analyze mathematical situations and structures using algebraic symbols;
- Use mathematical models to represent and understand quantitative relationships; and
- Analyze change in various contexts.

### **Data Analysis and Probability:**

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data; and
- Understand and apply basic concepts of probability.

### **Process Standards:**

Instructional programs for prekindergarten through grade 12 should enable students to –

### **Problem Solving Standard:**

- Build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts;
- Apply and adapt a variety of appropriate strategies to solve problems; and
- Monitor and reflect on the process of mathematical problem solving

### **Reasoning and Proof Standard:**

- Recognize reasoning and proof as fundamental aspects of mathematics; make and investigate mathematical conjectures;
- Develop and evaluate mathematical arguments and proofs; and
- Select and use various types of reasoning and methods of proof.

### **Communication Standard:**

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others; and
- Use the language of mathematics to express mathematical ideas precisely.

**Connections Standard:**

- Recognize and use connections among mathematical ideas;
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- Recognize and apply mathematics in contexts outside of mathematics.

**Representation Standard:**

- Create and use representations to organize, record, and communicate mathematical ideas;
- Select, apply and translate among mathematical representations to solve problems; and
- Use representations to model and interpret physical, social, and mathematical phenomena.

## Looking at Ohio's Mathematics Standards and National Standards



Ohio Standards	NCTM Standards
<p><b>Number, Number Sense and Operations Standard</b>                      Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.</p>	<p><b>Number and Operations Standard</b></p> <ul style="list-style-type: none"> <li>• Understand numbers, ways of representing numbers, relationships among numbers, and number systems;</li> <li>• Understand meanings of operations and how they relate to one another; and</li> <li>• compute fluently and make reasonable estimates.</li> </ul>
<p><b>Measurement Standard</b>                      Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.</p>	<p><b>Measurement Standard</b></p> <ul style="list-style-type: none"> <li>• Understand measurable attributes of objects and the units, systems, and processes of measurement; and</li> <li>• Apply appropriate techniques, tools, and formulas to determine measurements.</li> </ul>
<p><b>Geometry and Spatial Sense Standard</b>                      Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.</p>	<p><b>Geometry Standard</b></p> <ul style="list-style-type: none"> <li>• Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;</li> <li>• Specify locations and describe spatial relationships using coordinate geometry and other representational systems;</li> <li>• Apply transformations and use symmetry to analyze mathematical situations; and</li> <li>• Use visualization, spatial reasoning, and geometric modeling to solve problems.</li> </ul>

Ohio Standards	NCTM Standards
<p><b>Patterns, Functions and Algebra Standard</b></p> <p>Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.</p>	<p><b>Algebra Standard</b></p> <ul style="list-style-type: none"> <li>• Understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; and</li> <li>• Use mathematical models to represent and understand quantitative relationships; analyze change in various contexts.</li> </ul>
<p><b>Data Analysis and Probability Standard</b></p> <p>Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.</p>	<p><b>Data Analysis and Probability Standard</b></p> <ul style="list-style-type: none"> <li>• Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;</li> <li>• Select and use appropriate statistical methods to analyze data;</li> <li>• Develop and evaluate inferences and predictions that are based on data; and</li> <li>• Understand and apply basic concepts of probability.</li> </ul>
<p><b>Mathematical Processes Standard</b></p> <p>Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.</p>	<p><b>Problem Solving Standard</b>  <b>Reasoning and Proof Standard</b>  <b>Communication Standard</b>  <b>Connections Standard</b>  <b>Representation Standard</b></p>



K-12 Mathematics

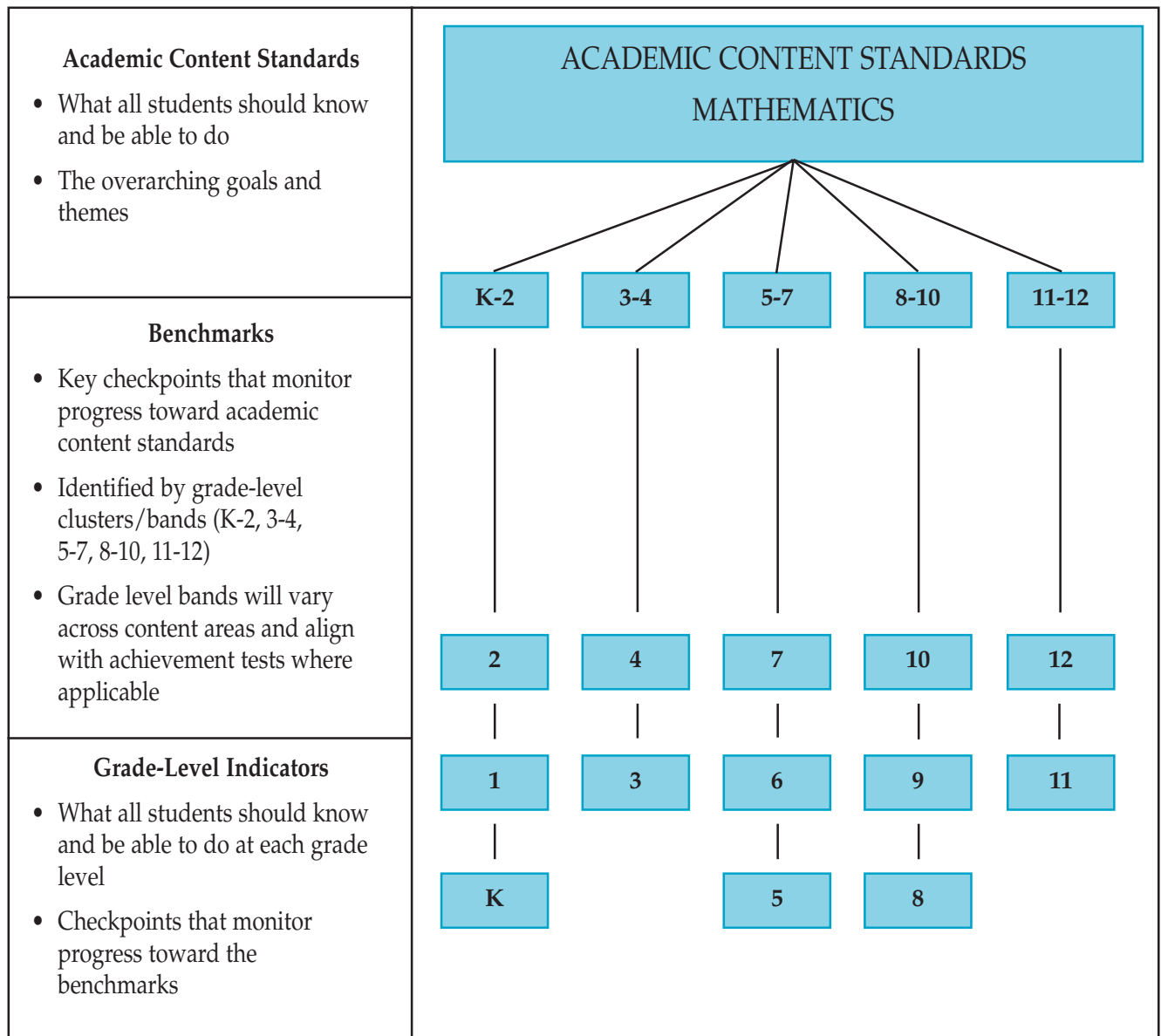
# Structure and Format



ACADEMIC CONTENT STANDARDS

## Academic Content Standards Framework Mathematics K-12

Standards are made up of several component parts. Those parts are outlined below:



## How to Read the Benchmarks

The benchmarks are key checkpoints that monitor student progress toward meeting the mathematics standards. Benchmarks are organized in grade-level bands.

ACADEMIC CONTENT STANDARDS

### K-12 Mathematics Benchmarks

By the end of the K-2 program:

Grade-Level Band	Standard	Benchmark
<p><b>K</b></p> <p><b>1</b></p> <p><b>2</b></p> <p>Number, Number Sense and Operations</p> <p>A. Use place value concepts to represent whole numbers using numerals, words and physical models.</p> <p>B. Recognize, classify, compare and order whole numbers.</p> <p>C. Represent commonly used fractions using words and physical models.</p> <p>D. Determine the value of a collection of coins and dollar bills.</p> <p>E. Make change using coins for values up to one dollar.</p> <p>F. Count, using numerals and ordinal numbers.</p>	<p>Measurement</p> <p>A. Explain the need for standard units of measure.</p> <p>B. Select appropriate units for length, weight, volume (capacity) and time, using:</p> <ul style="list-style-type: none"> <li>• U.S. customary units: inch, foot, yard, ounce, pound, cup, quart, gallon, minute, hour, day, week and year;</li> <li>• metric units: centimeter, meter, gram and liter.</li> </ul> <p>C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and</p>	<p>Geometry and Spatial Sense</p> <p>A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment.</p> <p>B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone, pyramid, and identify them in the environment.</p> <p>C. Sort and compare two-dimensional and three-dimensional objects according to characteristics and properties.</p> <p>D. Identify, explain and model (superimposing) the shapes being compared and similar.</p>

## How to Read the Indicators

The grade-level indicators represent specific statements of what all students should know and be able to do at each grade-level. The indicators serve as checkpoints for monitoring progress toward the benchmarks and standards.

The diagram shows a tilted page from the 'ACADEMIC CONTENT STANDARDS' document. The page is titled 'Grade Two' and contains the 'Number, Number Sense and Operations Standard'. A callout labeled 'Standard' points to the title. A callout labeled 'Organizer' points to the sub-section title 'Number and Number Systems'. A callout labeled 'Grade-Level Indicator' points to a list of indicators. A small blue box with the number '2' is also visible on the right side of the page.

**ACADEMIC CONTENT STANDARDS**

**Grade Two**

**Number, Number Sense and Operations Standard**

*Number and Number Systems*

1. Understand the concepts to represent, compare and order numbers using physical models, numerals and words, up to thousands and hundreds. For example:

- a. Recognize 10 can mean "10 ones" or a single entity (1 ten) through physical models and trading games.
- b. Read and write 3-digit numerals (e.g., 243 as two hundred forty three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each.

2. Recognize and classify numbers as even or odd.

3. Count money and make change using coins and a dollar bill.

4. Represent and write the value of money using the ¢ sign and in decimal form when using the \$ sign.

5. Represent fractions (halves, thirds, fourths, sixths and eighths), using words, numerals and physical models. For example:

- a. Recognize that a fractional part can mean different amounts depending on the original quantity.
- b. Recognize that a fractional part of a rectangle does not have to be shaded with contiguous parts.
- c. Identify and illustrate parts of a whole in real-world objects.

**Standard**

**Organizer**

**Grade-Level Indicator**

2

ACADEMIC CONTENT STANDARDS

## K-12 Mathematics

# Philosophy and Principles



## Philosophy and Guiding Assumptions



Ohio's Mathematics academic content standards serve as a basis for what all students should know and be able to do by the time they graduate from high school. These standards, benchmarks and grade-level indicators are intended to provide Ohio educators with a set of common expectations from which to base mathematics curriculum.

### Philosophy of Ohio's Mathematics Academic Content Standards

The intent of Ohio's Mathematics academic content standards is to ensure ALL Ohio students understand and use mathematics effectively in everyday life and the workplace.

- Mathematics literacy is needed to make everyday decisions such as choosing which product to purchase, interpreting information in news reports, and selecting insurance or health plans.
- Mathematical thinking and problem-solving are needed in the workplace, and those who understand and can use mathematics have significantly enhanced opportunities and options.
- Mathematics plays a central role in modern culture, including aesthetic and recreational aspects, and an essential role in the scientific and technical community.

### Assumptions for Mathematics Academic Content Standards

Ohio's academic content standards:

- Set high expectations and strong support for mathematics achievement by ALL students.
- Represent mathematics knowledge and skills needed to make a successful transition to post-secondary education, workplace and daily life.
- Reflect sound application of research on how students learn mathematics.
- Align with national standards documents and major studies in mathematics.



# A C A D E M I C   C O N T E N T   S T A N D A R D S

- Address mathematics content knowledge and mathematical processes, including problem-solving, mathematical reasoning, communication, representation and connections.
- Focus on important mathematics topics that are well-articulated through benchmarks and grade-level indicators.
- Represent rigorous progression across grades and in-depth study within each grade.
- Provide an appropriate balance among conceptual understanding, procedural knowledge and skills, and application and problem-solving.
- Incorporate use of technology by ALL students in learning mathematics.
- Serve as the basis for classroom, district and state assessments.
- Guide the development of local mathematics curriculum and instructional programs.

## Mathematics for All



The Ohio Department of Education believes that Ohio's academic content standards are for all students. Clearly defined standards delineate what all children, college and career-bound, should know and be able to do as they progress through the grade levels. Well-defined standards ensure that parents, teachers, and administrators will be able to monitor students' development. Students, as stakeholders in their own learning, will be capable of tracking their own learning.

No individual or group should be excluded from the opportunity to learn, and all students are presumed capable of learning. Every Ohio student, regardless of race, gender, ethnicity, socioeconomic status, limited English proficiency, learning disability, or giftedness, should have access to a challenging, standards-based curriculum.

The knowledge and skills defined in Ohio academic content standards are within the reach of all students. Students, however, develop at different rates. All children learn and experience success given time and opportunity, but the degree to which the standards are met and the time it takes to reach the standards will vary from student to student.

Students with disabilities, apart from a few profoundly handicapped students, should have Individual Education Plans aligned with the standards. Students with disabilities are first and foremost students of the regular curriculum, yet they may require specific supports and interventions to progress in the curriculum. These adaptations are not intended to compromise the content standards. Rather, adaptations provide students with disabilities the opportunity to maximize their strengths, compensate for their learning difficulties, and participate and progress in the standards-based curriculum.

Students who can exceed the grade-level indicators and benchmarks set for in the standards must be afforded the opportunity and be encouraged to do so. Gifted and talented students may require special services or activities in order to fully develop their intellectual, creative, artistic and leadership capabilities or to excel in a specific content area. Again, the point of departure is the standards-based curriculum.

Students with limited English proficiency (LEP) may also need specific supports and adaptive instructional delivery in order to achieve Ohio's academic content

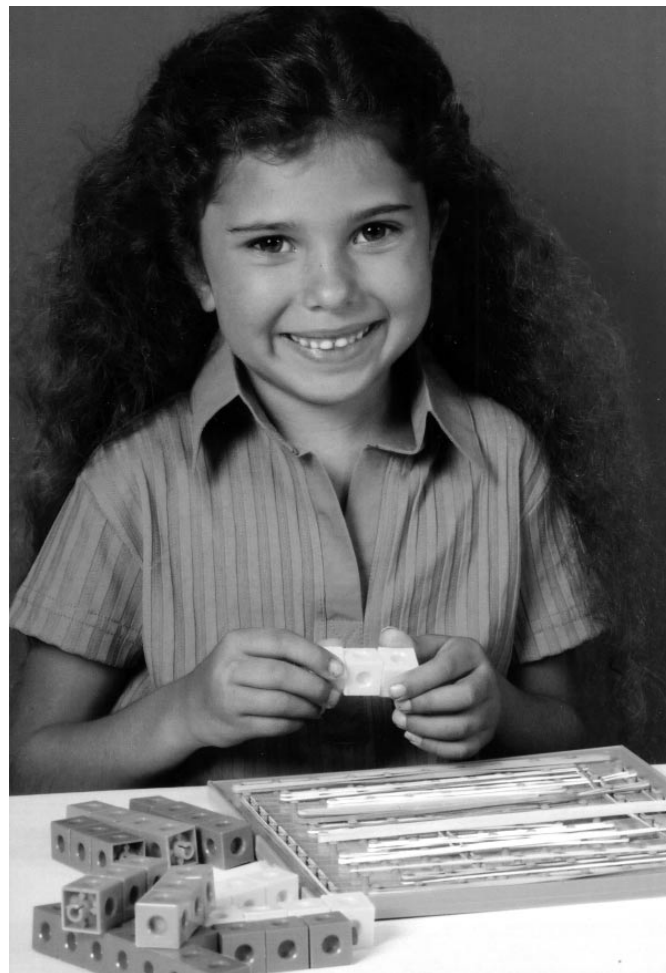
standards. An instructional delivery plan for a student with LEP needs to take into account the student's level of English language proficiency as well as his or her cultural experiences.

All children deserve adjustments in order to address their individual needs, regardless of whether they have been designated as learning disabled or gifted. Identifying and nurturing the talents of all students, and strategizing with students to overcome their weaknesses, will enable all children to reach the standards. The Department encourages school districts to align their programs with the standards to ensure that all of Ohio's students reach their full potential.

ACADEMIC CONTENT STANDARDS

K-12 Mathematics

# Benchmarks and Indicators by Standard



## Benchmarks

### Number, Number Sense and Operations Standard

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

Notes	<b>By the end of the K-2 program:</b>	<b>By the end of the 3-4 program:</b>
	<p>A. Use place value concepts to represent whole numbers using numerals, words and physical models.</p> <p>B. Recognize, classify, compare and order whole numbers.</p> <p>C. Represent commonly used fractions using words and physical models.</p> <p>D. Determine the value of a collection of coins and dollar bills.</p> <p>E. Make change using coins for values up to one dollar.</p> <p>F. Count, using numerals and ordinal numbers.</p> <p>G. Model, represent and explain addition as combining sets and counting on.</p> <p>H. Model, represent and explain subtraction as comparison, take-away and part-to-whole.</p>	<p>A. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers and decimals.</p> <p>B. Recognize and generate equivalent representations for whole numbers, fractions and decimals.</p> <p>C. Represent commonly used fractions and mixed numbers using words and physical models.</p> <p>D. Use models, points of reference and equivalent forms of commonly used fractions to judge the size of fractions and to compare, describe and order them.</p> <p>E. Recognize and classify numbers as prime or composite and list factors.</p> <p>F. Count money and make change using both coins and paper bills.</p>
	(continued page 30)	(continued page 30)

**By the end of the 5-7 program:**

- A. Represent and compare numbers less than 0 through familiar applications and extending the number line.
- B. Compare, order and convert among fractions, decimals and percents.
- C. Develop meaning for percents, including percents greater than 100 and less than 1.
- D. Use models and pictures to relate concepts of ratio, proportion and percent.
- E. Use order of operations, including use of parenthesis and exponents to solve multi-step problems, and verify and interpret the results.
- F. Apply number system properties when performing computations.

(continued page 31)

**By the end of the 8-10 program:**

- A. Use scientific notation to express large numbers and numbers less than one.
- B. Identify subsets of the real number system.
- C. Apply properties of operations and the real number system, and justify when they hold for a set of numbers.
- D. Connect physical, verbal and symbolic representations of integers, rational numbers and irrational numbers.
- E. Compare, order and determine equivalent forms of real numbers.
- F. Explain the effects of operations on the magnitude of quantities.
- G. Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.

(continued page 31)

**By the end of the 11-12 program:**

- A. Demonstrate that vectors and matrices are systems having some of the same properties of the real number system.
- B. Develop an understanding of properties of and representations for addition and multiplication of vectors and matrices.
- C. Apply factorials and exponents, including fractional exponents, to solve practical problems.
- D. Demonstrate fluency in operations with real numbers, vectors and matrices, using mental computation or paper and pencil calculations for simple cases and technology for more complicated cases.
- E. Represent and compute with complex numbers.

### Number, Number Sense and Operations Standard

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<p>I. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.</p> <p>J. Model, represent and explain division as sharing equally, repeated subtraction and rectangular arrays.</p> <p>K. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions.</p> <p>L. Demonstrate fluency in adding and subtracting multiples of 10, and recognize combinations that make 10.</p> <p>M. Add and subtract two-digit numbers with and without regrouping.</p>	<p>G. Model and use commutative and associative properties for addition and multiplication.</p> <p>H. Use relationships between operations, such as subtraction as the inverse of addition and division as the inverse of multiplication.</p> <p>I. Demonstrate fluency in multiplication facts with factors through 10 and corresponding divisions.</p> <p>J. Estimate the results of whole number computations using a variety of strategies, and judge the reasonableness.</p> <p>K. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division of whole numbers.</p> <p>L. Use a variety of methods and appropriate tools (mental math, paper and pencil, calculators) for computing with whole numbers.</p> <p>M. Add and subtract commonly used fractions with like denominators and decimals, using models and paper and pencil.</p>



**By the end of the 5-7 program:**

- G. Apply and explain the use of prime factorizations, common factors, and common multiples in problem situations.
- H. Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals and integers.
- I. Use a variety of strategies, including proportional reasoning, to estimate, compute, solve and explain solutions to problems involving integers, fractions, decimals and percents.

**By the end of the 8-10 program:**

- H. Find the square root of perfect squares, and approximate the square root of non-perfect squares.
- I. Estimate, compute and solve problems involving scientific notation, square roots and numbers with integer exponents.

**By the end of the 11-12 program:**

## Benchmarks

### Measurement Standard

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<p>A. Explain the need for standard units of measure.</p> <p>B. Select appropriate units for length, weight, volume (capacity) and time, using:</p> <ul style="list-style-type: none"> <li>• objects; i.e., non-standard units;</li> <li>• U.S. customary units: inch, foot, yard, ounce, pound, cup, quart, gallon, minute, hour, day, week and year;</li> <li>• metric units: centimeter, meter, gram and liter.</li> </ul> <p>C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.</p> <p>D. Apply measurement techniques to measure length, weight and volume (capacity).</p> <p>E. Recognize that using different units of measurement will yield different numbers for the same measurement.</p>	<p>A. Select appropriate units for perimeter, area, weight, volume (capacity), time and temperature, using:</p> <ul style="list-style-type: none"> <li>• objects of uniform size;</li> <li>• U.S. customary units; e.g., mile, square inch, cubic inch, second, degree Fahrenheit, and other units as appropriate;</li> <li>• metric units; e.g., millimeter, kilometer, square centimeter, kilogram, cubic centimeter, degree Celsius, and other units as appropriate.</li> </ul> <p>B. Know that the number of units is inversely related to the size of the unit for any item being measured.</p> <p>C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.</p> <p style="text-align: right;">(continued page 34)</p>

**By the end of the 5-7 program:**

- A. Select appropriate units to measure angles, circumference, surface area, mass and volume, using:
  - U.S. customary units; e.g., degrees, square feet, pounds, and other units as appropriate;
  - metric units; e.g., square meters, kilograms and other units as appropriate.
- B. Convert units of length, area, volume, mass and time within the same measurement system.
- C. Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.
- D. Select a tool and measure accurately to a specified level of precision.

(continued page 35)

**By the end of the 8-10 program:**

- A. Solve increasingly complex non-routine measurement problems and check for reasonableness of results.
- B. Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.
- C. Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and to find volume of prisms, cylinders, and pyramids.
- D. Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.

(continued page 35)

**By the end of the 11-12 program:**

- A. Explain differences among accuracy, precision and error, and describe how each of those can affect solutions in measurement situations.
- B. Apply various measurement scales to describe phenomena and solve problems.
- C. Estimate and compute areas and volume in increasingly complex problem situations.
- D. Solve problem situations involving derived measurements; e.g., density, acceleration.

**Measurement Standard**

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
		<p>D. Identify appropriate tools and apply counting techniques for measuring side lengths, perimeter and area of squares, rectangles, and simple irregular two-dimensional shapes, volume of rectangular prisms, and time and temperature.</p> <p>E. Tell time to the nearest minute.</p>

**By the end of the 5-7 program:**

- E. Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.
- F. Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.
- G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.

**By the end of the 8-10 program:**

- E. Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.
- F. Write and solve real-world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.

**By the end of the 11-12 program:**

## Benchmarks

### Geometry and Spatial Sense Standard

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects, and transformations to analyze mathematical situations and solve problems.

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<p>A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment.</p> <p>B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone and pyramid, and identify them in the environment.</p> <p>C. Sort and compare two-dimensional figures and three-dimensional objects according to their characteristics and properties.</p> <p>D. Identify, explain and model (superposition, copying) the concept of shapes being congruent and similar.</p> <p>E. Recognize two- and three-dimensional objects from different positions.</p> <p>(continued on page 38)</p>	<p>A. Provide rationale for groupings and comparisons of two-dimensional figures and three-dimensional objects.</p> <p>B. Describe and identify points, lines and planes in the environment.</p> <p>C. Describe and identify intersecting, parallel and perpendicular lines or segments in the environment.</p> <p>D. Identify and draw right, obtuse, acute and straight angles.</p> <p>E. Use attributes to describe, classify and sketch plane figures and build solid objects.</p> <p>F. Develop definitions of classes of shapes.</p> <p>G. Find and name locations in coordinate systems.</p> <p>(continued on page 38)</p>

**By the end of the 5-7 program:**

- A. Identify and label angle parts and the regions defined within the plane where the angle resides.
- B. Draw circles, and identify and determine the relationships among the radius, diameter, center and circumference.
- C. Specify locations and plot ordered pairs on a coordinate plane.
- D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.
- E. Use proportions to express relationships among corresponding parts of similar figures.
- F. Describe and use the concepts of congruence, similarity and symmetry to solve problems.

(continued on page 39)

**By the end of the 8-10 program:**

- A. Formally define geometric figures.
- B. Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.
- C. Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.
- D. Use coordinate geometry to represent and examine the properties of geometric figures.
- E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.
- F. Represent and model transformations in a coordinate plane and describe the results.

(continued on page 39)

**By the end of the 11-12 program:**

- A. Use trigonometric relationships to verify and determine solutions in problem situations.
- B. Represent transformations within a coordinate system using vectors and matrices.

### Geometry and Spatial Sense Standard

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<p>F. Describe location, using comparative (before, after), directional (above, below), and positional (first, last) words.</p> <p>G. Identify and draw figures with line symmetry.</p>	<p>H. Identify and describe line and rotational symmetry in two-dimensional shapes and designs.</p> <p>I. Describe, identify and model reflections, rotations and translations, using physical materials.</p> <p>J. Describe a motion or series of transformations that show two shapes are congruent.</p>



**By the end of the 5-7 program:**

- G. Describe and use properties of triangles to solve problems involving angle measures and side lengths of right triangles.
- H. Predict and describe results (size, position, orientation) of transformations of two-dimensional figures.
- I. Identify and draw three-dimensional objects from different views (top, side, front and perspective).
- J. Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing.

**By the end of the 8-10 program:**

- G. Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system.
- H. Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.
- I. Use right triangle trigonometric relationships to determine lengths and angle measures.

**By the end of the 11-12 program:**

## Benchmarks

### Patterns, Functions and Algebra Standard

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<ul style="list-style-type: none"> <li>A. Sort, classify and order objects by size, number and other properties, and describe the attributes used.</li> <li>B. Extend sequences of sounds and shapes or simple number patterns, and create and record similar patterns.</li> <li>C. Create and extend patterns, and describe the rule in words.</li> <li>D. Model problem situations, using objects, pictures, numbers and other symbols.</li> <li>E. Solve open sentences and explain strategies.</li> <li>F. Represent an unknown quantity as a variable using a symbol, such as <math>\square</math>, <math>\triangle</math>, <math>\circ</math>.</li> <li>G. Describe and compare qualitative and quantitative changes.</li> </ul>	<ul style="list-style-type: none"> <li>A. Analyze and extend patterns, and describe the rule in words.</li> <li>B. Use patterns to make predictions, identify relationships, and solve problems.</li> <li>C. Write and solve open sentences and explain strategies.</li> <li>D. Represent an unknown quantity as a variable using a symbol, including letters.</li> <li>E. Use variables to create and solve equations representing problem situations.</li> <li>F. Construct and use a table of values to solve problems associated with mathematical relationships.</li> <li>G. Describe how a change in one variable affects the value of a related variable.</li> </ul>

**By the end of the 5-7 program:**

- A. Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications.
- B. Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules.
- C. Use variables to create and solve equations and inequalities representing problem situations.
- D. Use symbolic algebra to represent and explain mathematical relationships.
- E. Use rules and variables to describe patterns, functions and other relationships.

(continued on page 43)

**By the end of the 8-10 program:**

- A. Generalize and explain patterns and sequences in order to find the next term and the  $n$ th term.
- B. Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.
- C. Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.
- D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.
- E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.
- F. Solve and graph linear equations and inequalities.

(continued on page 43)

**By the end of the 11-12 program:**

- A. Analyze functions by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- B. Use the quadratic formula to solve quadratic equations that have complex roots.
- C. Use recursive functions to model and solve problems; e.g., home mortgages, annuities.
- D. Apply algebraic methods to represent and generalize problem situations involving vectors and matrices.

**Patterns, Functions and Algebra Standard**

Notes	By the end of the K-2 program:	By the end of the 3-4 program:

**By the end of the 5-7 program:**

- F. Use representations, such as tables, graphs and equations, to model situations and to solve problems, especially those that involve linear relationships.
- G. Write, simplify and evaluate algebraic expressions.
- H. Solve linear equations and inequalities symbolically, graphically and numerically.
- I. Explain how inverse operations are used to solve linear equations.
- J. Use formulas in problem-solving situations.
- K. Graph linear equations and inequalities.
- L. Analyze functional relationships, and explain how a change in one quantity results in a change in the other.
- M. Approximate and interpret rates of change from graphical and numerical data.

**By the end of the 8-10 program:**


- G. Solve quadratic equations with real roots by graphing, formula and factoring.
- H. Solve systems of linear equations involving two variables graphically and symbolically.
- I. Model and solve problem situations involving direct and inverse variation.
- J. Describe and interpret rates of change from graphical and numerical data.

**By the end of the 11-12 program:**

# Benchmarks

## Data Analysis and Probability Standard

Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<p>A. Pose questions and gather data about everyday situations and familiar objects.</p> <p>B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart.</p> <p>C. Represent data using objects, picture graphs and bar graphs.</p> <p>D. Describe the probability of chance events as more, less or equally likely to occur.</p>	<p>A. Gather and organize data from surveys and classroom experiments, including data collected over a period of time.</p> <p>B. Read and interpret tables, charts, graphs (bar, picture, line, line plot), and timelines as sources of information, identify main idea, draw conclusions, and make predictions.</p> <p>C. Construct charts, tables and graphs to represent data, including picture graphs, bar graphs, line graphs, line plots and Venn diagrams.</p> <p>D. Read, interpret and construct graphs in which icons represent more than a single unit or intervals greater than one; e.g., each  = 10 bicycles or the intervals on an axis are multiples of 10.</p> <p>E. Describe data using mode, median and range.</p> <p style="text-align: right;">(continued page 46)</p>

**By the end of the 5-7 program:**

- A. Read, create and use line graphs, histograms, circle graphs, box-and-whisker plots, stem-and-leaf plots, and other representations when appropriate.
- B. Interpret data by looking for patterns and relationships, draw and justify conclusions, and answer related questions.
- C. Evaluate interpretations and conclusions as additional data are collected, modify conclusions and predictions, and justify new findings.
- D. Compare increasingly complex displays of data, such as multiple sets of data on the same graph.
- E. Collect, organize, display and interpret data for a specific purpose or need.

(continued page 47)

**By the end of the 8-10 program:**

- A. Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.
- B. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.
- C. Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.
- D. Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.

(continued page 47)

**By the end of the 11-12 program:**

- A. Create and analyze tabular and graphical displays of data using appropriate tools, including spreadsheets and graphing calculators.
- B. Use descriptive statistics to analyze and summarize data, including measures of center, dispersion, correlation and variability.
- C. Design and perform a statistical experiment, simulation or study; collect and interpret data; and use descriptive statistics to communicate and support predictions and conclusions.
- D. Connect statistical techniques to applications in workplace and consumer situations.

**Data Analysis and Probability Standard**

Notes	By the end of the K - 2 program:	By the end of the 3 - 4 program:
		<p>F. Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes.</p> <p>G. Identify and represent possible outcomes, such as arrangements of a set of up to four members and possible combinations from several sets, each containing 2 or 3 members.</p> <p>H. Use the set of possible outcomes to describe and predict events.</p>



**By the end of the 5-7 program:**

- F. Determine and use the range, mean, median and mode to analyze and compare data, and explain what each indicates about the data.
- G. Evaluate conjectures and predictions based upon data presented in tables and graphs, and identify misuses of statistical data and displays.
- H. Find all possible outcomes of simple experiments or problem situations, using methods such as lists, arrays and tree diagrams.
- I. Describe the probability of an event using ratios, including fractional notation.
- J. Compare experimental and theoretical results for a variety of simple experiments.
- K. Make and justify predictions based on experimental and theoretical probabilities.

**By the end of the 8-10 program:**

- E. Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.
- F. Construct convincing arguments based on analysis of data and interpretation of graphs.
- G. Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.
- H. Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.
- I. Design an experiment to test a theoretical probability, and record and explain results.
- J. Compute probabilities of compound events, independent events, and simple dependent events.
- K. Make predictions based on theoretical probabilities and experimental results.

**By the end of the 11-12 program:**

## Benchmarks

### Mathematical Processes Standard

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<p>A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.</p> <p>B. Identify and restate in own words the question or problem and the information needed to solve the problem.</p> <p>C. Generate alternative strategies to solve problems.</p> <p>D. Evaluate the reasonableness of predictions, estimations and solutions.</p> <p>E. Explain to others how a problem was solved.</p> <p>F. Draw pictures and use physical models to represent problem situations and solutions.</p> <p>(continued on page 50)</p>	<p>A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.</p> <p>B. Use an organized approach and appropriate strategies to solve multi-step problems.</p> <p>C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.</p> <p>D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.</p> <p>E. Link concepts to procedures and to symbolic notation; e.g., model <math>3 \times 4</math> with a geometric array, represent one-third by dividing an object into three equal parts.</p> <p>(continued on page 50)</p>

The benchmarks for mathematical processes articulate what students should demonstrate in problem solving, representation, communication, reasoning and connections at key points in their mathematics program. Specific grade-level indicators have not been included for the mathematical processes standard because content and processes should be interconnected at the indicator level. Therefore, mathematical processes have been embedded within the grade-level indicators for the five content standards.

<b>By the end of the 5-7 program:</b>	<b>By the end of the 8-10 program:</b>	<b>By the end of the 11-12 program:</b>
<ul style="list-style-type: none"> <li>A. Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.</li> <li>B. Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.</li> <li>C. Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.</li> <li>D. Recognize whether an estimate or an exact solution is appropriate for a given problem situation.</li> <li>E. Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.</li> </ul> <p style="text-align: center;">(continued on page 51)</p>	<ul style="list-style-type: none"> <li>A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.</li> <li>B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.</li> <li>C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the <math>x</math>-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.</li> </ul> <p style="text-align: center;">(continued on page 51)</p>	<ul style="list-style-type: none"> <li>A. Construct algorithms for multi-step and non-routine problems.</li> <li>B. Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.</li> <li>C. Assess the adequacy and reliability of information available to solve a problem.</li> <li>D. Select and use various types of reasoning and methods of proof.</li> <li>E. Evaluate a mathematical argument and use reasoning and logic to judge its validity.</li> <li>F. Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.</li> </ul> <p style="text-align: center;">(continued on page 51)</p>

### Mathematical Processes Standard

Notes	By the end of the K-2 program:	By the end of the 3-4 program:
	<p>G. Use invented and conventional symbols and common language to describe a problem situation and solution.</p> <p>H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.</p> <p>I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.</p>	<p>F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.</p> <p>G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.</p> <p>H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.</p> <p>I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.</p> <p>J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.</p> <p>K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions.</p>

**By the end of the 5-7 program:**

- F. Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.
- G. Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.
- H. Use representations to organize and communicate mathematical thinking and problem solutions.
- I. Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem.
- J. Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.
- K. Recognize and use mathematical language and symbols when reading, writing and conversing with others.

**By the end of the 8-10 program:**

- D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.
- E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
- F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
- G. Write clearly and coherently about mathematical thinking and ideas.
- H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.

**By the end of the 11-12 program:**

- G. Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.
- H. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
- I. Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.
- J. Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.

# K-12 Mathematics

## Grade-Level Indicators

### Number, Number Sense and Operations Standard

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

### Kindergarten

#### *Number and Number Systems*

1. Compare and order whole numbers up to 10.
2. Explain rules of counting, such as each object should be counted once and that order does not change the number.
3. Count to twenty; e.g., in play situations or while reading number books.
4. Determine “how many” in sets (groups) of 10 or fewer objects.
5. Relate, read and write numerals for single-digit numbers (0 to 9).
6. Construct multiple sets of objects each containing the same number of objects.
7. Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects.
8. Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.
9. Identify and state the value of a penny, nickel and dime.



- Meaning of Operations*
10. Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
    - a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.
    - b. Count on (forward) and count back (backward) on a number line between 0 and 10.
  11. Demonstrate joining multiple groups of objects, each containing the same number of objects; e.g., combining 3 bags of candy, each containing 2 pieces.
  12. Partition or share a small set of objects into groups of equal size; e.g., sharing 6 stickers equally among 3 children.
- Computation and Estimation*
13. Recognize the number or quantity of sets up to 5 without counting; e.g., recognize without counting the dot arrangement on a domino as 5.

## Grade One

- Number and Number Systems*
1. Use ordinal numbers to order objects; e.g., first, second, third.
  2. Recognize and generate equivalent forms for the same number using physical models, words and number expressions; e.g., concept of ten is described by “10 blocks”, full tens frame, numeral 10,  $5 + 5$ ,  $15 - 5$ , one less than 11, my brother’s age.
  3. Read and write the numerals for numbers to 100.
  4. Count forward to 100, count backwards from 100, and count forward or backward starting at any number between 1 and 100.

5. Use place value concepts to represent whole numbers using numerals, words, expanded notation and physical models with ones and tens. For example:
  - a. Develop a system to group and count by twos, fives and tens.
  - b. Identify patterns and groupings in a 100's chart and relate to place value concepts.
  - c. Recognize the first digit of a two-digit number as the most important to indicate size of a number and the nearness to 10 or 100.
6. Identify and state the value of a penny, nickel, dime, quarter and dollar.
7. Determine the value of a small collection of coins (with a total value up to one dollar) using 1 or 2 different type coins, including pennies, nickels, dimes and quarters.
8. Show different combinations of coins that have the same value.
9. Represent commonly used fractions using words and physical models for halves, thirds and fourths, recognizing fractions are represented by equal size parts of a whole and of a set of objects.

*Meaning of Operations*

10. Model, represent and explain addition as combining sets (part + part = whole) and counting on. For example:
  - a. Model and explain addition using physical materials in contextual situations.
  - b. Draw pictures to model addition.
  - c. Write number sentences to represent addition.
  - d. Explain that adding two whole numbers yields a larger whole number.
11. Model, represent and explain subtraction as take-away and comparison. For example:
  - a. Model and explain subtraction using physical materials in contextual situations.
  - b. Draw pictures to model subtraction.



- c. Write number sentences to represent subtraction.
  - d. Explain that subtraction of whole numbers yields an answer smaller than the original number.
12. Use conventional symbols to represent the operations of addition and subtraction.
  13. Model and represent multiplication as repeated addition and rectangular arrays in contextual situations; e.g., four people will be at my party and if I want to give 3 balloons to each person, how many balloons will I need to buy?
  14. Model and represent division as sharing equally in contextual situations; e.g., sharing cookies.
  15. Demonstrate that equal means “the same as” using visual representations.
  16. Develop strategies for basic addition facts, such as:
    - a. counting all;
    - b. counting on;
    - c. one more, two more;
    - d. doubles;
    - e. doubles plus or minus one;
    - f. make ten;
    - g. using tens frames;
    - h. identity property (adding zero).
  17. Develop strategies for basic subtraction facts, such as:
    - a. relating to addition (for example, think of  $7 - 3 = ?$  as “3 plus ? equals 7”);
    - b. one less, two less;
    - c. all but one (for example,  $8 - 7, 5 - 4$ );
    - d. using tens frames;
    - e. missing addends.

*Computation and Estimation*

## Grade Two

### *Number and Number Systems*

1. Use place value concepts to represent, compare and order whole numbers using physical models, numerals and words, with ones, tens and hundreds. For example:
  - a. Recognize 10 can mean “10 ones” or a single entity (1 ten) through physical models and trading games.
  - b. Read and write 3-digit numerals (e.g., 243 as two hundred forty three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each.
2. Recognize and classify numbers as even or odd.
3. Count money and make change using coins and a dollar bill.
4. Represent and write the value of money using the ¢ sign and in decimal form when using the \$ sign.
5. Represent fractions (halves, thirds, fourths, sixths and eighths), using words, numerals and physical models. For example:
  - a. Recognize that a fractional part can mean different amounts depending on the original quantity.
  - b. Recognize that a fractional part of a rectangle does not have to be shaded with contiguous parts.
  - c. Identify and illustrate parts of a whole and parts of sets of objects.
  - d. Compare and order physical models of halves, thirds and fourths in relation to 0 and 1.

### *Meaning of Operations*

6. Model, represent and explain subtraction as comparison, take-away and part-to-whole; e.g., solve missing addend problems by counting up or subtracting, such as “I had six baseball cards, my sister gave me more, and I now have ten. How many did she give me?” can be represented as  $6 + ? = 10$  or  $10 - 6 = ?$ .
7. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.

*Computation and Estimation*

8. Model, represent and explain division as sharing equally and repeated subtraction.
9. Model and use the commutative property for addition.
10. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions; e.g.,  $9 + 9 = 18$ ,  $18 - 9 = 9$ .
11. Add and subtract multiples of 10.
12. Demonstrate multiple strategies for adding and subtracting 2- or 3-digit whole numbers, such as:
  - a. compatible numbers;
  - b. compensatory numbers;
  - c. informal use of commutative and associative properties of addition.
13. Estimate the results of whole number addition and subtraction problems using front-end estimation, and judge the reasonableness of the answers.

## Grade Three

*Number and Number Systems*

1. Identify and generate equivalent forms of whole numbers; e.g., 36,  $30 + 6$ ,  $9 \times 4$ ,  $46 - 10$ , number of inches in a yard.
2. Use place value concepts to represent whole numbers and decimals using numerals, words, expanded notation and physical models. For example:
  - a. Recognize 100 means “10 tens” as well as a single entity (1 hundred) through physical models and trading games.
  - b. Describe the multiplicative nature of the number system; e.g., the structure of 3205 as  $3 \times 1000$  plus  $2 \times 100$  plus  $5 \times 1$ .
  - c. Model the size of 1000 in multiple ways; e.g., packaging 1000 objects into 10 boxes of 100, modeling a meter with centimeter and decimeter strips, or gathering 1000 pop-can tabs.

- d. Explain the concept of tenths and hundredths using physical models, such as metric pieces, base ten blocks, decimal squares or money.
- 3. Use mathematical language and symbols to compare and order; e.g., less than, greater than, at most, at least,  $<$ ,  $>$ ,  $=$ ,  $\leq$ ,  $\geq$ .
- 4. Count money and make change using coins and paper bills to ten dollars.
- 5. Represent fractions and mixed numbers using words, numerals and physical models.
- 6. Compare and order commonly used fractions and mixed numbers using number lines, models (such as fraction circles or bars), points of reference (such as more or less than  $\frac{1}{2}$ ), and equivalent forms using physical or visual models.
- 7. Recognize and use decimal and fraction concepts and notations as related ways of representing parts of a whole or a set; e.g., 3 of 10 marbles are red can also be described as  $\frac{3}{10}$  and 3 tenths are red.

*Meaning of Operations*

- 8. Model, represent and explain multiplication; e.g., repeated addition, skip counting, rectangular arrays and area model. For example:
  - a. Use conventional mathematical symbols to write equations for word problems involving multiplication.
  - b. Understand that, unlike addition and subtraction, the factors in multiplication and division may have different units; e.g., 3 boxes of 5 cookies each.
- 9. Model, represent and explain division; e.g., sharing equally, repeated subtraction, rectangular arrays and area model. For example:
  - a. Translate contextual situations involving division into conventional mathematical symbols.
  - b. Explain how a remainder may impact an answer in a real-world situation; e.g., 14 cookies being shared by 4 children.

- Computation and Estimation*
10. Explain and use relationships between operations, such as:
    - a. relate addition and subtraction as inverse operations;
    - b. relate multiplication and division as inverse operations;
    - c. relate addition to multiplication (repeated addition);
    - d. relate subtraction to division (repeated subtraction).
  11. Model and use the commutative and associative properties for addition and multiplication.
  12. Add and subtract whole numbers with and without regrouping.
  13. Demonstrate fluency in multiplication facts through 10 and corresponding division facts.
  14. Multiply and divide 2- and 3-digit numbers by a single-digit number, without remainders for division.
  15. Evaluate the reasonableness of computations based upon operations and the numbers involved; e.g., considering relative size, place value and estimates.

## Grade Four

*Number and Number Systems*

1. Identify and generate equivalent forms of fractions and decimals. For example:
  - a. Connect physical, verbal and symbolic representations of fractions, decimals and whole numbers; e.g.,  $\frac{1}{2}$ ,  $\frac{5}{10}$ , “five tenths,” 0.5, shaded rectangles with half, and five tenths.
  - b. Understand and explain that ten tenths is the same as one whole in both fraction and decimal form.
2. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers through millions and decimals through thousandths.
3. Round whole numbers to a given place value.

- |                                   |  |
|-----------------------------------|--|
| <i>Meaning of Operations</i>      | <ol style="list-style-type: none"> <li>4. Identify and represent factors and multiples of whole numbers through 100, and classify numbers as prime or composite.</li> <li>5. Use models and points of reference to compare commonly used fractions.</li> <li>6. Use associative and distributive properties to simplify and perform computations; e.g., use left to right multiplication and the distributive property to find an exact answer without paper and pencil, such as <math>5 \times 47 = 5 \times 40 + 5 \times 7 = 200 + 35 = 235</math>.</li> <li>7. Recognize that division may be used to solve different types of problem situations and interpret the meaning of remainders; e.g., situations involving measurement, money.</li> </ol>   |
| <i>Computation and Estimation</i> | <ol style="list-style-type: none"> <li>8. Solve problems involving counting money and making change, using both coins and paper bills.</li> <li>9. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.</li> <li>10. Use physical models, visual representations, and paper and pencil to add and subtract decimals and commonly used fractions with like denominators.</li> <li>11. Develop and explain strategies for performing computations mentally.</li> <li>12. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division using an organized approach, and verify and interpret results with respect to the original problem.</li> <li>13. Use a variety of methods and appropriate tools for computing with whole numbers; e.g., mental math, paper and pencil, and calculator.</li> <li>14. Demonstrate fluency in adding and subtracting whole numbers and in multiplying and dividing whole numbers by 1- and 2-digit numbers and multiples of ten.</li> </ol> |

## Grade Five

*Number and  
Number Systems*

1. Use models and visual representation to develop the concept of ratio as part-to-part and part-to-whole, and the concept of percent as part-to-whole.
2. Use various forms of “one” to demonstrate the equivalence of fractions; e.g.,  $\frac{18}{24} = \frac{9}{12} \times \frac{2}{2} = \frac{3}{4} \times \frac{6}{6}$ .
3. Identify and generate equivalent forms of fractions, decimals and percents.
4. Round decimals to a given place value and round fractions (including mixed numbers) to the nearest half.
5. Recognize and identify perfect squares and their roots.

*Meaning of Operations*

6. Represent and compare numbers less than 0 by extending the number line and using familiar applications; e.g., temperature, owing money.
7. Use commutative, associative, distributive, identity and inverse properties to simplify and perform computations.
8. Identify and use relationships between operations to solve problems.
9. Use order of operations, including use of parentheses, to simplify numerical expressions.
10. Justify why fractions need common denominators to be added or subtracted.
11. Explain how place value is related to addition and subtraction of decimals; e.g.,  $0.2 + 0.14$ ; the two tenths is added to the one tenth because they are both tenths.

*Computation and  
Estimation*

12. Use physical models, points of reference, and equivalent forms to add and subtract commonly used fractions with like and unlike denominators and decimals.
13. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

## Grade Six

*Number and  
Number Systems*

1. Decompose and recompose whole numbers using factors and exponents (e.g.,  $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ ), and explain why “squared” means “second power” and “cubed” means “third power.”
2. Find and use the prime factorization of composite numbers. For example:
  - a. Use the prime factorization to recognize the greatest common factor (GCF).
  - b. Use the prime factorization to recognize the least common multiple (LCM).
  - c. Apply the prime factorization to solve problems and explain solutions.
3. Explain why a number is referred to as being “rational,” and recognize that the expression  $\frac{a}{b}$  can mean  $a$  parts of size  $\frac{1}{b}$  each,  $a$  divided by  $b$ , or the ratio of  $a$  to  $b$ .
4. Describe what it means to find a specific percent of a number, using real-life examples.
5. Use models and pictures to relate concepts of ratio, proportion and percent, including percents less than 1 and greater than 100.

*Meaning of Operations*

6. Use the order of operations, including the use of exponents, decimals and rational numbers, to simplify numerical expressions.
7. Use simple expressions involving integers to represent and solve problems; e.g., if a running back loses 15 yards on the first carry but gains 8 yards on the second carry, what is the net gain/loss?
8. Represent multiplication and division situations involving fractions and decimals with models and visual representations; e.g., show with pattern blocks what it means to take  $2\frac{2}{3} \div \frac{1}{6}$ .



9. Give examples of how ratios are used to represent comparisons; e.g., part-to-part, part-to-whole, whole-to-part.
10. Recognize that a quotient may be larger than the dividend when the divisor is a fraction; e.g.,  
 $6 \div \frac{1}{2} = 12$ .
- Computation and Estimation*
11. Perform fraction and decimal computations and justify their solutions; e.g., using manipulatives, diagrams, mathematical reasoning.
12. Develop and analyze algorithms for computing with fractions and decimals, and demonstrate fluency in their use.
13. Estimate reasonable solutions to problem situations involving fractions and decimals; e.g.,  
 $\frac{7}{8} + \frac{12}{13} \approx 2$  and  $4.23 \times 5.8 \approx 25$ .
14. Use proportional reasoning, ratios and percents to represent problem situations and determine the reasonableness of solutions.
15. Determine the percent of a number and solve related problems; e.g., find the percent markdown if the original price was \$140, and the sale price is \$100.

## Grade Seven

*Number and Number Systems*

1. Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.
2. Explain the meaning of exponents that are negative or 0.
3. Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.

*Meaning of Operations*

4. Use order of operations and properties to simplify numerical expressions involving integers, fractions and decimals.

- Computation and Estimation*
5. Explain the meaning and effect of adding, subtracting, multiplying and dividing integers; e.g., how adding two integers can result in a lesser value.
  6. Simplify numerical expressions involving integers and use integers to solve real-life problems.
  7. Solve problems using the appropriate form of a rational number (fraction, decimal or percent).
  8. Develop and analyze algorithms for computing with percents and integers, and demonstrate fluency in their use.
  9. Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents and square roots (for perfect squares).

## Grade Eight

- Number and Number Systems*
1. Use scientific notation to express large numbers and small numbers between 0 and 1.
  2. Recognize that natural numbers, whole numbers, integers, rational numbers and irrational numbers are subsets of the real number system.
- Meaning of Operations*
3. Apply order of operations to simplify expressions and perform computations involving integer exponents and radicals.
  4. Explain and use the inverse and identity properties and use inverse relationships (addition/subtraction, multiplication/division, squaring/square roots) in problem solving situations.
- Computation and Estimation*
5. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.
  6. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

7. Find the square root of perfect squares, and approximate the square root of non-perfect squares as consecutive integers between which the root lies; e.g.,  $\sqrt{130}$  is between 11 and 12.
8. Add, subtract, multiply, divide and compare numbers written in scientific notation.

## Grade Nine

*Number and  
Number Systems*

1. Identify and justify whether properties (closure, identity, inverse, commutative and associative) hold for a given set and operations; e.g., even integers and multiplication.

2. Compare, order and determine equivalent forms for rational and irrational numbers.

*Meaning of Operations*

3. Explain the effects of operations such as multiplication or division, and of computing powers and roots on the magnitude of quantities.

*Computation and  
Estimation*

4. Demonstrate fluency in computations using real numbers.
5. Estimate the solutions for problem situations involving square and cube roots.

## Grade Ten

*Number and  
Number Systems*

1. Connect physical, verbal and symbolic representations of irrational numbers; e.g., construct  $\sqrt{2}$  as a hypotenuse or on a number line.

*Meaning of Operations*

2. Explain the meaning of the  $n$ th root.

*Computation and  
Estimation*

3. Use factorial notation and computations to represent and solve problem situations involving arrangements.
4. Approximate the  $n$ th root of a given number greater than zero between consecutive integers when  $n$  is an integer; e.g., the 4<sup>th</sup> root of 50 is between 2 and 3.

## Grade Eleven

*Number and  
Number Systems*

1. Determine what properties hold for matrix addition and matrix multiplication; e.g., use examples to show addition is commutative and when multiplication is not commutative.

2. Determine what properties hold for vector addition and multiplication, and for scalar multiplication.

3. Represent complex numbers on the complex plane.

*Meaning of Operations*

4. Use matrices to represent given information in a problem situation.

5. Model, using the coordinate plane, vector addition and scalar multiplication.

*Computation and  
Estimation*

6. Compute sums, differences and products of matrices using paper and pencil calculations for simple cases, and technology for more complicated cases.

7. Compute sums, differences, products and quotients of complex numbers.

8. Use fractional and negative exponents as optional ways of representing and finding solutions for problem situations; e.g.,  $27^{2/3} = (27^{1/3})^2 = 9$ .

9. Use vector addition and scalar multiplication to solve problems.

## Grade Twelve

*Number and  
Number Systems*

1. Determine what properties (closure, identity, inverse, commutative and associative) hold for operations with complex numbers.

*Computation and  
Estimation*

2. Apply combinations as a method to create coefficients for the Binomial Theorem, and make connections to everyday and workplace problem situations.

# K-12 Mathematics

## Grade-Level Indicators

### Measurement Standard

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

### Kindergarten

#### *Measurement Units*

1. Identify units of time (day, week, month, year) and compare calendar elements; e.g., weeks are longer than days.

#### *Use Measurement Techniques and Tools*

2. Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less.
3. Measure length and volume (capacity) using uniform objects in the environment. For example, find:
  - a. how many paper clips long is a pencil;
  - b. how many small containers it takes to fill one big container using sand, rice, beans.
4. Order events based on time. For example:
  - a. activities that take a long or short time;
  - b. review what we do first, next, last;
  - c. recall what we did or plan to do yesterday, today, tomorrow.



## Grade One

### *Measurement Units*

1. Recognize and explain the need for fixed units and tools for measuring length and weight; e.g., rulers and balance scales.
2. Tell time to the hour and half hour on digital and analog (dial) timepieces.
3. Order a sequence of events with respect to time; e.g., summer, fall, winter and spring; morning, afternoon and night.

### *Use Measurement Techniques and Tools*

4. Estimate and measure weight using non-standard units; e.g., blocks of uniform size.
5. Estimate and measure lengths using non-standard and standard units; i.e., centimeters, inches and feet.

## Grade Two

### *Measurement Units*

1. Identify and select appropriate units of measure for:
  - a. length – centimeters, meters, inches, feet or yards;
  - b. volume (capacity) – liters, cups, pints or quarts;
  - c. weight – grams, ounces or pounds;
  - d. time – hours, half-hours, quarter-hours or minutes and time designations, a.m. or p.m.
2. Establish personal or common referents for units of measure to make estimates and comparisons; e.g., the width of a finger is a centimeter, a large bottle of soda pop is 2 liters, a small paper clip weighs about one gram.
3. Describe and compare the relationships among units of measure, such as centimeters and meters; inches, feet and yards; cups, pints and quarts; ounces and pounds; and hours, half-hours, and quarter-hours; e.g., how many inches in a foot?
4. Tell time to the nearest minute interval on digital and to the nearest 5 minute interval on analog (dial) timepieces.

*Use Measurement Techniques and Tools*

5. Estimate and measure the length and weight of common objects, using metric and U.S. customary units, accurate to the nearest unit.
6. Select and use appropriate measurement tools; e.g., a ruler to draw a segment 3 inches long, a measuring cup to place 2 cups of rice in a bowl, a scale to weigh 50 grams of candy.
7. Make and test predictions about measurements, using different units to measure the same length or volume.

### Grade Three

*Measurement Units*

1. Identify and select appropriate units for measuring:
  - a. length – miles, kilometers and other units of measure as appropriate;
  - b. volume (capacity) – gallons;
  - c. weight – ounces, pounds, grams, or kilograms;
  - d. temperature – degrees (Fahrenheit or Celsius).
2. Establish personal or common referents to include additional units; e.g., a gallon container of milk; a postage stamp is about a square inch.
3. Tell time to the nearest minute and find elapsed time using a calendar or a clock.
4. Read thermometers in both Fahrenheit and Celsius scales.

*Use Measurement Techniques and Tools*

5. Estimate and measure length, weight and volume (capacity), using metric and U.S. customary units, accurate to the nearest  $\frac{1}{2}$  or  $\frac{1}{4}$  unit as appropriate.
6. Use appropriate measurement tools and techniques to construct a figure or approximate an amount of specified length, weight or volume (capacity); e.g., construct a rectangle with length  $2\frac{1}{2}$  inches and width 3 inches, fill a measuring cup to the  $\frac{3}{4}$  cup mark.

7. Make estimates for perimeter, area and volume using links, tiles, cubes and other models.

## Grade Four

### *Measurement Units*

1. Relate the number of units to the size of the units used to measure an object; e.g., compare the number of cups to fill a pitcher to the number of quarts to fill the same pitcher.
2. Demonstrate and describe perimeter as surrounding and area as covering a two-dimensional shape, and volume as filling a three-dimensional object.
3. Identify and select appropriate units to measure:
  - a. perimeter – string or links (inches or centimeters).
  - b. area – tiles (square inches or square centimeters).
  - c. volume – cubes (cubic inches or cubic centimeters).

### *Use Measurement Techniques and Tools*

4. Develop and use strategies to find perimeter using string or links, area using tiles or a grid, and volume using cubes; e.g., count squares to find area of regular or irregular shapes on a grid, layer cubes in a box to find its volume.
5. Make simple unit conversions within a measurement system; e.g., inches to feet, kilograms to grams, quarts to gallons.
6. Write, solve and verify solutions to multi-step problems involving measurement.

## Grade Five

### *Measurement Units*

1. Identify and select appropriate units to measure angles; i.e., degrees.
2. Identify paths between points on a grid or coordinate plane and compare the lengths of the paths; e.g., shortest path, paths of equal length.



*Use Measurement  
Techniques and Tools*

3. Demonstrate and describe the differences between covering the faces (surface area) and filling the interior (volume) of three-dimensional objects.
4. Demonstrate understanding of the differences among linear units, square units and cubic units.
5. Make conversions within the same measurement system while performing computations.
6. Use strategies to develop formulas for determining perimeter and area of triangles, rectangles and parallelograms, and volume of rectangular prisms.
7. Use benchmark angles (e.g.;  $45^\circ$ ,  $90^\circ$ ,  $120^\circ$ ) to estimate the measure of angles, and use a tool to measure and draw angles.

## Grade Six

*Measurement Units*

*Use Measurement  
Techniques and Tools*

1. Understand and describe the difference between surface area and volume.
2. Use strategies to develop formulas for finding circumference and area of circles, and to determine the area of sectors; e.g.,  $\frac{1}{2}$  circle,  $\frac{2}{3}$  circle,  $\frac{1}{3}$  circle,  $\frac{1}{4}$  circle.
3. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:
  - a. estimating lengths using string or links, areas using tiles or grid, and volumes using cubes;
  - b. measuring attributes (diameter, side lengths, or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.
4. Determine which measure (perimeter, area, surface area, volume) matches the context for a problem situation; e.g., perimeter is the context for fencing a garden, surface area is the context for painting a room.

5. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.
6. Describe what happens to the perimeter and area of a two-dimensional shape when the measurements of the shape are changed; e.g. length of sides are doubled.

## Grade Seven

### *Measurement Units*

1. Select appropriate units for measuring derived measurements; e.g., miles per hour, revolutions per minute.
2. Convert units of area and volume within the same measurement system using proportional reasoning and a reference table when appropriate; e.g., square feet to square yards, cubic meters to cubic centimeters.

### *Use Measurement Techniques and Tools*

3. Estimate a measurement to a greater degree of precision than the tool provides.
4. Solve problems involving proportional relationships and scale factors; e.g., scale models that require unit conversions within the same measurement system.
5. Analyze problem situations involving measurement concepts, select appropriate strategies, and use an organized approach to solve narrative and increasingly complex problems.
6. Use strategies to develop formulas for finding area of trapezoids and volume of cylinders and prisms.
7. Develop strategies to find the area of composite shapes using the areas of triangles, parallelograms, circles and sectors.
8. Understand the difference between surface area and volume and demonstrate that two objects may have the same surface area, but different volumes or may have the same volume, but different surface areas.
4. Give examples of how the same absolute error can be

9. Describe what happens to the surface area and volume of a three-dimensional object when the measurements of the object are changed; e.g., length of sides are doubled.

## Grade Eight

### *Measurement Units*

1. Compare and order the relative size of common U.S. customary units and metric units; e.g., mile and kilometer, gallon and liter, pound and kilogram.
2. Use proportional relationships and formulas to convert units from one measurement system to another; e.g., degrees Fahrenheit to degrees Celsius.

### *Use Measurement Techniques and Tools*

3. Use appropriate levels of precision when calculating with measurements.
4. Derive formulas for surface area and volume and justify them using geometric models and common materials. For example, find:
  - a. the surface area of a cylinder as a function of its height and radius;
  - b. that the volume of a pyramid (or cone) is one-third of the volume of a prism (or cylinder) with the same base area and height.
5. Determine surface area for pyramids by analyzing their parts.
6. Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.
7. Apply proportional reasoning to solve problems involving indirect measurements or rates.
8. Find the sum of the interior and exterior angles of regular convex polygons with and without measuring the angles with a protractor.

9. Demonstrate understanding of the concepts of perimeter, circumference and area by using established formulas for triangles, quadrilaterals, and circles to determine the surface area and volume of prisms, pyramids, cylinders, spheres and cones. (Note: Only volume should be calculated for spheres and cones.)
10. Use conventional formulas to find the surface area and volume of prisms, pyramids and cylinders and the volume of spheres and cones to a specified level of precision.

## Grade Nine

### *Measurement Units*

1. Convert rates within the same measurement system; e.g., miles per hour to feet per second; kilometers per hour to meters per second.

### *Use Measurement Techniques and Tools*

2. Use unit analysis to check computations involving measurement.
3. Use the ratio of lengths in similar two-dimensional figures or three-dimensional objects to calculate the ratio of their areas or volumes respectively.
4. Use scale drawings and right triangle trigonometry to solve problems that include unknown distances and angle measures.
5. Solve problems involving unit conversion for situations involving distances, areas, volumes and rates within the same measurement system.

## Grade Ten

### *Use Measurement Techniques and Tools*

1. Explain how a small error in measurement may lead to a large error in calculated results.
2. Calculate relative error.
3. Explain the difference between absolute error and relative error in measurement.

problematic in one situation but not in another; e.g., compare “accurate to the nearest foot” when measuring the height of a person versus when measuring the height of a mountain.

5. Determine the measures of central and inscribed angles and their associated major and minor arcs.

## Grade Eleven

*Measurement Units*

1. Determine the number of significant digits in a measurement.
2. Use radian and degree angle measures to solve problems and perform conversions as needed.

*Use Measurement Techniques and Tools*

3. Derive a formula for the surface area of a cone as a function of its slant height and the circumference of its base.
4. Calculate distances, areas, surface areas and volumes of composite three-dimensional objects to a specified number of significant digits.
5. Solve real-world problems involving area, surface area, volume and density to a specified degree of precision.

## Grade Twelve

*Use Measurement Techniques and Tools*

1. Solve problems involving derived measurements; e.g., acceleration and pressure.
2. Use radian measures in the solution of problems involving angular velocity and acceleration.
3. Apply informal concepts of successive approximation, upper and lower bounds, and limits in measurement situations; e.g., measurement of some quantities, such as volume of a cone, can be determined by sequences of increasingly accurate approximations.

# K-12 Mathematics

## Grade-Level Indicators

### Geometry and Spatial Sense Standard

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two-, and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects and transformations to analyze mathematical situations and solve problems.

### Kindergarten

#### *Characteristics and Properties*

1. Identify and sort two-dimensional shapes and three-dimensional objects. For example:
  - a. Identify and describe two-dimensional figures and three-dimensional objects from the environment using the child's own vocabulary.
  - b. Sort shapes and objects into groups based on student-defined categories.
  - c. Select all shapes or objects of one type from a group.
  - d. Build two-dimensional figures using paper shapes or tangrams; build simple three-dimensional objects using blocks.

#### *Spatial Relationships*

2. Name and demonstrate the relative position of objects as follows:
  - a. place objects over, under, inside, outside, on, beside, between, above, below, on top of, upside-down, behind, in back of, in front of;
  - b. describe placement of objects with terms, such as on, inside, outside, above, below, over, under, beside, between, in front of, behind.



## Grade One

*Characteristics and Properties*

1. Identify, compare and sort two-dimensional shapes; i.e., square, circle, ellipse, triangle, rectangle, rhombus, trapezoid, parallelogram, pentagon and hexagon. For example:
  - a. Recognize and identify triangles and rhombuses independent of position, shape or size;
  - b. Describe two-dimensional shapes using attributes such as number of sides and number of vertices (corners or angles).
2. Create new shapes by combining or cutting apart existing shapes.
3. Identify the shapes of the faces of three-dimensional objects.

*Spatial Relationships*

4. Extend the use of location words to include distance (near, far, close to) and directional words (left, right).
5. Copy figures and draw simple two-dimensional shapes from memory.

## Grade Two

*Characteristics and Properties*

1. Identify, describe, compare and sort three-dimensional objects (i.e., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the number of faces, edges or vertices.
2. Predict what new shapes will be formed by combining or cutting apart existing shapes.
3. Recognize two-dimensional shapes and three-dimensional objects from different positions.

*Spatial Relationships*

4. Identify and determine whether two-dimensional shapes are congruent (same shape and size) or similar (same shape different size) by copying or using superposition (lay one thing on top of another).

*Transformations and Symmetry*

5. Create and identify two-dimensional figures with line symmetry; e.g., what letter shapes, logos, polygons are symmetrical?

### Grade Three

*Characteristics and Properties*

1. Analyze and describe properties of two-dimensional shapes and three-dimensional objects using terms such as vertex, edge, angle, side and face.
2. Identify and describe the relative size of angles with respect to right angles as follows:
  - a. Use physical models, like straws, to make different sized angles by opening and closing the sides, not by changing the side lengths.
  - b. Identify, classify and draw right, acute, obtuse and straight angles.

*Spatial Relationships*

3. Find and name locations on a labeled grid or coordinate system; e.g., a map or graph.

*Transformations and Symmetry*

4. Draw lines of symmetry to verify symmetrical two-dimensional shapes.

*Visualization and Geometric Models*

5. Build a three-dimensional model of an object composed of cubes; e.g., construct a model based on an illustration or actual object.

### Grade Four

*Characteristics and Properties*

1. Identify, describe and model intersecting, parallel and perpendicular lines and line segments; e.g., use straws or other material to model lines.
2. Describe, classify, compare and model two- and three-dimensional objects using their attributes.
3. Identify similarities and differences of quadrilaterals; e.g., squares, rectangles, parallelograms and trapezoids.



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|---|--|
| <i>Spatial Relationships</i>              | <ul style="list-style-type: none"> <li>4. Identify and define triangles based on angle measures (equiangular, right, acute and obtuse triangles) and side lengths (isosceles, equilateral and scalene triangles).</li> <li>5. Describe points, lines and planes, and identify models in the environment.</li> <li>6. Specify locations and plot ordered pairs on a coordinate plane, using first quadrant points.</li> </ul> |
| <i>Transformations and Symmetry</i>       | <ul style="list-style-type: none"> <li>7. Identify, describe and use reflections (flips), rotations (turns), and translations (slides) in solving geometric problems; e.g., use transformations to determine if 2 shapes are congruent.</li> </ul>   |
| <i>Visualization and Geometric Models</i> | <ul style="list-style-type: none"> <li>8. Use geometric models to solve problems in other areas of mathematics, such as number (multiplication/division) and measurement (area, perimeter, border).</li> </ul>   |

## Grade Five

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|---|--|
| <i>Characteristics and Properties</i>     | <ul style="list-style-type: none"> <li>1. Draw circles, and identify and determine relationships among the radius, diameter, center and circumference; e.g., radius is half the diameter, the ratio of the circumference of a circle to its diameter is an approximation of <math>\pi</math>.</li> <li>2. Use standard language to describe line, segment, ray, angle, skew, parallel and perpendicular.</li> <li>3. Label vertex, rays, interior and exterior for an angle.</li> <li>4. Describe and use properties of congruent figures to solve problems.</li> <li>5. Use physical models to determine the sum of the interior angles of triangles and quadrilaterals.</li> </ul> |
| <i>Spatial Relationships</i>              | <ul style="list-style-type: none"> <li>6. Extend understanding of coordinate system to include points whose <math>x</math> or <math>y</math> values may be negative numbers.</li> </ul>  |
| <i>Visualization and Geometric Models</i> | <ul style="list-style-type: none"> <li>7. Understand that the measure of an angle is determined by the degree of rotation of an angle side rather than the length of either side.</li> </ul>   |

8. Predict what three-dimensional object will result from folding a two-dimensional net, then confirm the prediction by folding the net.

## Grade Six

### *Characteristics and Properties*

1. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.
2. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse and other vocabulary as appropriate.
3. Use multiple classification criteria to classify triangles; e.g., right scalene triangle.
4. Identify and define relationships between planes; i.e., parallel, perpendicular and intersecting.

### *Spatial Relationships*

5. Predict and describe sizes, positions and orientations of two-dimensional shapes after transformations such as reflections, rotations, translations and dilations.

### *Transformations and Symmetry*

6. Draw similar figures that model proportional relationships; e.g., model similar figures with a 1 to 2 relationship by sketching two of the same figure, one with corresponding sides twice the length of the other.

### *Visualization and Geometric Models*

7. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side; i.e., projection sets.

## Grade Seven

### *Characteristics and Properties*

1. Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.
2. Determine sufficient (not necessarily minimal) properties that define a specific two-dimensional figure or three-dimensional object. For example:

- a. Determine when one set of figures is a subset of another; e.g., all squares are rectangles.
- b. Develop a set of properties that eliminates all but the desired figure; e.g., only squares are quadrilaterals with all sides congruent and all angles congruent.
- 3. Use and demonstrate understanding of the properties of triangles. For example:
  - a. Use Pythagorean Theorem to solve problems involving right triangles.
  - b. Use triangle angle sum relationships to solve problems.
- 4. Determine necessary conditions for congruence of triangles.
- 5. Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.
- Spatial Relationships* 6. Determine and use scale factors for similar figures to solve problems using proportional reasoning.
- Transformations and Symmetry* 7. Identify the line and rotation symmetries of two-dimensional figures to solve problems.
- 8. Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).
- Visualization and Geometric Models* 9. Draw representations of three-dimensional geometric objects from different views.

## Grade Eight

- Characteristics and Properties* 1. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.
- 2. Recognize the angles formed and the relationship between the angles when two lines intersect and when parallel lines are cut by a transversal.

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| <i>Spatial Relationships</i>              | 3. Use proportions in several forms to solve problems involving similar figures (part-to-part, part-to-whole, corresponding sides between figures).  |
| <i>Transformations and Symmetry</i>       | 4. Represent and analyze shapes using coordinate geometry; e.g., given three vertices and the type of quadrilateral, find the coordinates of the fourth vertex.  |
| <i>Visualization and Geometric Models</i> | 5. Draw the results of translations, reflections, rotations and dilations of objects in the coordinate plane, and determine properties that remain fixed; e.g., lengths of sides remain the same under translations. |
| <i>Visualization and Geometric Models</i> | 6. Draw nets for a variety of prisms, pyramids, cylinders and cones.   |

## Grade Nine

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|---|--|
| <i>Characteristics and Properties</i>     | 1. Define the basic trigonometric ratios in right triangles: sine, cosine and tangent.   |
| <i>Visualization and Geometric Models</i> | 2. Apply proportions and right triangle trigonometric ratios to solve problems involving missing lengths and angle measures in similar figures.  |
| <i>Visualization and Geometric Models</i> | 3. Analyze two-dimensional figures in a coordinate plane; e.g., use slope and distance formulas to show that a quadrilateral is a parallelogram. |

## Grade Ten

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|---------------------------------------|--|
| <i>Characteristics and Properties</i> | 1. Formally define and explain key aspects of geometric figures, including: <ol style="list-style-type: none"> <li>a. interior and exterior angles of polygons;</li> <li>b. segments related to triangles (median, altitude, midsegment);</li> <li>c. points of concurrency related to triangles (centroid, incenter, orthocenter, circumcenter);</li> </ol> |
|---------------------------------------|--|

- d. circles (radius, diameter, chord, circumference, major arc, minor arc, sector, segment, inscribed angle).
  - 2. Recognize and explain the necessity for certain terms to remain undefined, such as point, line and plane.
  - 3. Make, test and establish the validity of conjectures about geometric properties and relationships using counterexample, inductive and deductive reasoning, and paragraph or two-column proof, including:
    - a. prove the Pythagorean Theorem;
    - b. prove theorems involving triangle similarity and congruence;
    - c. prove theorems involving properties of lines, angles, triangles and quadrilaterals;
    - d. test a conjecture using basic constructions made with a compass and straightedge or technology.
- Spatial Relationships*
- 4. Construct right triangles, equilateral triangles, parallelograms, trapezoids, rectangles, rhombuses, squares and kites, using compass and straightedge or dynamic geometry software.
  - 5. Construct congruent figures and similar figures using tools, such as compass, straightedge, and protractor or dynamic geometry software.
- Transformation and Symmetry*
- 6. Identify the reflection and rotation symmetries of two- and three-dimensional figures.
  - 7. Perform reflections and rotations using compass and straightedge constructions and dynamic geometry software.
  - 8. Derive coordinate rules for translations, reflections and rotations of geometric figures in the coordinate plane.
  - 9. Show and describe the results of combinations of translations, reflections and rotations (compositions); e.g., perform compositions and specify the result of a composition as the outcome of a single motion, when applicable.
- Visualization and Geometric Models*
- 10. Solve problems involving chords, radii and arcs within the same circle.

## Grade Eleven

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|---|---|
| <i>Spatial Relationships</i>              | 1. Use polar coordinates to specify locations on a plane.   |
| <i>Transformations and Symmetry</i>       | 2. Represent translations using vectors.  |
|   | 3. Describe multiplication of a vector and a scalar graphically and algebraically, and apply to problem situations. |
|   | 4. Use trigonometric relationships to determine lengths and angle measures; i.e., Law of Sines and Law of Cosines.  |
| <i>Visualization and Geometric Models</i> | 5. Identify, sketch and classify the cross sections of three-dimensional objects.                                   |

## Grade Twelve

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| <i>Transformations and Symmetry</i>       | 1. Use matrices to represent translations, reflections, rotations, dilations and their compositions.                   |
|   | 2. Derive and apply the basic trigonometric identities; i.e., angle addition, angle subtraction and double angle.      |
| <i>Visualization and Geometric Models</i> | 3. Relate graphical and algebraic representations of lines, simple curves and conic sections.                          |
|   | 4. Recognize and compare specific shapes and properties in multiple geometries; e.g., plane, spherical and hyperbolic. |

# K-12 Mathematics

## Grade-Level Indicators



### Patterns, Functions and Algebra Standard

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

### Kindergarten

*Use Patterns, Relations and Functions*

1. Sort, classify and order objects by size, number and other properties. For example:
  - a. Identify how objects are alike and different.
  - b. Order three events or objects according to a given attribute, such as time or size.
  - c. Recognize and explain how objects can be classified in more than one way.
  - d. Identify what attribute was used to sort groups of objects that have already been sorted.
2. Identify, create, extend and copy sequences of sounds (such as musical notes), shapes (such as buttons, leaves or blocks), motions (such as hops or skips), and numbers from 1 to 10.
3. Describe orally the pattern of a given sequence.
4. Model a problem situation using physical materials.

*Use Algebraic Representations*

## Grade One

*Use Patterns, Relations  
and Functions*

1. Sort, classify and order objects by two or more attributes, such as color and shape, and explain how objects were sorted.
2. Extend sequences of sounds, shapes or simple number patterns, and create and record similar patterns. For example:
  - a. Analyze and describe patterns with multiple attributes using numbers and shapes; e.g., AA, B, aa, b, AA, B, aa, b,...
  - b. Continue repeating and growing patterns with materials, pictures and geometric items; e.g., XO, XOO, XOOO, XOOOO.
3. Describe orally the basic unit or general plan of a repeating or growing pattern.

*Use Algebraic  
Representations*

4. Solve open sentences by representing an expression in more than one way using the commutative property; e.g.,  $4 + 5 = 5 + 4$  or the number of blue balls plus red balls is the same as the number of red balls plus blue balls ( $R + B = B + R$ ).
5. Describe orally and model a problem situation using words, objects or number phrase or sentence.

## Grade Two

*Use Patterns, Relations  
and Functions*

1. Extend simple number patterns (both repeating and growing patterns), and create similar patterns using different objects, such as using physical materials or shapes to represent numerical patterns.
2. Use patterns to make generalizations and predictions; e.g., determine a missing element in a pattern.
3. Create new patterns with consistent rules or plans, and describe the rule or general plan of existing patterns.

*Use Algebraic  
Representations*

4. Use objects, pictures, numbers and other symbols to represent a problem situation.



5. Understand equivalence and extend the concept to situations involving symbols; e.g.,  $4 + 5 = 9$  and  $9 = 4 + 5$ , and  $4 + 5 = 3 + 6 = \triangle + \square \dots$
6. Use symbols to represent unknown quantities and identify values for symbols in an expression or equation using addition and subtraction; e.g.,  $\square + \circ = 10$ ,  $\triangle - 2 = 4$ .
7. Describe qualitative and quantitative changes, especially those involving addition and subtraction; e.g., a student growing taller versus a student growing two inches in one year.

*Analyze Change*

### Grade Three

*Use Patterns, Relations and Functions*

1. Extend multiplicative and growing patterns, and describe the pattern or rule in words.
2. Analyze and replicate arithmetic sequences with and without a calculator.
3. Use patterns to make predictions, identify relationships, and solve problems.

*Use Algebraic Representations*

4. Model problem situations using objects, pictures, tables, numbers, letters and other symbols.
5. Write, solve and explain simple mathematical statements, such as  $7 + \square > 8$  or  $\triangle + 8 = 10$ .
6. Express mathematical relationships as equations and inequalities.

*Analyze Change*

7. Create tables to record, organize and analyze data to discover patterns and rules.
8. Identify and describe quantitative changes, especially those involving addition and subtraction; e.g., the height of water in a glass becoming 1 centimeter lower each week due to evaporation.

## Grade Four

*Use Patterns, Relations  
and Functions*

1. Use models and words to describe, extend and make generalizations of patterns and relationships occurring in computation, numerical patterns, geometry, graphs and other applications.

2. Represent and analyze patterns and functions using words, tables and graphs.

*Use Algebraic  
Representation*

3. Construct a table of values to solve problems associated with a mathematical relationship.

4. Use rules and variables to describe patterns and other relationships.

5. Represent mathematical relationships with equations or inequalities.

*Analyze Change*

6. Describe how a change in one variable affects the value of a related variable; e.g., as one increases the other increases or as one increases the other decreases.

## Grade Five

*Use Patterns, Relations  
and Functions*

1. Justify a general rule for a pattern or a function by using physical materials, visual representations, words, tables or graphs.

2. Use calculators or computers to develop patterns, and generalize them using tables and graphs.

*Use Algebraic  
Representation*

3. Use variables as unknown quantities in general rules when describing patterns and other relationships.

4. Create and interpret the meaning of equations and inequalities representing problem situations.

5. Model problems with physical materials and visual representations, and use models, graphs and tables to draw conclusions and make predictions.

*Analyze Change*

6. Describe how the quantitative change in a variable affects the value of a related variable; e.g., describe how the rate of growth varies over time, based upon data in a table or graph.

## Grade Six

*Use Patterns, Relations and Functions*

1. Represent and analyze patterns, rules and functions, using physical materials, tables and graphs.
2. Use words and symbols to describe numerical and geometric patterns, rules and functions.

*Use Algebraic Representations*

3. Recognize and generate equivalent forms of algebraic expressions, and explain how the commutative, associative and distributive properties can be used to generate equivalent forms; e.g., perimeter as  $2(l + w)$  or  $2l + 2w$ .
4. Solve simple linear equations and inequalities using physical models, paper and pencil, tables and graphs.
5. Produce and interpret graphs that represent the relationship between two variables.
6. Evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations.

*Analyze Change*

7. Identify and describe situations with constant or varying rates of change, and compare them.
8. Use technology to analyze change; e.g., use computer applications or graphing calculators to display and interpret rate of change.

## Grade Seven

*Use Patterns, Relations and Functions*

1. Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions.
2. Generalize patterns by describing in words how to find the next term.
3. Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1,3,5,7... is linear and 1,3,4,8,16... is nonlinear.

*Use Algebraic Representations*

4. Create visual representations of equation-solving processes that model the use of inverse operations.

5. Represent linear equations by plotting points in the coordinate plane.
6. Represent inequalities on a number line or a coordinate plane.
7. Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g.,  $4m = m + m + m + m$  or  $a \cdot 5 + 4 = 5a + 4$ .
8. Use formulas in problem-solving situations.
9. Recognize a variety of uses for variables; e.g., placeholder for an unknown quantity in an equation, generalization for a pattern, formula.
- Analyze Change* 10. Analyze linear and simple nonlinear relationships to explain how a change in one variable results in the change of another.
11. Use graphing calculators or computers to analyze change; e.g., distance-time relationships.

## Grade Eight

*Use Patterns, Relations  
and Functions*

1. Relate the various representations of a relationship; i.e., relate a table to graph, description and symbolic form.
2. Generalize patterns and sequences by describing how to find the  $n$ th term.
3. Identify functions as linear or nonlinear based on information given in a table, graph or equation.

*Use Algebraic  
Representations*

4. Extend the uses of variables to include covariants where  $y$  depends on  $x$ .
5. Use physical models to add and subtract monomials and polynomials, and to multiply a polynomial by a monomial.
6. Describe the relationship between the graph of a line and its equation, including being able to explain the meaning of slope as a constant rate of change and  $y$ -intercept in real-world problems.

7. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.
8. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.
9. Solve linear equations and inequalities graphically, symbolically and using technology.
10. Solve 2 by 2 systems of linear equations graphically and by simple substitution.
11. Interpret the meaning of the solution of a 2 by 2 system of equations; i.e., point, line, no solution.
12. Solve simple quadratic equations graphically; e.g.,  $y = x^2 - 16$ .
13. Compute and interpret slope, midpoint and distance given a set of ordered pairs.

*Analyze Change*

14. Differentiate and explain types of changes in mathematical relationships, such as linear vs. nonlinear, continuous vs. noncontinuous, direct variation vs. inverse variation.
15. Describe and compare how changes in an equation affects the related graphs; e.g., for a linear equation changing the coefficient of  $x$  affects the slope and changing the constant affects the intercepts.
16. Use graphing calculators or computers to analyze change; e.g., interest compounded over time as a nonlinear growth pattern.

## Grade Nine

*Use Patterns, Relations and Functions*

1. Define function with ordered pairs in which each domain element is assigned exactly one range element.

2. Generalize patterns using functions or relationships (linear, quadratic and exponential), and freely translate among tabular, graphical and symbolic representations.
  3. Describe problem situations (linear, quadratic and exponential) by using tabular, graphical and symbolic representations.
  4. Demonstrate the relationship among zeros of a function, roots of equations, and solutions of equations graphically and in words.
  5. Describe and compare characteristics of the following families of functions: linear, quadratic and exponential functions; e.g., general shape, number of roots, domain, range, rate of change, maximum or minimum.
- Use Algebraic Representations*
6. Write and use equivalent forms of equations and inequalities in problem situations; e.g., changing a linear equation to the slope-intercept form.
  7. Use formulas to solve problems involving exponential growth and decay.
  8. Find linear equations that represent lines that pass through a given set of ordered pairs, and find linear equations that represent lines parallel or perpendicular to a given line through a specific point.
  9. Solve and interpret the meaning of 2 by 2 systems of linear equations graphically, by substitution and by elimination, with and without technology.
  10. Solve quadratic equations with real roots by factoring, graphing, using the quadratic formula and with technology.
  11. Add, subtract, multiply and divide monomials and polynomials (division of polynomials by monomials only).
  12. Simplify rational expressions by eliminating common factors and applying properties of integer exponents.
- Analyze Change*
13. Model and solve problems involving direct and inverse variation using proportional reasoning.

14. Describe the relationship between slope and the graph of a direct variation and inverse variation.
15. Describe how a change in the value of a constant in a linear or quadratic equation affects the related graphs.

## Grade Ten

*Use Patterns, Relations  
and Functions*

1. Define function formally and with  $f(x)$  notation.
2. Describe and compare characteristics of the following families of functions: square root, cubic, absolute value and basic trigonometric functions; e.g., general shape, possible number of roots, domain and range.

*Use Algebraic  
Representations*

3. Solve equations and formulas for a specified variable; e.g., express the base of a triangle in terms of the area and height.
4. Use algebraic representations and functions to describe and generalize geometric properties and relationships.
5. Solve simple linear and nonlinear equations and inequalities having square roots as coefficients and solutions.
6. Solve equations and inequalities having rational expressions as coefficients and solutions.
7. Solve systems of linear inequalities.
8. Graph the quadratic relationship that defines circles.
9. Recognize and explain that the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals.
10. Solve real-world problems that can be modeled using linear, quadratic, exponential or square root functions.
11. Solve real-world problems that can be modeled, using systems of linear equations and inequalities.

*Analyze Change*

12. Describe the relationship between slope of a line through the origin and the tangent function of the angle created by the line and the positive  $x$ -axis.

## Grade Eleven

*Use Patterns, Relations  
and Functions*

1. Identify and describe problem situations involving an iterative process that can be represented as a recursive function; e.g., compound interest.
2. Translate a recursive function into a closed form expression or formula for the  $n$ th term to solve a problem situation involving an iterative process; e.g., find the value of an annuity after 7 years.
3. Describe and compare the characteristics of the following families of functions: quadratics with complex roots, polynomials of any degree, logarithms, and rational functions; e.g., general shape, number of roots, domain and range, asymptotic behavior.
4. Identify the maximum and minimum points of polynomial, rational and trigonometric functions graphically and with technology.
5. Identify families of functions with graphs that have rotation symmetry or reflection symmetry about the  $y$ -axis,  $x$ -axis or  $y = x$ .

*Use Algebraic  
Representations*

6. Represent the inverse of a function symbolically and graphically as a reflection about  $y = x$ .
7. Model and solve problems with matrices and vectors.
8. Solve equations involving radical expressions and complex roots.
9. Solve 3 by 3 systems of linear equations by elimination and using technology, and interpret graphically what the solution means (a point, line, plane, or no solution).
10. Describe the characteristics of the graphs of conic sections.

*Analyze Change*

11. Describe how a change in the value of a constant in an exponential, logarithmic or radical equation affects the graph of the equation.



## Grade Twelve

*Use Patterns, Relations  
and Functions*

1. Analyze the behavior of arithmetic and geometric sequences and series as the number of terms increases.
2. Translate between the numeric and symbolic form of a sequence or series.
3. Describe and compare the characteristics of transcendental and periodic functions; e.g., general shape, number of roots, domain and range, asymptotic behavior, extrema, local and global behavior.
4. Represent the inverse of a transcendental function symbolically.

*Use Algebraic  
Representations*

5. Set up and solve systems of equations using matrices and graphs, with and without technology.
6. Make arguments about mathematical properties using mathematical induction.
7. Make mathematical arguments using the concepts of limit.
8. Compare estimates of the area under a curve over a bounded interval by partitioning the region with rectangles; e.g., make successive estimates using progressively smaller rectangles.
9. Translate freely between polar and Cartesian coordinate systems.

*Analyze Change*

10. Use the concept of limit to find instantaneous rate of change for a point on a graph as the slope of a tangent at a point.

# K-12 Mathematics

## Grade-Level Indicators



### Data Analysis and Probability Standard

Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

### Kindergarten

#### *Data Collection*

1. Gather and sort data in response to questions posed by teacher and students; e.g., how many sisters and brothers, what color shoes.
2. Arrange objects in a floor or table graph according to attributes, such as use, size, color or shape.

#### *Statistical Methods*

3. Select the category or categories that have the most or fewest objects in a floor or table graph.

### Grade One

#### *Data Collection*

1. Identify multiple categories for sorting data.
2. Collect and organize data into charts using tally marks.
3. Display data in picture graphs with units of 1 and bar graphs with intervals of 1.
4. Read and interpret charts, picture graphs and bar graphs as sources of information to identify main ideas, draw conclusions, and make predictions.

- 5. Construct a question that can be answered by using information from a graph.
- Statistical Methods*
  - 6. Arrange five objects by an attribute, such as size or weight, and identify the ordinal position of each object.
  - 7. Answer questions about the number of objects represented in a picture graph, bar graph or table graph; e.g., category with most, how many more in a category compared to another, how many altogether in two categories.
- Probability*
  - 8. Describe the likelihood of simple events as possible/impossible and more likely/less likely; e.g., when using spinners or number cubes in classroom activities.

## Grade Two

- Data Collection*
  - 1. Pose questions, use observations, interviews and surveys to collect data, and organize data in charts, picture graphs and bar graphs.
  - 2. Read, interpret and make comparisons and predictions from data represented in charts, line plots, picture graphs and bar graphs.
  - 3. Read and construct simple timelines to sequence events.
- Statistical Methods*
  - 4. Write a few sentences to describe and compare categories of data represented in a chart or graph, and make statements about the data as a whole.
  - 5. Identify untrue or inappropriate statements about a given set of data.
  - 6. Recognize that data may vary from one population to another; e.g., favorite TV shows of students and of parents.
- Probability*
  - 7. List some of the possible outcomes of a simple experiment, and predict whether given outcomes are more, less or equally likely to occur.
  - 8. Use physical models and pictures to represent possible arrangements of 2 or 3 objects.

## Grade Three

### *Data Collection*

1. Collect and organize data from an experiment, such as recording and classifying observations or measurements, in response to a question posed.
2. Draw and interpret picture graphs in which a symbol or picture represents more than one object.
3. Read, interpret and construct bar graphs with intervals greater than one.
4. Support a conclusion or prediction orally and in writing, using information in a table or graph.
5. Match a set of data with a graphical representation of the data.
6. Translate information freely among charts, tables, line plots, picture graphs and bar graphs; e.g., create a bar graph from the information in a chart.
7. Analyze and interpret information represented on a timeline.

### *Statistical Methods*

8. Identify the mode of a data set and describe the information it gives about a data set.

### *Probability*

9. Conduct a simple experiment or simulation of a simple event, record the results in a chart, table or graph, and use the results to draw conclusions about the likelihood of possible outcomes.
10. Use physical models, pictures, diagrams and lists to solve problems involving possible arrangements or combinations of two to four objects.

## Grade Four

### *Data Collection*

1. Create a plan for collecting data for a specific purpose.
2. Represent and interpret data using tables, bar graphs, line plots and line graphs.

3. Interpret and construct Venn diagrams to sort and describe data.
4. Compare different representations of the same data to evaluate how well each representation shows important aspects of the data, and identify appropriate ways to display the data.
5. Propose and explain interpretations and predictions based on data displayed in tables, charts and graphs.
6. Describe the characteristics of a set of data based on a graphical representation, such as range of the data, clumps of data, and holes in the data.
7. Identify the median of a set of data and describe what it indicates about the data.
8. Use range, median and mode to make comparisons among related sets of data.

*Statistical Methods*

9. Conduct simple probability experiments and draw conclusions from the results; e.g., rolling number cubes or drawing marbles from a bag.
10. Represent the likelihood of possible outcomes for chance situations; e.g., probability of selecting a red marble from a bag containing 3 red and 5 white marbles.
11. Relate the concepts of impossible and certain-to-happen events to the numerical values of 0 (impossible) and 1 (certain).
12. Place events in order of likelihood and use a diagram or appropriate language to compare the chance of each event occurring; e.g., impossible, unlikely, equal, likely, certain.
13. List and count all possible combinations using one member from each of several sets, each containing 2 or 3 members; e.g., the number of possible outfits from 3 shirts, 2 shorts and 2 pairs of shoes.

*Probability*

## Grade Five

### *Data Collection*

1. Read, construct and interpret frequency tables, circle graphs and line graphs.
2. Select and use a graph that is appropriate for the type of data to be displayed; e.g., numerical vs. categorical data, discrete vs. continuous data.
3. Read and interpret increasingly complex displays of data, such as double bar graphs.
4. Determine appropriate data to be collected to answer questions posed by students or teacher, collect and display data, and clearly communicate findings.
5. Modify initial conclusions, propose and justify new interpretations and predictions as additional data are collected.

### *Statistical Methods*

6. Determine and use the range, mean, median and mode, and explain what each does and does not indicate about the set of data.

### *Probability*

7. List and explain all possible outcomes in a given situation.
8. Identify the probability of events within a simple experiment, such as three chances out of eight.
9. Use 0, 1 and ratios between 0 and 1 to represent the probability of outcomes for an event, and associate the ratio with the likelihood of the outcome.
10. Compare what should happen (theoretical/expected results) with what did happen (experimental/actual results) in a simple experiment.
11. Make predictions based on experimental and theoretical probabilities.

## Grade Six

### *Data Collection*

1. Read, construct and interpret line graphs, circle graphs and histograms.

- 2. Select, create and use graphical representations that are appropriate for the type of data collected.
- 3. Compare representations of the same data in different types of graphs, such as a bar graph and circle graph.
- Statistical Methods* 4. Understand the different information provided by measures of center (mean, mode and median) and measures of spread (range).
- 5. Describe the frequency distribution of a set of data, as shown in a histogram or frequency table, by general appearance or shape; e.g., number of modes, middle of data, level of symmetry, outliers.
- 6. Make logical inferences from statistical data.
- Probability* 7. Design an experiment to test a theoretical probability and explain how the results may vary.

## Grade Seven

- Data Collection* 1. Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs, when appropriate.
- 2. Analyze how decisions about graphing affect the graphical representation; e.g., scale, size of classes in a histogram, number of categories in a circle graph.
- Statistical Methods* 3. Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range), and describe how the inclusion or exclusion of outliers affects those measures.
- 4. Construct opposing arguments based on analysis of the same data, using different graphical representations.
- 5. Compare data from two or more samples to determine how sample selection can influence results.
- 6. Identify misuses of statistical data in articles, advertisements, and other media.

- Probability*
7. Compute probabilities of compound events; e.g., multiple coin tosses or multiple rolls of number cubes, using such methods as organized lists, tree diagrams and area models.
  8. Make predictions based on theoretical probabilities, design and conduct an experiment to test the predictions, compare actual results to predicted results, and explain differences.

## Grade Eight

*Data Collection*

1. Use, create and interpret scatterplots and other types of graphs as appropriate.
2. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose; e.g., line graph for change over time, circle graph for part-to-whole comparison, scatterplot for relationship between two variants.
3. Differentiate between discrete and continuous data and appropriate ways to represent each.

*Statistical Methods*

4. Compare two sets of data using measures of center (mean, mode, median) and measures of spread (range, quartiles, interquartile range, percentiles).
5. Explain the mean's sensitivity to extremes and its use in comparison with the median and mode.
6. Make conjectures about possible relationship in a scatterplot and approximate line of best fit.
7. Identify different ways of selecting samples, such as survey response, random sample, representative sample and convenience sample.
8. Describe how the relative size of a sample compared to the target population affects the validity of predictions.
9. Construct convincing arguments based on analysis of data and interpretation of graphs.



- Probability*
10. Calculate the number of possible outcomes for a situation, recognizing and accounting for when items may occur more than once or when order is important.
  11. Demonstrate an understanding that the probability of either of two disjoint events occurring can be found by adding the probabilities for each and that the probability of one independent event following another can be found by multiplying the probabilities.

## Grade Nine

- Data Collection*
1. Classify data as univariate (single variable) or bivariate (two variables) and as quantitative (measurement) or qualitative (categorical) data.
  2. Create a scatterplot for a set of bivariate data, sketch the line of best fit, and interpret the slope of the line of best fit.

- Statistical Methods*
3. Analyze and interpret frequency distributions based on spread, symmetry, skewness, clusters and outliers.
  4. Describe and compare various types of studies (survey, observation, experiment), and identify possible misuses of statistical data.
  5. Describe characteristics and limitations of sampling methods, and analyze the effects of random versus biased sampling; e.g., determine and justify whether the sample is likely to be representative of the population.
  6. Make inferences about relationships in bivariate data, and recognize the difference between evidence of relationship (correlation) and causation.

- Probability*
7. Use counting techniques and the Fundamental Counting principle to determine the total number of possible outcomes for mathematical situations.
  8. Describe, create and analyze a sample space and use it to calculate probability.

9. Identify situations involving independent and dependent events, and explain differences between, and common misconceptions about, probabilities associated with those events.
10. Use theoretical and experimental probability, including simulations or random numbers, to estimate probabilities and to solve problems dealing with uncertainty; e.g., compound events, independent events, simple dependent events.

## Grade Ten

### *Data Collection*

1. Describe measures of center and the range verbally, graphically and algebraically.
2. Represent and analyze bivariate data using appropriate graphical displays (scatterplots, parallel box-and-whisker plots, histograms with more than one set of data, tables, charts, spreadsheets) with and without technology.
3. Display bivariate data where at least one variable is categorical.
4. Identify outliers on a data display; e.g., use interquartile range to identify outliers on a box-and-whisker plot.

### *Statistical Methods*

5. Provide examples and explain how a statistic may or may not be an attribute of the entire population; e.g., intentional or unintentional bias may be present.
6. Interpret the relationship between two variables using multiple graphical displays and statistical measures; e.g., scatterplots, parallel box-and-whisker plots, and measures of center and spread.

### *Probability*

7. Model problems dealing with uncertainty with area models (geometric probability).
8. Differentiate and explain the relationship between the probability of an event and the odds of an event, and compute one given the other.

## Grade Eleven

### *Data Collection*

1. Design a statistical experiment, survey or study for a problem; collect data for the problem; and interpret the data with appropriate graphical displays, descriptive statistics, concepts of variability, causation, correlation and standard deviation.
2. Describe the role of randomization in a well-designed study, especially as compared to a convenience sample, and the generalization of results from each.

### *Statistical Methods*

3. Describe how a linear transformation of univariate data affects range, mean, mode and median.
4. Create a scatterplot of bivariate data, identify trends, and find a function to model the data.
5. Use technology to find the Least Squares Regression Line, the regression coefficient, and the correlation coefficient for bivariate data with a linear trend, and interpret each of these statistics in the context of the problem situation.
6. Use technology to compute the standard deviation for a set of data, and interpret standard deviation in relation to the context or problem situation.
7. Describe the standard normal curve and its general properties, and answer questions dealing with data assumed to be normal.
8. Analyze and interpret univariate and bivariate data to identify patterns, note trends, draw conclusions, and make predictions.
9. Evaluate validity of results of a study based on characteristics of the study design, including sampling method, summary statistics and data analysis techniques.

### *Probability*

10. Understand and use the concept of random variable, and compute and interpret the expected value for a random variable in simple cases.
11. Examine statements and decisions involving risk; e.g., insurance rates and medical decisions.

## Grade Twelve

*Data Collection*

1. Identify and use various sampling methods (voluntary response, convenience sample, random sample, stratified random sample, census) in a study.

*Statistical Methods*

2. Transform bivariate data so it can be modeled by a function; e.g., use logarithms to allow nonlinear relationship to be modeled by linear function.
3. Describe the shape and find all summary statistics for a set of univariate data, and describe how a linear transformation affects shape, center and spread.
4. Apply the concept of a random variable to generate and interpret probability distributions, including binomial, normal and uniform.
5. Use sampling distributions as the basis for informal inference.

*Probability*

6. Use theoretical or experimental probability, including simulations, to determine probabilities in real-world problem situations involving uncertainty, such as mutually exclusive events, complementary events, and conditional probability.

*Note: The benchmarks for mathematical processes articulate what students should demonstrate in problem solving, representation, communication, reasoning and connections at key points in their mathematics program. Specific grade-level indicators have not been included for the mathematical processes standard because content and processes should be interconnected at the indicator level. Therefore, mathematical processes have been embedded within the grade-level indicators for the five content standards.*

## K-12 Mathematics

# Benchmarks and Indicators by Grade-Level



## K-12 Mathematics Benchmarks

By the end of the K-2 program:

K

1

2

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>A. Use place value concepts to represent whole numbers using numerals, words and physical models.</p> <p>B. Recognize, classify, compare and order whole numbers.</p> <p>C. Represent commonly used fractions using words and physical models.</p> <p>D. Determine the value of a collection of coins and dollar bills.</p> <p>E. Make change using coins for values up to one dollar.</p> <p>F. Count, using numerals and ordinal numbers.</p> <p>G. Model, represent and explain addition as combining sets and counting on.</p> <p>H. Model, represent and explain subtraction as comparison, take-away and part-to-whole.</p>	<p>A. Explain the need for standard units of measure.</p> <p>B. Select appropriate units for length, weight, volume (capacity) and time, using:</p> <ul style="list-style-type: none"> <li>• objects; i.e., non-standard units;</li> <li>• U.S. customary units: inch, foot, yard, ounce, pound, cup, quart, gallon, minute, hour, day, week and year;</li> <li>• metric units: centimeter, meter, gram and liter.</li> </ul> <p>C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.</p> <p>D. Apply measurement techniques to measure length, weight and volume (capacity).</p> <p>E. Recognize that using different units of measurement will yield different numbers for the same measurement.</p>	<p>A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment.</p> <p>B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone and pyramid, and identify them in the environment.</p> <p>C. Sort and compare two-dimensional figures and three-dimensional objects according to their characteristics and properties.</p> <p>D. Identify, explain and model (superposition, copying) the concept of shapes being congruent and similar.</p> <p>E. Recognize two- and three-dimensional objects from different positions.</p> <p>F. Describe location, using comparative (before, after), directional (above, below), and positional (first, last) words.</p> <p>G. Identify and draw figures with line symmetry.</p>

## K-12 Mathematics Benchmarks

By the end of the K-2 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
<ul style="list-style-type: none"> <li>A. Sort, classify and order objects by size, number and other properties, and describe the attributes used.</li> <li>B. Extend sequences of sounds and shapes or simple number patterns, and create and record similar patterns.</li> <li>C. Create and extend patterns, and describe the rule in words.</li> <li>D. Model problem situations, using objects, pictures, numbers and other symbols.</li> <li>E. Solve open sentences and explain strategies.</li> <li>F. Represent an unknown quantity as a variable using a symbol, such as <math>\square</math>, <math>\triangle</math>, <math>\circ</math>.</li> <li>G. Describe and compare qualitative and quantitative changes.</li> </ul>	<ul style="list-style-type: none"> <li>A. Pose questions and gather data about everyday situations and familiar objects.</li> <li>B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart.</li> <li>C. Represent data using objects, picture graphs and bar graphs.</li> <li>D. Describe the probability of chance events as more, less or equally likely to occur.</li> </ul>	<ul style="list-style-type: none"> <li>A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.</li> <li>B. Identify and restate in own words the question or problem and the information needed to solve the problem.</li> <li>C. Generate alternative strategies to solve problems.</li> <li>D. Evaluate the reasonableness of predictions, estimations and solutions.</li> <li>E. Explain to others how a problem was solved.</li> <li>F. Draw pictures and use physical models to represent problem situations and solutions.</li> <li>G. Use invented and conventional symbols and common language to describe a problem situation and solution.</li> </ul>



**K-12 Mathematics Benchmarks (continued)**

By the end of the K-2 program:

K

1

2

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>I. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.</p> <p>J. Model, represent and explain division as sharing equally, repeated subtraction and rectangular arrays.</p> <p>K. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions.</p> <p>L. Demonstrate fluency in adding and subtracting multiples of 10, and recognize combinations that make 10.</p> <p>M. Add and subtract two-digit numbers with and without regrouping.</p>		



**K-12 Mathematics Benchmarks (continued)**

By the end of the K-2 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
		<p>H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.</p> <p>I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.</p>

K

1

2

## K-12 Mathematics

K



## Kindergarten

## Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Compare and order whole numbers up to 10.
2. Explain rules of counting, such as each object should be counted once and that order does not change the number.
3. Count to twenty; e.g., in play situations or while reading number books.
4. Determine “how many” in sets (groups) of 10 or fewer objects.
5. Relate, read and write numerals for single-digit numbers (0 to 9).
6. Construct multiple sets of objects each containing the same number of objects.
7. Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects.
8. Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.
9. Identify and state the value of a penny, nickel and dime.
10. Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
  - a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.

*Meaning of  
Operations*

Kindergarten



- Computation and Estimation*
- b. Count on (forward) and count back (backward) on a number line between 0 and 10.
  - 11. Demonstrate joining multiple groups of objects, each containing the same number of objects; e.g., combining 3 bags of candy, each containing 2 pieces.
  - 12. Partition or share a small set of objects into groups of equal size; e.g., sharing 6 stickers equally among 3 children.
  - 13. Recognize the number or quantity of sets up to 5 without counting; e.g., recognize without counting the dot arrangement on a domino as 5.

### Measurement Standard

- Measurement Units*
- 1. Identify units of time (day, week, month, year) and compare calendar elements; e.g., weeks are longer than days.
- Use Measurement Techniques and Tools*
- 2. Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less.
  - 3. Measure length and volume (capacity) using uniform objects in the environment. For example, find:
    - a. how many paper clips long is a pencil;
    - b. how many small containers it takes to fill one big container using sand, rice, beans.
  - 4. Order events based on time. For example:
    - a. activities that take a long or short time;
    - b. review what we do first, next, last;
    - c. recall what we did or plan to do yesterday, today, tomorrow.

## Geometry and Spatial Sense Standard

K

*Characteristics and Properties*

1. Identify and sort two-dimensional shapes and three-dimensional objects. For example:
  - a. Identify and describe two-dimensional figures and three-dimensional objects from the environment using the child's own vocabulary.
  - b. Sort shapes and objects into groups based on student-defined categories.
  - c. Select all shapes or objects of one type from a group.
  - d. Build two-dimensional figures using paper shapes or tangrams; build simple three-dimensional objects using blocks.

*Spatial Relationships*

2. Name and demonstrate the relative position of objects as follows:
  - a. place objects over, under, inside, outside, on, beside, between, above, below, on top of, upside-down, behind, in back of, in front of;
  - b. describe placement of objects with terms, such as on, inside, outside, above, below, over, under, beside, between, in front of, behind.

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Sort, classify and order objects by size, number and other properties. For example:
  - a. Identify how objects are alike and different.
  - b. Order three events or objects according to a given attribute, such as time or size.
  - c. Recognize and explain how objects can be classified in more than one way.
  - d. Identify what attribute was used to sort groups of objects that have already been sorted.
2. Identify, create, extend and copy sequences of sounds (such as musical notes), shapes (such as buttons, leaves or blocks), motions (such as hops or skips), and numbers from 1 to 10.

Kindergarten

*Use Algebraic  
Representations*

3. Describe orally the pattern of a given sequence.
4. Model a problem situation using physical materials.



### **Data Analysis and Probability Standard**

*Data Collection*

1. Gather and sort data in response to questions posed by teacher and students; e.g., how many sisters and brothers, what color shoes.
2. Arrange objects in a floor or table graph according to attributes, such as use, size, color or shape.

*Statistical Methods*

3. Select the category or categories that have the most or fewest objects in a floor or table graph.

## Grade One

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Use ordinal numbers to order objects; e.g., first, second, third.
2. Recognize and generate equivalent forms for the same number using physical models, words and number expressions; e.g., concept of ten is described by “10 blocks,” full tens frame, numeral 10,  $5 + 5$ ,  $15 - 5$ , one less than 11, my brother’s age.
3. Read and write the numerals for numbers to 100.
4. Count forward to 100, count backwards from 100, and count or backward starting at any number between 1 and 100.
5. Use place value concepts to represent whole numbers using numerals, words, expanded notation and physical models with ones and tens. For example:
  - a. Develop a system to group and count by twos, fives and tens.
  - b. Identify patterns and groupings in a 100's chart and relate to place value concepts.
  - c. Recognize the first digit of a two-digit number as the most important to indicate size of a number and the nearness to 10 or 100.
6. Identify and state the value of a penny, nickel, dime, quarter and dollar.
7. Determine the value of a small collection of coins (with a total value up to one dollar) using 1 or 2 different type coins, including pennies, nickels, dimes and quarters.
8. Show different combinations of coins that have the same value.
9. Represent commonly used fractions using words and physical models for halves, thirds and fourths, recognizing fractions are represented by equal size parts of a whole and of a set of objects.
10. Model, represent and explain addition as combining sets (part + part = whole) and counting on. For example:
  - a. Model and explain addition using physical materials in contextual situations.
  - b. Draw pictures to model addition.

*Meaning of  
Operations*

- c. Write number sentences to represent addition.
  - d. Explain that adding two whole numbers yields a larger whole number.
11. Model, represent and explain subtraction as take-away and comparison. For example:
    - a. Model and explain subtraction using physical materials in contextual situations.
    - b. Draw pictures to model subtraction.
    - c. Write number sentences to represent subtraction.
    - d. Explain that subtraction of whole numbers yields an answer smaller than the original number.
  12. Use conventional symbols to represent the operations of addition and subtraction.
  13. Model and represent multiplication as repeated addition and rectangular arrays in contextual situations; e.g., four people will be at my party and if I want to give 3 balloons to each person, how many balloons will I need to buy?
  14. Model and represent division as sharing equally in contextual situations; e.g., sharing cookies.
  15. Demonstrate that equal means “the same as” using visual representations.
  16. Develop strategies for basic addition facts, such as:
    - a. counting all;
    - b. counting on;
    - c. one more, two more;
    - d. doubles;
    - e. doubles plus or minus one;
    - f. make ten;
    - g. using tens frames;
    - h. identity property (adding zero).

*Computation and Estimation*



17. Develop strategies for basic subtraction facts, such as:
  - a. relating to addition (for example, think of  $7 - 3 = ?$  as “3 plus ? equals 7”);
  - b. one less, two less;
  - c. all but one (for example,  $8 - 7, 5 - 4$ );
  - d. using tens frames;
  - e. missing addends.

### Measurement Standard

*Measurement Units*

1. Recognize and explain the need for fixed units and tools for measuring length and weight; e.g., rulers and balance scales.
2. Tell time to the hour and half hour on digital and analog (dial) timepieces.
3. Order a sequence of events with respect to time; e.g., summer, fall, winter and spring; morning, afternoon and night.

*Use Measurement Techniques and Tools*

4. Estimate and measure weight using non-standard units; e.g., blocks of uniform size.
5. Estimate and measure lengths using non-standard and standard units; i.e., centimeters, inches and feet.

### Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Identify, compare and sort two-dimensional shapes; i.e., square, circle, ellipse, triangle, rectangle, rhombus, trapezoid, parallelogram, pentagon and hexagon. For example:
  - a. Recognize and identify triangles and rhombuses independent of position, shape or size;
  - b. Describe two-dimensional shapes using attributes such as number of sides and number of vertices (corners or angles).
2. Create new shapes by combining or cutting apart existing shapes.
3. Identify the shapes of the faces of three-dimensional objects.



*Spatial Relationships*

4. Extend the use of location words to include distance (near, far, close to) and directional words (left, right).
5. Copy figures and draw simple two-dimensional shapes from memory.

## Patterns, Functions and Algebra Standard

*Use Patterns,  
Relations  
and Functions*

1. Sort, classify and order objects by two or more attributes, such as color and shape, and explain how objects were sorted.
2. Extend sequences of sounds, shapes or simple number patterns, and create and record similar patterns. For example:
  - a. Analyze and describe patterns with multiple attributes using numbers and shapes; e.g., AA, B, aa, b, AA, B, aa, b,...
  - b. Continue repeating and growing patterns with materials, pictures and geometric items; e.g., XO, XOO, XOOO, XOOOO.
3. Describe orally the basic unit or general plan of a repeating or growing pattern.

*Use Algebraic  
Representations*

4. Solve open sentences by representing an expression in more than one way using the commutative property; e.g.,  $4 + 5 = 5 + 4$  or the number of blue balls plus red balls is the same as the number of red balls plus blue balls ( $R + B = B + R$ ).
5. Describe orally and model a problem situation using words, objects or number phrase or sentence.

## Data Analysis and Probability Standard

*Data Collection*

1. Identify multiple categories for sorting data.
2. Collect and organize data into charts using tally marks.
3. Display data in picture graphs with units of 1 and bar graphs with intervals of 1.
4. Read and interpret charts, picture graphs and bar graphs as sources of information to identify main ideas, draw conclusions, and make predictions.



- Statistical Methods*
5. Construct a question that can be answered by using information from a graph.
  6. Arrange five objects by an attribute, such as size or weight, and identify the ordinal position of each object.
  7. Answer questions about the number of objects represented in a picture graph, bar graph or table graph; e.g., category with most, how many more in a category compared to another, how many altogether in two categories.
- Probability*
8. Describe the likelihood of simple events as possible/impossible and more likely/less likely; e.g., when using spinners or number cubes in classroom activities.

## Grade Two

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Use place value concepts to represent, compare and order whole numbers using physical models, numerals and words, with ones, tens and hundreds. For example:
  - a. Recognize 10 can mean “10 ones” or a single entity (1 ten) through physical models and trading games.
  - b. Read and write 3-digit numerals (e.g., 243 as two hundred forty three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each.
2. Recognize and classify numbers as even or odd.
3. Count money and make change using coins and a dollar bill.
4. Represent and write the value of money using the ¢ sign and in decimal form when using the \$ sign.
5. Represent fractions (halves, thirds, fourths, sixths and eighths), using words, numerals and physical models. For example:
  - a. Recognize that a fractional part can mean different amounts depending on the original quantity.
  - b. Recognize that a fractional part of a rectangle does not have to be shaded with contiguous parts.
  - c. Identify and illustrate parts of a whole and parts of sets of objects.
  - d. Compare and order physical models of halves, thirds and fourths in relation to 0 and 1.

*Meaning of  
Operations*

6. Model, represent and explain subtraction as comparison, take-away and part-to-whole; e.g., solve missing addend problems by counting up or subtracting, such as “I had six baseball cards, my sister gave me more, and I now have ten. How many did she give me?” can be represented as  $6 + ? = 10$  or  $10 - 6 = ?$ .

*Computation and Estimation*

7. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.
8. Model, represent and explain division as sharing equally and repeated subtraction.
9. Model and use the commutative property for addition.
10. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions; e.g.,  $9 + 9 = 18$ ,  $18 - 9 = 9$ .
11. Add and subtract multiples of 10.
12. Demonstrate multiple strategies for adding and subtracting 2- or 3-digit whole numbers, such as:
  - a. compatible numbers;
  - b. compensatory numbers;
  - c. informal use of commutative and associative properties of addition.
13. Estimate the results of whole number addition and subtraction problems using front-end estimation, and judge the reasonableness of the answers.

### Measurement Standard

*Measurement Units*

1. Identify and select appropriate units of measure for:
  - a. length – centimeters, meters, inches, feet or yards;
  - b. volume (capacity) – liters, cups, pints or quarts;
  - c. weight – grams, ounces or pounds;
  - d. time – hours, half-hours, quarter-hours or minutes and time designations, a.m. or p.m.
2. Establish personal or common referents for units of measure to make estimates and comparisons; e.g., the width of a finger is a centimeter, a large bottle of soda pop is 2 liters, a small paper clip weighs about one gram.

*Use Measurement  
Techniques and  
Tools*

3. Describe and compare the relationships among units of measure, such as centimeters and meters; inches, feet and yards; cups, pints and quarts; ounces and pounds; and hours, half-hours, and quarter-hours; e.g., how many inches in a foot?
4. Tell time to the nearest minute interval on digital and to the nearest 5 minute interval on analog (dial) timepieces.
5. Estimate and measure the length and weight of common objects, using metric and U.S. customary units, accurate to the nearest unit.
6. Select and use appropriate measurement tools; e.g., a ruler to draw a segment 3 inches long, a measuring cup to place 2 cups of rice in a bowl, a scale to weigh 50 grams of candy.
7. Make and test predictions about measurements, using different units to measure the same length or volume.

2

### Geometry and Spatial Sense Standard

*Characteristics and  
Properties*

1. Identify, describe, compare and sort three-dimensional objects (i.e., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the number of faces, edges or vertices.
2. Predict what new shapes will be formed by combining or cutting apart existing shapes.
3. Recognize two-dimensional shapes and three-dimensional objects from different positions.

*Spatial Relationships*

4. Identify and determine whether two-dimensional shapes are congruent (same shape and size) or similar (same shape different size) by copying or using superposition (lay one thing on top of another).

*Transformations and  
Symmetry*

5. Create and identify two-dimensional figures with line symmetry; e.g., what letter shapes, logos, polygons are symmetrical?

## Patterns, Functions and Algebra Standard

*Use Patterns,  
Relations  
and Functions*

1. Extend simple number patterns (both repeating and growing patterns), and create similar patterns using different objects, such as using physical materials or shapes to represent numerical patterns.
2. Use patterns to make generalizations and predictions; e.g., determine a missing element in a pattern.
3. Create new patterns with consistent rules or plans, and describe the rule or general plan of existing patterns.

*Use Algebraic  
Representations*

4. Use objects, pictures, numbers and other symbols to represent a problem situation.
5. Understand equivalence and extend the concept to situations involving symbols; e.g.,  $4 + 5 = 9$  and  $9 = 4 + 5$ , and  $4 + 5 = 3 + 6 = \triangle + \square \dots$
6. Use symbols to represent unknown quantities and identify values for symbols in an expression or equation using addition and subtraction; e.g.,  $\square + \circ = 10$ ,  $\triangle - 2 = 4$ .

*Analyze Change*

7. Describe qualitative and quantitative changes, especially those involving addition and subtraction; e.g., a student growing taller versus a student growing two inches in one year.

## Data Analysis and Probability Standard

*Data Collection*

1. Pose questions, use observations, interviews and surveys to collect data, and organize data in charts, picture graphs and bar graphs.
2. Read, interpret and make comparisons and predictions from data represented in charts, line plots, picture graphs and bar graphs.
3. Read and construct simple timelines to sequence events.

*Statistical Methods*

4. Write a few sentences to describe and compare categories of data represented in a chart or graph, and make statements about the data as a whole.
5. Identify untrue or inappropriate statements about a given set of data.

Grade Two

- Probability*
6. Recognize that data may vary from one population to another; e.g., favorite TV shows of students and of parents.
  7. List some of the possible outcomes of a simple experiment, and predict whether given outcomes are more, less or equally likely to occur.
  8. Use physical models and pictures to represent possible arrangements of 2 or 3 objects.

## K-12 Mathematics Benchmarks

By the end of the 3-4 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>A. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers and decimals.</p> <p>B. Recognize and generate equivalent representations for whole numbers, fractions and decimals.</p> <p>C. Represent commonly used fractions and mixed numbers using words and physical models.</p> <p>D. Use models, points of reference and equivalent forms of commonly used fractions to judge the size of fractions and to compare, describe and order them.</p> <p>E. Recognize and classify numbers as prime or composite and list factors.</p> <p>F. Count money and make change using both coins and paper bills.</p> <p>G. Model and use commutative and associative properties for addition and multiplication.</p>	<p>A. Select appropriate units for perimeter, area, weight, volume (capacity), time and temperature, using:</p> <ul style="list-style-type: none"> <li>• objects of uniform size;</li> <li>• U.S. customary units; e.g., mile, square inch, cubic inch, second, degree Fahrenheit, and other units as appropriate;</li> <li>• metric units; e.g., millimeter, kilometer, square centimeter, kilogram, cubic centimeter, degree Celsius, and other units as appropriate.</li> </ul> <p>B. Know that the number of units is inversely related to the size of the unit for any item being measured.</p> <p>C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.</p>	<p>A. Provide rationale for groupings and comparisons of two-dimensional figures and three-dimensional objects.</p> <p>B. Describe and identify points, lines and planes in the environment.</p> <p>C. Describe and identify intersecting, parallel and perpendicular lines or segments in the environment.</p> <p>D. Identify and draw right, obtuse, acute and straight angles.</p> <p>E. Use attributes to describe, classify and sketch plane figures and build solid objects.</p> <p>F. Develop definitions of classes of shapes.</p> <p>G. Find and name locations in coordinate systems.</p> <p>H. Identify and describe line and rotational symmetry in two-dimensional shapes and designs.</p>


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## K-12 Mathematics Benchmarks

By the end of the 3-4 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
<p>A. Analyze and extend patterns, and describe the rule in words.</p> <p>B. Use patterns to make predictions, identify relationships, and solve problems.</p> <p>C. Write and solve open sentences and explain strategies.</p> <p>D. Represent an unknown quantity as a variable using a symbol, including letters.</p> <p>E. Use variables to create and solve equations representing problem situations.</p> <p>F. Construct and use a table of values to solve problems associated with mathematical relationships.</p> <p>G. Describe how a change in one variable affects the value of a related variable.</p>	<p>A. Gather and organize data from surveys and classroom experiments, including data collected over a period of time.</p> <p>B. Read and interpret tables, charts, graphs (bar, picture, line, line plot), and timelines as sources of information, identify main idea, draw conclusions, and make predictions.</p> <p>C. Construct charts, tables and graphs to represent data, including picture graphs, bar graphs, line graphs, line plots and Venn diagrams.</p> <p>D. Read, interpret and construct graphs in which icons represent more than a single unit or intervals greater than one; e.g., each  = 10 bicycles or the intervals on an axis are multiples of 10.</p> <p>E. Describe data using mode, median and range.</p>	<p>A. Apply and justify the use of a variety of problem-solving strategies; e.g., make an organized list, guess and check.</p> <p>B. Use an organized approach and appropriate strategies to solve multi-step problems.</p> <p>C. Interpret results in the context of the problem being solved; e.g., the solution must be a whole number of buses when determining the number of buses necessary to transport students.</p> <p>D. Use mathematical strategies to solve problems that relate to other curriculum areas and the real world; e.g., use a timeline to sequence events; use symmetry in artwork.</p> <p>E. Link concepts to procedures and to symbolic notation; e.g., model <math>3 \times 4</math> with a geometric array, represent one-third by dividing an object into three equal parts.</p>

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**K-12 Mathematics Benchmarks (continued)**

By the end of the 3-4 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>H. Use relationships between operations, such as subtraction as the inverse of addition and division as the inverse of multiplication.</p> <p>I. Demonstrate fluency in multiplication facts with factors through 10 and corresponding divisions.</p> <p>J. Estimate the results of whole number computations using a variety of strategies, and judge the reasonableness.</p> <p>K. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division of whole numbers.</p> <p>L. Use a variety of methods and appropriate tools (mental math, paper and pencil, calculators) for computing with whole numbers.</p> <p>M. Add and subtract commonly used fractions with like denominators and decimals, using models and paper and pencil.</p>	<p>D. Identify appropriate tools and apply counting techniques for measuring side lengths, perimeter and area of squares, rectangles, and simple irregular two-dimensional shapes, volume of rectangular prisms, and time and temperature.</p> <p>E. Tell time to the nearest minute.</p>	<p>I. Describe, identify and model reflections, rotations and translations, using physical materials.</p> <p>J. Describe a motion or series of transformations that show two shapes are congruent.</p>

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## K-12 Mathematics Benchmarks (continued)

By the end of the 3-4 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
	<p>F. Conduct a simple probability experiment and draw conclusions about the likelihood of possible outcomes.</p> <p>G. Identify and represent possible outcomes, such as arrangements of a set of up to four members and possible combinations from several sets, each containing 2 or 3 members.</p> <p>H. Use the set of possible outcomes to describe and predict events.</p>	<p>F. Recognize relationships among different topics within mathematics; e.g., the length of an object can be represented by a number.</p> <p>G. Use reasoning skills to determine and explain the reasonableness of a solution with respect to the problem situation.</p> <p>H. Recognize basic valid and invalid arguments, and use examples and counter examples, models, number relationships, and logic to support or refute.</p> <p>I. Represent problem situations in a variety of forms (physical model, diagram, in words or symbols), and recognize when some ways of representing a problem may be more helpful than others.</p> <p>J. Read, interpret, discuss and write about mathematical ideas and concepts using both everyday and mathematical language.</p> <p>K. Use mathematical language to explain and justify mathematical ideas, strategies and solutions.</p>

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## Grade Three

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Identify and generate equivalent forms of whole numbers; e.g., 36,  $30 + 6$ ,  $9 \times 4$ ,  $46 - 10$ , number of inches in a yard.
2. Use place value concepts to represent whole numbers and decimals using numerals, words, expanded notation and physical models. For example:
  - a. Recognize 100 means “10 tens” as well as a single entity (1 hundred) through physical models and trading games.
  - b. Describe the multiplicative nature of the number system; e.g., the structure of 3205 as  $3 \times 1000$  plus  $2 \times 100$  plus  $5 \times 1$ .
  - c. Model the size of 1000 in multiple ways; e.g., packaging 1000 objects into 10 boxes of 100, modeling a meter with centimeter and decimeter strips, or gathering 1000 pop-can tabs.
  - d. Explain the concept of tenths and hundredths using physical models, such as metric pieces, base ten blocks, decimal squares or money.
3. Use mathematical language and symbols to compare and order; e.g., less than, greater than, at most, at least,  $<$ ,  $>$ ,  $=$ ,  $\leq$ ,  $\geq$ .
4. Count money and make change using coins and paper bills to ten dollars.
5. Represent fractions and mixed numbers using words, numerals and physical models.
6. Compare and order commonly used fractions and mixed numbers using number lines, models (such as fraction circles or bars), points of reference (such as more or less than  $\frac{1}{2}$ ), and equivalent forms using physical or visual models.
7. Recognize and use decimal and fraction concepts and notations as related ways of representing parts of a whole or a set; e.g., 3 of 10 marbles are red can also be described as  $\frac{3}{10}$  and 3 tenths are red.

*Meaning of Operations*

8. Model, represent and explain multiplication; e.g., repeated addition, skip counting, rectangular arrays and area model. For example:
  - a. Use conventional mathematical symbols to write equations for word problems involving multiplication.
  - b. Understand that, unlike addition and subtraction, the factors in multiplication and division may have different units; e.g., 3 boxes of 5 cookies each.
9. Model, represent and explain division; e.g., sharing equally, repeated subtraction, rectangular arrays and area model. For example:
  - a. Translate contextual situations involving division into conventional mathematical symbols.
  - b. Explain how a remainder may impact an answer in a real-world situation; e.g., 14 cookies being shared by 4 children.
10. Explain and use relationships between operations, such as:
  - a. relate addition and subtraction as inverse operations;
  - b. relate multiplication and division as inverse operations;
  - c. relate addition to multiplication (repeated addition);
  - d. relate subtraction to division (repeated subtraction).
11. Model and use the commutative and associative properties for addition and multiplication.

*Computation and Estimation*

12. Add and subtract whole numbers with and without regrouping.
13. Demonstrate fluency in multiplication facts through 10 and corresponding division facts.
14. Multiply and divide 2- and 3-digit numbers by a single-digit number, without remainders for division.
15. Evaluate the reasonableness of computations based upon operations and the numbers involved; e.g., considering relative size, place value and estimates.



## Measurement Standard

### *Measurement Units*

1. Identify and select appropriate units for measuring:
  - a. length – miles, kilometers and other units of measure as appropriate;
  - b. volume (capacity) – gallons;
  - c. weight – ounces, pounds, grams, or kilograms;
  - d. temperature – degrees (Fahrenheit or Celsius).
2. Establish personal or common referents to include additional units; e.g., a gallon container of milk; a postage stamp is about a square inch.
3. Tell time to the nearest minute and find elapsed time using a calendar or a clock.
4. Read thermometers in both Fahrenheit and Celsius scales.

### *Use Measurement Techniques and Tools*

5. Estimate and measure length, weight and volume (capacity), using metric and U.S. customary units, accurate to the nearest  $\frac{1}{2}$  or  $\frac{1}{4}$  unit as appropriate.
6. Use appropriate measurement tools and techniques to construct a figure or approximate an amount of specified length, weight or volume (capacity); e.g., construct a rectangle with length  $2\frac{1}{2}$  inches and width 3 inches, fill a measuring cup to the  $\frac{3}{4}$  cup mark.
7. Make estimates for perimeter, area and volume using links, tiles, cubes and other models.

## Geometry and Spatial Sense Standard

### *Characteristics and Properties*

1. Analyze and describe properties of two-dimensional shapes and three-dimensional objects using terms such as vertex, edge, angle, side and face.

- 2. Identify and describe the relative size of angles with respect to right angles as follows:
  - a. Use physical models, like straws, to make different sized angles by opening and closing the sides, not by changing the side lengths.
  - b. Identify, classify and draw right, acute, obtuse and straight angles.
- Spatial Relationships* 3. Find and name locations on a labeled grid or coordinate system; e.g., a map or graph.
- Transformations and Symmetry* 4. Draw lines of symmetry to verify symmetrical two-dimensional shapes.
- Visualization and Geometric Models* 5. Build a three-dimensional model of an object composed of cubes; e.g., construct a model based on an illustration or actual object.



### Patterns, Functions and Algebra Standard

- Use Patterns, Relations and Functions* 1. Extend multiplicative and growing patterns, and describe the pattern or rule in words.
- 2. Analyze and replicate arithmetic sequences with and without a calculator.
- 3. Use patterns to make predictions, identify relationships, and solve problems.
- Use Algebraic Representations* 4. Model problem situations using objects, pictures, tables, numbers, letters and other symbols.
- 5. Write, solve and explain simple mathematical statements, such as  $7 + \square > 8$  or  $\triangle + 8 = 10$ .
- 6. Express mathematical relationships as equations and inequalities.
- Analyze Change* 7. Create tables to record, organize and analyze data to discover patterns and rules.
- 8. Identify and describe quantitative changes, especially those involving addition and subtraction; e.g., the height of water in a glass becoming 1 centimeter lower each week due to evaporation.

## Data Analysis and Probability Standard

### *Data Collection*

1. Collect and organize data from an experiment, such as recording and classifying observations or measurements, in response to a question posed.
2. Draw and interpret picture graphs in which a symbol or picture represents more than one object.
3. Read, interpret and construct bar graphs with intervals greater than one.
4. Support a conclusion or prediction orally and in writing, using information in a table or graph.
5. Match a set of data with a graphical representation of the data.
6. Translate information freely among charts, tables, line plots, picture graphs and bar graphs; e.g., create a bar graph from the information in a chart.
7. Analyze and interpret information represented on a timeline.

### *Statistical Methods*

8. Identify the mode of a data set and describe the information it gives about a data set.

### *Probability*

9. Conduct a simple experiment or simulation of a simple event, record the results in a chart, table or graph, and use the results to draw conclusions about the likelihood of possible outcomes.
10. Use physical models, pictures, diagrams and lists to solve problems involving possible arrangements or combinations of two to four objects.



## Grade Four

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Identify and generate equivalent forms of fractions and decimals.  
For example:
  - a. Connect physical, verbal and symbolic representations of fractions, decimals and whole numbers; e.g.,  $\frac{1}{2}$ ,  $\frac{5}{10}$ , “five tenths,” 0.5, shaded rectangles with half, and five tenths.
  - b. Understand and explain that ten tenths is the same as one whole in both fraction and decimal form.
2. Use place value structure of the base-ten number system to read, write, represent and compare whole numbers through millions and decimals through thousandths.
3. Round whole numbers to a given place value.
4. Identify and represent factors and multiples of whole numbers through 100, and classify numbers as prime or composite.
5. Use models and points of reference to compare commonly used fractions.

*Meaning of  
Operations*

6. Use associative and distributive properties to simplify and perform computations; e.g., use left to right multiplication and the distributive property to find an exact answer without paper and pencil, such as  $5 \times 47 = 5 \times 40 + 5 \times 7 = 200 + 35 = 235$ .
7. Recognize that division may be used to solve different types of problem situations and interpret the meaning of remainders; e.g., situations involving measurement, money.

*Computation and  
Estimation*

8. Solve problems involving counting money and making change, using both coins and paper bills.
9. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.
10. Use physical models, visual representations, and paper and pencil to add and subtract decimals and commonly used fractions with like denominators.

11. Develop and explain strategies for performing computations mentally.
12. Analyze and solve multi-step problems involving addition, subtraction, multiplication and division using an organized approach, and verify and interpret results with respect to the original problem.
13. Use a variety of methods and appropriate tools for computing with whole numbers; e.g., mental math, paper and pencil, and calculator.
14. Demonstrate fluency in adding and subtracting whole numbers and in multiplying and dividing whole numbers by 1- and 2-digit numbers and multiples of ten.

## Measurement Standard

4

### *Measurement Units*

1. Relate the number of units to the size of the units used to measure an object; e.g., compare the number of cups to fill a pitcher to the number of quarts to fill the same pitcher.
2. Demonstrate and describe perimeter as surrounding and area as covering a two-dimensional shape, and volume as filling a three-dimensional object.
3. Identify and select appropriate units to measure:
  - a. perimeter – string or links (inches or centimeters).
  - b. area – tiles (square inches or square centimeters).
  - c. volume – cubes (cubic inches or cubic centimeters).

### *Use Measurement Techniques and Tools*

4. Develop and use strategies to find perimeter using string or links, area using tiles or a grid, and volume using cubes; e.g., count squares to find area of regular or irregular shapes on a grid, layer cubes in a box to find its volume.
5. Make simple unit conversions within a measurement system; e.g., inches to feet, kilograms to grams, quarts to gallons.
6. Write, solve and verify solutions to multi-step problems involving measurement.

## Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Identify, describe and model intersecting, parallel and perpendicular lines and line segments; e.g., use straws or other material to model lines.
2. Describe, classify, compare and model two- and three-dimensional objects using their attributes.
3. Identify similarities and differences of quadrilaterals; e.g., squares, rectangles, parallelograms and trapezoids.
4. Identify and define triangles based on angle measures (equiangular, right, acute and obtuse triangles) and side lengths (isosceles, equilateral and scalene triangles).

*Spacial Relationships*

5. Describe points, lines and planes, and identify models in the environment.
6. Specify locations and plot ordered pairs on a coordinate plane, using first quadrant points.

*Transformations and Symmetry*

7. Identify, describe and use reflections (flips), rotations (turns), and translations (slides) in solving geometric problems; e.g., use transformations to determine if 2 shapes are congruent.

*Visualization and Geometric Models*

8. Use geometric models to solve problems in other areas of mathematics, such as number (multiplication/division) and measurement (area, perimeter, border).

4

## Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Use models and words to describe, extend and make generalizations of patterns and relationships occurring in computation, numerical patterns, geometry, graphs and other applications.
2. Represent and analyze patterns and functions using words, tables and graphs.

*Use Algebraic Representation*

3. Construct a table of values to solve problems associated with a mathematical relationship.
4. Use rules and variables to describe patterns and other relationships.

- Analyze Change*
5. Represent mathematical relationships with equations or inequalities.
  6. Describe how a change in one variable affects the value of a related variable; e.g., as one increases the other increases or as one increases the other decreases.

## Data Analysis and Probability Standard

- Data Collection*
1. Create a plan for collecting data for a specific purpose.
  2. Represent and interpret data using tables, bar graphs, line plots and line graphs.
  3. Interpret and construct Venn diagrams to sort and describe data.
  4. Compare different representations of the same data to evaluate how well each representation shows important aspects of the data, and identify appropriate ways to display the data.
  5. Propose and explain interpretations and predictions based on data displayed in tables, charts and graphs.
- Statistical Methods*
6. Describe the characteristics of a set of data based on a graphical representation, such as range of the data, clumps of data, and holes in the data.
  7. Identify the median of a set of data and describe what it indicates about the data.
  8. Use range, median and mode to make comparisons among related sets of data.
- Probability*
9. Conduct simple probability experiments and draw conclusions from the results; e.g., rolling number cubes or drawing marbles from a bag.
  10. Represent the likelihood of possible outcomes for chance situations; e.g., probability of selecting a red marble from a bag containing 3 red and 5 white marbles.
  11. Relate the concepts of impossible and certain-to-happen events to the numerical values of 0 (impossible) and 1 (certain).

12. Place events in order of likelihood and use a diagram or appropriate language to compare the chance of each event occurring; e.g., impossible, unlikely, equal, likely, certain.
13. List and count all possible combinations using one member from each of several sets, each containing 2 or 3 members; e.g., the number of possible outfits from 3 shirts, 2 shorts and 2 pairs of shoes.

## K-12 Mathematics Benchmarks

By the end of the 5-7 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>A. Represent and compare numbers less than 0 through familiar applications and extending the number line.</p> <p>B. Compare, order and convert among fractions, decimals and percents.</p> <p>C. Develop meaning for percents, including percents greater than 100 and less than 1.</p> <p>D. Use models and pictures to relate concepts of ratio, proportion and percent.</p> <p>E. Use order of operations, including use of parenthesis and exponents to solve multi-step problems, and verify and interpret the results.</p> <p>F. Apply number system properties when performing computations.</p> <p>G. Apply and explain the use of prime factorizations, common factors, and common multiples in problem situations.</p>	<p>A. Select appropriate units to measure angles, circumference, surface area, mass and volume, using:</p> <ul style="list-style-type: none"> <li>• U.S. customary units; e.g., degrees, square feet, pounds, and other units as appropriate;</li> <li>• metric units; e.g., square meters, kilograms and other units as appropriate.</li> </ul> <p>B. Convert units of length, area, volume, mass and time within the same measurement system.</p> <p>C. Identify appropriate tools and apply appropriate techniques for measuring angles, perimeter or circumference and area of triangles, quadrilaterals, circles and composite shapes, and surface area and volume of prisms and cylinders.</p> <p>D. Select a tool and measure accurately to a specified level of precision.</p>	<p>A. Identify and label angle parts and the regions defined within the plane where the angle resides.</p> <p>B. Draw circles, and identify and determine the relationships among the radius, diameter, center and circumference.</p> <p>C. Specify locations and plot ordered pairs on a coordinate plane.</p> <p>D. Identify, describe and classify types of line pairs, angles, two-dimensional figures and three-dimensional objects using their properties.</p> <p>E. Use proportions to express relationships among corresponding parts of similar figures.</p> <p>F. Describe and use the concepts of congruence, similarity and symmetry to solve problems.</p> <p>G. Describe and use properties of triangles to solve problems involving angle measures and side lengths of right triangles.</p>

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## K-12 Mathematics Benchmarks

By the end of the 5-7 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
<p>A. Describe, extend and determine the rule for patterns and relationships occurring in numeric patterns, computation, geometry, graphs and other applications.</p> <p>B. Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules.</p> <p>C. Use variables to create and solve equations and inequalities representing problem situations.</p> <p>D. Use symbolic algebra to represent and explain mathematical relationships.</p> <p>E. Use rules and variables to describe patterns, functions and other relationships.</p> <p>F. Use representations, such as tables, graphs and equations, to model situations and to solve problems, especially those that involve linear relationships.</p>	<p>A. Read, create and use line graphs, histograms, circle graphs, box-and-whisker plots, stem-and-leaf plots, and other representations when appropriate.</p> <p>B. Interpret data by looking for patterns and relationships, draw and justify conclusions, and answer related questions.</p> <p>C. Evaluate interpretations and conclusions as additional data are collected, modify conclusions and predictions, and justify new findings.</p> <p>D. Compare increasingly complex displays of data, such as multiple sets of data on the same graph.</p> <p>E. Collect, organize, display and interpret data for a specific purpose or need.</p> <p>F. Determine and use the range, mean, median and mode to analyze and compare data, and explain what each indicates about the data.</p>	<p>A. Clarify problem-solving situation and identify potential solution processes; e.g., consider different strategies and approaches to a problem, restate problem from various perspectives.</p> <p>B. Apply and adapt problem-solving strategies to solve a variety of problems, including unfamiliar and non-routine problem situations.</p> <p>C. Use more than one strategy to solve a problem, and recognize there are advantages associated with various methods.</p> <p>D. Recognize whether an estimate or an exact solution is appropriate for a given problem situation.</p> <p>E. Use deductive thinking to construct informal arguments to support reasoning and to justify solutions to problems.</p>



**K-12 Mathematics Benchmarks (continued)**

By the end of the 5-7 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>H. Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals and integers.</p> <p>I. Use a variety of strategies, including proportional reasoning, to estimate, compute, solve and explain solutions to problems involving integers, fractions, decimals and percents.</p>	<p>E. Use problem solving techniques and technology as needed to solve problems involving length, weight, perimeter, area, volume, time and temperature.</p> <p>F. Analyze and explain what happens to area and perimeter or surface area and volume when the dimensions of an object are changed.</p> <p>G. Understand and demonstrate the independence of perimeter and area for two-dimensional shapes and of surface area and volume for three-dimensional shapes.</p>	<p>H. Predict and describe results (size, position, orientation) of transformations of two-dimensional figures.</p> <p>I. Identify and draw three-dimensional objects from different views (top, side, front and perspective).</p> <p>J. Apply properties of equality and proportionality to solve problems involving congruent or similar figures; e.g., create a scale drawing.</p>

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## K-12 Mathematics Benchmarks (continued)

By the end of the 5-7 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
<p>G. Write, simplify and evaluate algebraic expressions.</p> <p>H. Solve linear equations and inequalities symbolically, graphically and numerically.</p> <p>I. Explain how inverse operations are used to solve linear equations.</p> <p>J. Use formulas in problem-solving situations.</p> <p>K. Graph linear equations and inequalities.</p> <p>L. Analyze functional relationships, and explain how a change in one quantity results in a change in the other.</p> <p>M. Approximate and interpret rates of change from graphical and numerical data.</p>	<p>G. Evaluate conjectures and predictions based upon data presented in tables and graphs, and identify misuses of statistical data and displays.</p> <p>H. Find all possible outcomes of simple experiments or problem situations, using methods such as lists, arrays and tree diagrams.</p> <p>I. Describe the probability of an event using ratios, including fractional notation.</p> <p>J. Compare experimental and theoretical results for a variety of simple experiments.</p> <p>K. Make and justify predictions based on experimental and theoretical probabilities.</p>	<p>F. Use inductive thinking to generalize a pattern of observations for particular cases, make conjectures, and provide supporting arguments for conjectures.</p> <p>G. Relate mathematical ideas to one another and to other content areas; e.g., use area models for adding fractions, interpret graphs in reading, science and social studies.</p> <p>H. Use representations to organize and communicate mathematical thinking and problem solutions.</p> <p>I. Select, apply, and translate among mathematical representations to solve problems; e.g., representing a number as a fraction, decimal or percent as appropriate for a problem.</p> <p>J. Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.</p> <p>K. Recognize and use mathematical language and symbols when reading, writing and conversing with others.</p>



## Grade Five

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Use models and visual representation to develop the concept of ratio as part-to-part and part-to-whole, and the concept of percent as part-to-whole.
2. Use various forms of “one” to demonstrate the equivalence of fractions; e.g.,  $\frac{18}{24} = \frac{9}{12} \times \frac{2}{2} = \frac{3}{4} \times \frac{6}{6}$ .
3. Identify and generate equivalent forms of fractions, decimals and percents.
4. Round decimals to a given place value and round fractions (including mixed numbers) to the nearest half.
5. Recognize and identify perfect squares and their roots.

*Meaning of  
Operations*

6. Represent and compare numbers less than 0 by extending the number line and using familiar applications; e.g., temperature, owing money.
7. Use commutative, associative, distributive, identity and inverse properties to simplify and perform computations.
8. Identify and use relationships between operations to solve problems.
9. Use order of operations, including use of parentheses, to simplify numerical expressions.
10. Justify why fractions need common denominators to be added or subtracted.
11. Explain how place value is related to addition and subtraction of decimals; e.g.,  $0.2 + 0.14$ ; the two tenths is added to the one tenth because they are both tenths.

*Computation and  
Estimation*

12. Use physical models, points of reference, and equivalent forms to add and subtract commonly used fractions with like and unlike denominators and decimals.
13. Estimate the results of computations involving whole numbers, fractions and decimals, using a variety of strategies.

## Measurement Standard

### *Measurement Units*

1. Identify and select appropriate units to measure angles; i.e., degrees.
2. Identify paths between points on a grid or coordinate plane and compare the lengths of the paths; e.g., shortest path, paths of equal length.
3. Demonstrate and describe the differences between covering the faces (surface area) and filling the interior (volume) of three-dimensional objects.
4. Demonstrate understanding of the differences among linear units, square units and cubic units.

### *Use Measurement Techniques and Tools*

5. Make conversions within the same measurement system while performing computations.
6. Use strategies to develop formulas for determining perimeter and area of triangles, rectangles and parallelograms, and volume of rectangular prisms.
7. Use benchmark angles (e.g.;  $45^\circ$ ,  $90^\circ$ ,  $120^\circ$ ) to estimate the measure of angles, and use a tool to measure and draw angles.

## Geometry and Spatial Sense Standard

### *Characteristics and Properties*

1. Draw circles, and identify and determine relationships among the radius, diameter, center and circumference; e.g., radius is half the diameter, the ratio of the circumference of a circle to its diameter is an approximation of  $\pi$ .
2. Use standard language to describe line, segment, ray, angle, skew, parallel and perpendicular.
3. Label vertex, rays, interior and exterior for an angle.
4. Describe and use properties of congruent figures to solve problems.
5. Use physical models to determine the sum of the interior angles of triangles and quadrilaterals.

### *Spatial Relationships*

6. Extend understanding of coordinate system to include points whose  $x$  or  $y$  values may be negative numbers.

*Visualization and Geometric Models*

7. Understand that the measure of an angle is determined by the degree of rotation of an angle side rather than the length of either side.
8. Predict what three-dimensional object will result from folding a two-dimensional net, then confirm the prediction by folding the net.

### Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Justify a general rule for a pattern or a function by using physical materials, visual representations, words, tables or graphs.
2. Use calculators or computers to develop patterns, and generalize them using tables and graphs.
3. Use variables as unknown quantities in general rules when describing patterns and other relationships.
4. Create and interpret the meaning of equations and inequalities representing problem situations.
5. Model problems with physical materials and visual representations, and use models, graphs and tables to draw conclusions and make predictions.

*Use Algebraic Representation*

6. Describe how the quantitative change in a variable affects the value of a related variable; e.g., describe how the rate of growth varies over time, based upon data in a table or graph.

*Analyze Change*

### Data Analysis and Probability Standard

*Data Collection*

1. Read, construct and interpret frequency tables, circle graphs and line graphs.
2. Select and use a graph that is appropriate for the type of data to be displayed; e.g., numerical vs. categorical data, discrete vs. continuous data.
3. Read and interpret increasingly complex displays of data, such as double bar graphs.
4. Determine appropriate data to be collected to answer questions posed by students or teacher, collect and display data, and clearly communicate findings.

Grade Five

5. Modify initial conclusions, propose and justify new interpretations and predictions as additional data are collected.
- Statistical Methods* 6. Determine and use the range, mean, median and mode, and explain what each does and does not indicate about the set of data.
- Probability* 7. List and explain all possible outcomes in a given situation.
8. Identify the probability of events within a simple experiment, such as three chances out of eight.
9. Use 0, 1 and ratios between 0 and 1 to represent the probability of outcomes for an event, and associate the ratio with the likelihood of the outcome.
10. Compare what should happen (theoretical/expected results) with what did happen (experimental/actual results) in a simple experiment.
11. Make predictions based on experimental and theoretical probabilities.

## Grade Six

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Decompose and recompose whole numbers using factors and exponents (e.g.,  $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ ), and explain why “squared” means “second power” and “cubed” means “third power.”
2. Find and use the prime factorization of composite numbers. For example:
  - a. Use the prime factorization to recognize the greatest common factor (GCF).
  - b. Use the prime factorization to recognize the least common multiple (LCM).
  - c. Apply the prime factorization to solve problems and explain solutions.
3. Explain why a number is referred to as being “rational,” and recognize that the expression  $\frac{a}{b}$  can mean  $a$  parts of size  $\frac{1}{b}$  each,  $a$  divided by  $b$ , or the ratio of  $a$  to  $b$ .
4. Describe what it means to find a specific percent of a number, using real-life examples.
5. Use models and pictures to relate concepts of ratio, proportion and percent, including percents less than 1 and greater than 100.

*Meaning of  
Operations*

6. Use the order of operations, including the use of exponents, decimals and rational numbers, to simplify numerical expressions.
7. Use simple expressions involving integers to represent and solve problems; e.g., if a running back loses 15 yards on the first carry but gains 8 yards on the second carry, what is the net gain/loss?
8. Represent multiplication and division situations involving fractions and decimals with models and visual representations; e.g., show with pattern blocks what it means to take  $2\frac{2}{3} \div \frac{1}{6}$ .
9. Give examples of how ratios are used to represent comparisons; e.g., part-to-part, part-to-whole, whole-to-part.

- Computation and Estimation*
10. Recognize that a quotient may be larger than the dividend when the divisor is a fraction; e.g.,  $6 \div \frac{1}{2} = 12$ .
  11. Perform fraction and decimal computations and justify their solutions; e.g., using manipulatives, diagrams, mathematical reasoning.
  12. Develop and analyze algorithms for computing with fractions and decimals, and demonstrate fluency in their use.
  13. Estimate reasonable solutions to problem situations involving fractions and decimals; e.g.,  $\frac{7}{8} + \frac{12}{13} \approx 2$  and  $4.23 \times 5.8 \approx 25$ .
  14. Use proportional reasoning, ratios and percents to represent problem situations and determine the reasonableness of solutions.
  15. Determine the percent of a number and solve related problems; e.g., find the percent markdown if the original price was \$140, and the sale price is \$100.

### Measurement Standard

- Measurement Units*
1. Understand and describe the difference between surface area and volume.
- Use Measurement Techniques and Tools*
2. Use strategies to develop formulas for finding circumference and area of circles, and to determine the area of sectors; e.g.,  $\frac{1}{2}$  circle,  $\frac{2}{3}$  circle,  $\frac{1}{3}$  circle,  $\frac{1}{4}$  circle.
  3. Estimate perimeter or circumference and area for circles, triangles and quadrilaterals, and surface area and volume for prisms and cylinders by:
    - a. estimating lengths using string or links, areas using tiles or grid, and volumes using cubes;
    - b. measuring attributes (diameter, side lengths, or heights) and using established formulas for circles, triangles, rectangles, parallelograms and rectangular prisms.



4. Determine which measure (perimeter, area, surface area, volume) matches the context for a problem situation; e.g., perimeter is the context for fencing a garden, surface area is the context for painting a room.
5. Understand the difference between perimeter and area, and demonstrate that two shapes may have the same perimeter, but different areas or may have the same area, but different perimeters.
6. Describe what happens to the perimeter and area of a two-dimensional shape when the measurements of the shape are changed; e.g. length of sides are doubled.

### Geometry and Spatial Sense Standard

*Characteristics and Properties*

1. Classify and describe two-dimensional and three-dimensional geometric figures and objects by using their properties; e.g., interior angle measures, perpendicular/parallel sides, congruent angles/sides.
2. Use standard language to define geometric vocabulary: vertex, face, altitude, diagonal, isosceles, equilateral, acute, obtuse and other vocabulary as appropriate.
3. Use multiple classification criteria to classify triangles; e.g., right scalene triangle.
4. Identify and define relationships between planes; i.e., parallel, perpendicular and intersecting.

*Spatial Relationships*

5. Predict and describe sizes, positions and orientations of two-dimensional shapes after transformations such as reflections, rotations, translations and dilations.

*Transformations and Symmetry*

6. Draw similar figures that model proportional relationships; e.g., model similar figures with a 1 to 2 relationship by sketching two of the same figure, one with corresponding sides twice the length of the other.

*Visualization and Geometric Models*

7. Build three-dimensional objects with cubes, and sketch the two-dimensional representations of each side; i.e., projection sets.

Grade Six



## Patterns, Functions and Algebra Standard

*Use Patterns,  
Relations  
and Functions*

1. Represent and analyze patterns, rules and functions, using physical materials, tables and graphs.
2. Use words and symbols to describe numerical and geometric patterns, rules and functions.

*Use Algebraic  
Representations*

3. Recognize and generate equivalent forms of algebraic expressions, and explain how the commutative, associative and distributive properties can be used to generate equivalent forms; e.g., perimeter as  $2(l + w)$  or  $2l + 2w$ .
4. Solve simple linear equations and inequalities using physical models, paper and pencil, tables and graphs.
5. Produce and interpret graphs that represent the relationship between two variables.
6. Evaluate simple expressions by replacing variables with given values, and use formulas in problem-solving situations.

*Analyze Change*

7. Identify and describe situations with constant or varying rates of change, and compare them.
8. Use technology to analyze change; e.g., use computer applications or graphing calculators to display and interpret rate of change.

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## Data Analysis and Probability Standard

*Data Collection*

1. Read, construct and interpret line graphs, circle graphs and histograms.
2. Select, create and use graphical representations that are appropriate for the type of data collected.
3. Compare representations of the same data in different types of graphs, such as a bar graph and circle graph.

*Statistical Methods*

4. Understand the different information provided by measures of center (mean, mode and median) and measures of spread (range).

- Probability*
5. Describe the frequency distribution of a set of data, as shown in a histogram or frequency table, by general appearance or shape; e.g., number of modes, middle of data, level of symmetry, outliers.
  6. Make logical inferences from statistical data.
  7. Design an experiment to test a theoretical probability and explain how the results may vary.

## Grade Seven

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Demonstrate an understanding of place value using powers of 10 and write large numbers in scientific notation.
2. Explain the meaning of exponents that are negative or 0.
3. Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.

*Meaning of  
Operations*

4. Use order of operations and properties to simplify numerical expressions involving integers, fractions and decimals.
5. Explain the meaning and effect of adding, subtracting, multiplying and dividing integers; e.g., how adding two integers can result in a lesser value.

*Computation and  
Estimation*

6. Simplify numerical expressions involving integers and use integers to solve real-life problems.
7. Solve problems using the appropriate form of a rational number (fraction, decimal or percent).
8. Develop and analyze algorithms for computing with percents and integers, and demonstrate fluency in their use.
9. Represent and solve problem situations that can be modeled by and solved using concepts of absolute value, exponents and square roots (for perfect squares).

### Measurement Standard

*Measurement Units*

1. Select appropriate units for measuring derived measurements; e.g., miles per hour, revolutions per minute.
2. Convert units of area and volume within the same measurement system using proportional reasoning and a reference table when appropriate; e.g., square feet to square yards, cubic meters to cubic centimeters.

*Use Measurement  
Techniques and  
Tools*

3. Estimate a measurement to a greater degree of precision than the tool provides.
4. Solve problems involving proportional relationships and scale factors; e.g., scale models that require unit conversions within the same measurement system.
5. Analyze problem situations involving measurement concepts, select appropriate strategies, and use an organized approach to solve narrative and increasingly complex problems.
6. Use strategies to develop formulas for finding area of trapezoids and volume of cylinders and prisms.
7. Develop strategies to find the area of composite shapes using the areas of triangles, parallelograms, circles and sectors.
8. Understand the difference between surface area and volume and demonstrate that two objects may have the same surface area, but different volumes or may have the same volume, but different surface areas.
9. Describe what happens to the surface area and volume of a three-dimensional object when the measurements of the object are changed; e.g., length of sides are doubled.

## Geometry and Spatial Sense Standard

*Characteristics and  
Properties*

1. Use proportional reasoning to describe and express relationships between parts and attributes of similar and congruent figures.
2. Determine sufficient (not necessarily minimal) properties that define a specific two-dimensional figure or three-dimensional object. For example:
  - a. Determine when one set of figures is a subset of another; e.g., all squares are rectangles.
  - b. Develop a set of properties that eliminates all but the desired figure; e.g., only squares are quadrilaterals with all sides congruent and all angles congruent.

- 3. Use and demonstrate understanding of the properties of triangles.  
For example:
  - a. Use Pythagorean Theorem to solve problems involving right triangles.
  - b. Use triangle angle sum relationships to solve problems.
- 4. Determine necessary conditions for congruence of triangles.
- 5. Apply properties of congruent or similar triangles to solve problems involving missing lengths and angle measures.
- Spatial Relationships* 6. Determine and use scale factors for similar figures to solve problems using proportional reasoning.
- Transformations and Symmetry* 7. Identify the line and rotation symmetries of two-dimensional figures to solve problems.
- 8. Perform translations, reflections, rotations and dilations of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).
- Visualization and Geometric Models* 9. Draw representations of three-dimensional geometric objects from different views.

### Patterns, Functions and Algebra Standard

- Use Patterns, Relations and Functions* 1. Represent and analyze patterns, rules and functions with words, tables, graphs and simple variable expressions.
- 2. Generalize patterns by describing in words how to find the next term.
- 3. Recognize and explain when numerical patterns are linear or nonlinear progressions; e.g., 1, 3, 5, 7... is linear and 1, 3, 4, 8, 16... is nonlinear.
- Use Algebraic Representations* 4. Create visual representations of equation-solving processes that model the use of inverse operations.
- 5. Represent linear equations by plotting points in the coordinate plane.
- 6. Represent inequalities on a number line or a coordinate plane.



7. Justify that two forms of an algebraic expression are equivalent, and recognize when an expression is simplified; e.g.,  $4m = m + m + m + m$  or  $a \cdot 5 + 4 = 5a + 4$ .
8. Use formulas in problem-solving situations.
9. Recognize a variety of uses for variables; e.g., placeholder for an unknown quantity in an equation, generalization for a pattern, formula.
10. Analyze linear and simple nonlinear relationships to explain how a change in one variable results in the change of another.
11. Use graphing calculators or computers to analyze change; e.g., distance-time relationships.

*Analyze Change*

### Data Analysis and Probability Standard

*Data Collection*

1. Read, create and interpret box-and-whisker plots, stem-and-leaf plots, and other types of graphs, when appropriate.
2. Analyze how decisions about graphing affect the graphical representation; e.g., scale, size of classes in a histogram, number of categories in a circle graph.

*Statistical Methods*

3. Analyze a set of data by using and comparing combinations of measures of center (mean, mode, median) and measures of spread (range, quartile, interquartile range), and describe how the inclusion or exclusion of outliers affects those measures.
4. Construct opposing arguments based on analysis of the same data, using different graphical representations.
5. Compare data from two or more samples to determine how sample selection can influence results.
6. Identify misuses of statistical data in articles, advertisements, and other media.

*Probability*

7. Compute probabilities of compound events; e.g., multiple coin tosses or multiple rolls of number cubes, using such methods as organized lists, tree diagrams and area models.
8. Make predictions based on theoretical probabilities, design and conduct an experiment to test the predictions, compare actual results to predicted results, and explain differences.

## K-12 Mathematics Benchmarks

By the end of the 8-10 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>A. Use scientific notation to express large numbers and numbers less than one.</p> <p>B. Identify subsets of the real number system.</p> <p>C. Apply properties of operations and the real number system, and justify when they hold for a set of numbers.</p> <p>D. Connect physical, verbal and symbolic representations of integers, rational numbers and irrational numbers.</p> <p>E. Compare, order and determine equivalent forms of real numbers.</p> <p>F. Explain the effects of operations on the magnitude of quantities.</p> <p>G. Estimate, compute and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.</p>	<p>A. Solve increasingly complex non-routine measurement problems and check for reasonableness of results.</p> <p>B. Use formulas to find surface area and volume for specified three-dimensional objects accurate to a specified level of precision.</p> <p>C. Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and to find volume of prisms, cylinders, and pyramids.</p> <p>D. Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.</p>	<p>A. Formally define geometric figures.</p> <p>B. Describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence.</p> <p>C. Recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines and parallel lines.</p> <p>D. Use coordinate geometry to represent and examine the properties of geometric figures.</p> <p>E. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools, such as straightedge, compass and technology.</p> <p>F. Represent and model transformations in a coordinate plane and describe the results.</p>

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## K-12 Mathematics Benchmarks

By the end of the 8-10 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
<p>A. Generalize and explain patterns and sequences in order to find the next term and the <math>n</math>th term.</p> <p>B. Identify and classify functions as linear or nonlinear, and contrast their properties using tables, graphs or equations.</p> <p>C. Translate information from one representation (words, table, graph or equation) to another representation of a relation or function.</p> <p>D. Use algebraic representations, such as tables, graphs, expressions, functions and inequalities, to model and solve problem situations.</p> <p>E. Analyze and compare functions and their graphs using attributes, such as rates of change, intercepts and zeros.</p> <p>F. Solve and graph linear equations and inequalities.</p>	<p>A. Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.</p> <p>B. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose.</p> <p>C. Compare the characteristics of the mean, median and mode for a given set of data, and explain which measure of center best represents the data.</p> <p>D. Find, use and interpret measures of center and spread, such as mean and quartiles, and use those measures to compare and draw conclusions about sets of data.</p> <p>E. Evaluate the validity of claims and predictions that are based on data by examining the appropriateness of the data collection and analysis.</p>	<p>A. Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.</p> <p>B. Apply mathematical knowledge and skills routinely in other content areas and practical situations.</p> <p>C. Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the <math>x</math>-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.</p>

**K-12 Mathematics Benchmarks (continued)**

By the end of the 8-10 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<p>H. Find the square root of perfect squares, and approximate the square root of non-perfect squares.</p> <p>I. Estimate, compute and solve problems involving scientific notation, square roots and numbers with integer exponents.</p>	<p>E. Estimate and compute various attributes, including length, angle measure, area, surface area and volume, to a specified level of precision.</p> <p>F. Write and solve real-world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.</p>	<p>G. Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system.</p> <p>H. Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.</p> <p>I. Use right triangle trigonometric relationships to determine lengths and angle measures.</p>

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**K-12 Mathematics Benchmarks (continued)**

By the end of the 8-10 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
<p>G. Solve quadratic equations with real roots by graphing, formula and factoring.</p> <p>H. Solve systems of linear equations involving two variables graphically and symbolically.</p> <p>I. Model and solve problem situations involving direct and inverse variation.</p> <p>J. Describe and interpret rates of change from graphical and numerical data.</p>	<p>F. Construct convincing arguments based on analysis of data and interpretation of graphs.</p> <p>G. Describe sampling methods and analyze the effects of method chosen on how well the resulting sample represents the population.</p> <p>H. Use counting techniques, such as permutations and combinations, to determine the total number of options and possible outcomes.</p> <p>I. Design an experiment to test a theoretical probability, and record and explain results.</p> <p>J. Compute probabilities of compound events, independent events, and simple dependent events.</p> <p>K. Make predictions based on theoretical probabilities and experimental results.</p>	<p>D. Apply reasoning processes and skills to construct logical verifications or counter-examples to test conjectures and to justify and defend algorithms and solutions.</p> <p>E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.</p> <p>F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.</p> <p>G. Write clearly and coherently about mathematical thinking and ideas.</p> <p>H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solutions in a complete and easily understood manner.</p>

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## Grade Eight

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Use scientific notation to express large numbers and small numbers between 0 and 1.
2. Recognize that natural numbers, whole numbers, integers, rational numbers and irrational numbers are subsets of the real number system.

*Meaning of  
Operations*

3. Apply order of operations to simplify expressions and perform computations involving integer exponents and radicals.
4. Explain and use the inverse and identity properties and use inverse relationships (addition/subtraction, multiplication/division, squaring/square roots) in problem solving situations.

*Computation and  
Estimation*

5. Determine when an estimate is sufficient and when an exact answer is needed in problem situations, and evaluate estimates in relation to actual answers; e.g., very close, less than, greater than.
6. Estimate, compute and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.
7. Find the square root of perfect squares, and approximate the square root of non-perfect squares as consecutive integers between which the root lies; e.g.,  $\sqrt{130}$  is between 11 and 12.
8. Add, subtract, multiply, divide and compare numbers written in scientific notation.



### Measurement Standard

*Measurement Units*

1. Compare and order the relative size of common U.S. customary units and metric units; e.g., mile and kilometer, gallon and liter, pound and kilogram.
2. Use proportional relationships and formulas to convert units from one measurement system to another; e.g., degrees Fahrenheit to degrees Celsius.

*Use Measurement  
Techniques and  
Tools*

3. Use appropriate levels of precision when calculating with measurements.
4. Derive formulas for surface area and volume and justify them using geometric models and common materials. For example, find:
  - a. the surface area of a cylinder as a function of its height and radius;
  - b. that the volume of a pyramid (or cone) is one-third of the volume of a prism (or cylinder) with the same base area and height.
5. Determine surface area for pyramids by analyzing their parts.
6. Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.
7. Apply proportional reasoning to solve problems involving indirect measurements or rates.
8. Find the sum of the interior and exterior angles of regular convex polygons with and without measuring the angles with a protractor.
9. Demonstrate understanding of the concepts of perimeter, circumference and area by using established formulas for triangles, quadrilaterals, and circles to determine the surface area and volume of prisms, pyramids, cylinders, spheres and cones. (Note: Only volume should be calculated for spheres and cones.)
10. Use conventional formulas to find the surface area and volume of prisms, pyramids and cylinders and the volume of spheres and cones to a specified level of precision.



## Geometry and Spatial Sense Standard

*Characteristics and  
Properties*

1. Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.
2. Recognize the angles formed and the relationship between the angles when two lines intersect and when parallel lines are cut by a transversal.

Grade Eight

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|---|--|
| <i>Spatial Relationships</i>              | 3. Use proportions in several forms to solve problems involving similar figures (part-to-part, part-to-whole, corresponding sides between figures).  |
| <i>Transformations and Symmetry</i>       | 4. Represent and analyze shapes using coordinate geometry; e.g., given three vertices and the type of quadrilateral, find the coordinates of the fourth vertex.  |
| <i>Visualization and Geometric Models</i> | 5. Draw the results of translations, reflections, rotations and dilations of objects in the coordinate plane, and determine properties that remain fixed; e.g., lengths of sides remain the same under translations. |
|   | 6. Draw nets for a variety of prisms, pyramids, cylinders and cones.   |

### Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Relate the various representations of a relationship; i.e., relate a table to graph, description and symbolic form.
2. Generalize patterns and sequences by describing how to find the  $n$ th term.
3. Identify functions as linear or nonlinear based on information given in a table, graph or equation.

*Use Algebraic Representations*

4. Extend the uses of variables to include covariants where  $y$  depends on  $x$ .
5. Use physical models to add and subtract monomials and polynomials, and to multiply a polynomial by a monomial.
6. Describe the relationship between the graph of a line and its equation, including being able to explain the meaning of slope as a constant rate of change and  $y$ -intercept in real-world problems.
7. Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems.
8. Write, simplify and evaluate algebraic expressions (including formulas) to generalize situations and solve problems.
9. Solve linear equations and inequalities graphically, symbolically and using technology.

10. Solve 2 by 2 systems of linear equations graphically and by simple substitution.
11. Interpret the meaning of the solution of a 2 by 2 system of equations; i.e., point, line, no solution.
12. Solve simple quadratic equations graphically; e.g.,  $y = x^2 - 16$ .
13. Compute and interpret slope, midpoint and distance given a set of ordered pairs.
14. Differentiate and explain types of changes in mathematical relationships, such as linear vs. nonlinear, continuous vs. noncontinuous, direct variation vs. inverse variation.
15. Describe and compare how changes in an equation affects the related graphs; e.g., for a linear equation changing the coefficient of  $x$  affects the slope and changing the constant affects the intercepts.
16. Use graphing calculators or computers to analyze change; e.g., interest compounded over time as a nonlinear growth pattern.

*Analyze Change*

### Data Analysis and Probability Standard

*Data Collection*

1. Use, create and interpret scatterplots and other types of graphs as appropriate.
2. Evaluate different graphical representations of the same data to determine which is the most appropriate representation for an identified purpose; e.g., line graph for change over time, circle graph for part-to-whole comparison, scatterplot for relationship between two variants.
3. Differentiate between discrete and continuous data and appropriate ways to represent each.

*Statistical Methods*

4. Compare two sets of data using measures of center (mean, mode, median) and measures of spread (range, quartiles, interquartile range, percentiles).
5. Explain the mean's sensitivity to extremes and its use in comparison with the median and mode.
6. Make conjectures about possible relationship in a scatterplot and approximate line of best fit.

Grade Eight



7. Identify different ways of selecting samples, such as survey response, random sample, representative sample and convenience sample.
  8. Describe how the relative size of a sample compared to the target population affects the validity of predictions.
  9. Construct convincing arguments based on analysis of data and interpretation of graphs.
- Probability*
10. Calculate the number of possible outcomes for a situation, recognizing and accounting for when items may occur more than once or when order is important.
  11. Demonstrate an understanding that the probability of either of two disjoint events occurring can be found by adding the probabilities for each and that the probability of one independent event following another can be found by multiplying the probabilities.



## Grade Nine

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Identify and justify whether properties (closure, identity, inverse, commutative and associative) hold for a given set and operations; e.g., even integers and multiplication.

2. Compare, order and determine equivalent forms for rational and irrational numbers.

*Meaning of  
Operations*

3. Explain the effects of operations such as multiplication or division, and of computing powers and roots on the magnitude of quantities.

*Computation and  
Estimation*

4. Demonstrate fluency in computations using real numbers.

5. Estimate the solutions for problem situations involving square and cube roots.

### Measurement Standard

*Measurement Units*

1. Convert rates within the same measurement system; e.g., miles per hour to feet per second; kilometers per hour to meters per second.

*Use Measurement  
Techniques and  
Tools*

2. Use unit analysis to check computations involving measurement.

3. Use the ratio of lengths in similar two-dimensional figures or three-dimensional objects to calculate the ratio of their areas or volumes respectively.

4. Use scale drawings and right triangle trigonometry to solve problems that include unknown distances and angle measures.

5. Solve problems involving unit conversion for situations involving distances, areas, volumes and rates within the same measurement system.

### Geometry and Spatial Sense Standard

*Characteristics and  
Properties*

1. Define the basic trigonometric ratios in right triangles: sine, cosine and tangent.



*Visualization and Geometric Models*

2. Apply proportions and right triangle trigonometric ratios to solve problems involving missing lengths and angle measures in similar figures.
3. Analyze two-dimensional figures in a coordinate plane; e.g., use slope and distance formulas to show that a quadrilateral is a parallelogram.

### Patterns, Functions and Algebra Standard

*Use Patterns, Relations and Functions*

1. Define function with ordered pairs in which each domain element is assigned exactly one range element.
2. Generalize patterns using functions or relationships (linear, quadratic and exponential), and freely translate among tabular, graphical and symbolic representations.
3. Describe problem situations (linear, quadratic and exponential) by using tabular, graphical and symbolic representations.
4. Demonstrate the relationship among zeros of a function, roots of equations, and solutions of equations graphically and in words.
5. Describe and compare characteristics of the following families of functions: linear, quadratic and exponential functions; e.g., general shape, number of roots, domain, range, rate of change, maximum or minimum.

*Use Algebraic Representations*

6. Write and use equivalent forms of equations and inequalities in problem situations; e.g., changing a linear equation to the slope-intercept form.
7. Use formulas to solve problems involving exponential growth and decay.
8. Find linear equations that represent lines that pass through a given set of ordered pairs, and find linear equations that represent lines parallel or perpendicular to a given line through a specific point.
9. Solve and interpret the meaning of 2 by 2 systems of linear equations graphically, by substitution and by elimination, with and without technology.

- Analyze Change*
10. Solve quadratic equations with real roots by factoring, graphing, using the quadratic formula and with technology.
  11. Add, subtract, multiply and divide monomials and polynomials (division of polynomials by monomials only).
  12. Simplify rational expressions by eliminating common factors and applying properties of integer exponents.
  13. Model and solve problems involving direct and inverse variation using proportional reasoning.
  14. Describe the relationship between slope and the graph of a direct variation and inverse variation.
  15. Describe how a change in the value of a constant in a linear or quadratic equation affects the related graphs.

## Data Analysis and Probability Standard

- Data Collection*
1. Classify data as univariate (single variable) or bivariate (two variables) and as quantitative (measurement) or qualitative (categorical) data.
  2. Create a scatterplot for a set of bivariate data, sketch the line of best fit, and interpret the slope of the line of best fit.
- Statistical Methods*
3. Analyze and interpret frequency distributions based on spread, symmetry, skewness, clusters and outliers.
  4. Describe and compare various types of studies (survey, observation, experiment), and identify possible misuses of statistical data.
  5. Describe characteristics and limitations of sampling methods, and analyze the effects of random versus biased sampling; e.g., determine and justify whether the sample is likely to be representative of the population.
  6. Make inferences about relationships in bivariate data, and recognize the difference between evidence of relationship (correlation) and causation.
- Probability*
7. Use counting techniques and the Fundamental Counting principle to determine the total number of possible outcomes for mathematical situations.

8. Describe, create and analyze a sample space and use it to calculate probability.
9. Identify situations involving independent and dependent events, and explain differences between, and common misconceptions about, probabilities associated with those events.
10. Use theoretical and experimental probability, including simulations or random numbers, to estimate probabilities and to solve problems dealing with uncertainty; e.g., compound events, independent events, simple dependent events.

## Grade Ten

### Number, Number Sense and Operations Standard

- |                                       |  |
|---------------------------------------|--|
| <i>Number and<br/>Number Systems</i>  | 1. Connect physical, verbal and symbolic representations of irrational numbers; e.g., construct $\sqrt{2}$ as a hypotenuse or on a number line.                                  |
| <i>Meaning of<br/>Operations</i>      | 2. Explain the meaning of the $n$ th root.   |
| <i>Computation and<br/>Estimation</i> | 3. Use factorial notation and computations to represent and solve problem situations involving arrangements.   |
|                                       | 4. Approximate the $n$ th root of a given number greater than zero between consecutive integers when $n$ is an integer; e.g., the 4 <sup>th</sup> root of 50 is between 2 and 3. |

### Measurement Standard

- |   |  |
|---|--|
| <i>Use Measurement<br/>Techniques and<br/>Tools</i> | 1. Explain how a small error in measurement may lead to a large error in calculated results.   |
|   | 2. Calculate relative error.   |
|   | 3. Explain the difference between absolute error and relative error in measurement.  |
|   | 4. Give examples of how the same absolute error can be problematic in one situation but not in another; e.g., compare “accurate to the nearest foot” when measuring the height of a person versus when measuring the height of a mountain. |
|   | 5. Determine the measures of central and inscribed angles and their associated major and minor arcs.   |

### Geometry and Spatial Sense Standard

- |   |  |
|---|--|
| <i>Characteristics and<br/>Properties</i> | 1. Formally define and explain key aspects of geometric figures, including: <ol style="list-style-type: none"> <li>a. interior and exterior angles of polygons;</li> <li>b. segments related to triangles (median, altitude, midsegment);</li> </ol> |
|---|--|

- c. points of concurrency related to triangles (centroid, incenter, orthocenter, circumcenter);
  - d. circles (radius, diameter, chord, circumference, major arc, minor arc, sector, segment, inscribed angle).
2. Recognize and explain the necessity for certain terms to remain undefined, such as point, line and plane.
  3. Make, test and establish the validity of conjectures about geometric properties and relationships using counterexample, inductive and deductive reasoning, and paragraph or two-column proof, including:
    - a. prove the Pythagorean Theorem;
    - b. prove theorems involving triangle similarity and congruence;
    - c. prove theorems involving properties of lines, angles, triangles and quadrilaterals;
    - d. test a conjecture using basic constructions made with a compass and straightedge or technology.
- Spatial Relationships*
4. Construct right triangles, equilateral triangles, parallelograms, trapezoids, rectangles, rhombuses, squares and kites, using compass and straightedge or dynamic geometry software.
  5. Construct congruent figures and similar figures using tools, such as compass, straightedge, and protractor or dynamic geometry software.
- Transformation and Symmetry*
6. Identify the reflection and rotation symmetries of two- and three-dimensional figures.
  7. Perform reflections and rotations using compass and straightedge constructions and dynamic geometry software.
  8. Derive coordinate rules for translations, reflections and rotations of geometric figures in the coordinate plane.
  9. Show and describe the results of combinations of translations, reflections and rotations (compositions); e.g., perform compositions and specify the result of a composition as the outcome of a single motion, when applicable.
- Visualization and Geometric Models*
10. Solve problems involving chords, radii and arcs within the same circle.

Grade Ten

## Patterns, Functions and Algebra Standard

- |  |   |
|--|---|
| <i>Use Patterns, Relations and Functions</i> | <ol style="list-style-type: none"> <li>1. Define function formally and with <math>f(x)</math> notation.</li> <li>2. Describe and compare characteristics of the following families of functions: square root, cubic, absolute value and basic trigonometric functions; e.g., general shape, possible number of roots, domain and range.</li> </ol>  |
| <i>Use Algebraic Representations</i>         | <ol style="list-style-type: none"> <li>3. Solve equations and formulas for a specified variable; e.g., express the base of a triangle in terms of the area and height.</li> <li>4. Use algebraic representations and functions to describe and generalize geometric properties and relationships.</li> <li>5. Solve simple linear and nonlinear equations and inequalities having square roots as coefficients and solutions.</li> <li>6. Solve equations and inequalities having rational expressions as coefficients and solutions.</li> <li>7. Solve systems of linear inequalities.</li> <li>8. Graph the quadratic relationship that defines circles.</li> <li>9. Recognize and explain that the slopes of parallel lines are equal and the slopes of perpendicular lines are negative reciprocals.</li> <li>10. Solve real-world problems that can be modeled using linear, quadratic, exponential or square root functions.</li> <li>11. Solve real-world problems that can be modeled, using systems of linear equations and inequalities.</li> </ol> |
| <i>Analyze Change</i>                        | <ol style="list-style-type: none"> <li>12. Describe the relationship between slope of a line through the origin and the tangent function of the angle created by the line and the positive <math>x</math>-axis.</li> </ol>  |

## Data Analysis and Probability Standard

- |                        |   |
|------------------------|---|
| <i>Data Collection</i> | <ol style="list-style-type: none"> <li>1. Describe measures of center and the range verbally, graphically and algebraically.</li> </ol> |
|------------------------|---|

- 2. Represent and analyze bivariate data using appropriate graphical displays (scatterplots, parallel box-and-whisker plots, histograms with more than one set of data, tables, charts, spreadsheets) with and without technology.
  - 3. Display bivariate data where at least one variable is categorical.
  - 4. Identify outliers on a data display; e.g., use interquartile range to identify outliers on a box-and-whisker plot.
- Statistical Methods*
- 5. Provide examples and explain how a statistic may or may not be an attribute of the entire population; e.g., intentional or unintentional bias may be present.
  - 6. Interpret the relationship between two variables using multiple graphical displays and statistical measures; e.g., scatterplots, parallel box-and-whisker plots, and measures of center and spread.
- Probability*
- 7. Model problems dealing with uncertainty with area models (geometric probability).
  - 8. Differentiate and explain the relationship between the probability of an event and the odds of an event, and compute one given the other.





## K-12 Mathematics Benchmarks

By the end of the 11-12 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense
<ul style="list-style-type: none"> <li>A. Demonstrate that vectors and matrices are systems having some of the same properties of the real number system.</li> <li>B. Develop an understanding of properties of and representations for addition and multiplication of vectors and matrices.</li> <li>C. Apply factorials and exponents, including fractional exponents, to solve practical problems.</li> <li>D. Demonstrate fluency in operations with real numbers, vectors and matrices, using mental computation or paper and pencil calculations for simple cases and technology for more complicated cases.</li> <li>E. Represent and compute with complex numbers.</li> </ul>	<ul style="list-style-type: none"> <li>A. Explain differences among accuracy, precision and error, and describe how each of those can affect solutions in measurement situations.</li> <li>B. Apply various measurement scales to describe phenomena and solve problems.</li> <li>C. Estimate and compute areas and volume in increasingly complex problem situations.</li> <li>D. Solve problem situations involving derived measurements; e.g., density, acceleration.</li> </ul>	<ul style="list-style-type: none"> <li>A. Use trigonometric relationships to verify and determine solutions in problem situations.</li> <li>B. Represent transformations within a coordinate system using vectors and matrices.</li> </ul>

## K-12 Mathematics Benchmarks

By the end of the 11-12 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
<ul style="list-style-type: none"> <li>A. Analyze functions by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.</li> <li>B. Use the quadratic formula to solve quadratic equations that have complex roots.</li> <li>C. Use recursive functions to model and solve problems; e.g., home mortgages, annuities.</li> <li>D. Apply algebraic methods to represent and generalize problem situations involving vectors and matrices.</li> </ul>	<ul style="list-style-type: none"> <li>A. Create and analyze tabular and graphical displays of data using appropriate tools, including spreadsheets and graphing calculators.</li> <li>B. Use descriptive statistics to analyze and summarize data, including measures of center, dispersion, correlation and variability.</li> <li>C. Design and perform a statistical experiment, simulation or study; collect and interpret data; and use descriptive statistics to communicate and support predictions and conclusions.</li> <li>D. Connect statistical techniques to applications in workplace and consumer situations.</li> </ul>	<ul style="list-style-type: none"> <li>A. Construct algorithms for multi-step and non-routine problems.</li> <li>B. Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems.</li> <li>C. Assess the adequacy and reliability of information available to solve a problem.</li> <li>D. Select and use various types of reasoning and methods of proof.</li> <li>E. Evaluate a mathematical argument and use reasoning and logic to judge its validity.</li> <li>F. Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences.</li> </ul>

**K-12 Mathematics Benchmarks (continued)**

By the end of the 11-12 program:

Number, Number Sense and Operations	Measurement	Geometry and Spatial Sense

**K-12 Mathematics Benchmarks (continued)**

By the end of the 11-12 program:

Patterns, Functions and Algebra	Data Analysis and Probability	Mathematical Processes
		<p>G. Understand the difference between a statement that is verified by mathematical proof, such as a theorem, and one that is verified empirically using examples or data.</p> <p>H. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.</p> <p>I. Communicate mathematical ideas orally and in writing with a clear purpose and appropriate for a specific audience.</p> <p>J. Apply mathematical modeling to workplace and consumer situations, including problem formulation, identification of a mathematical model, interpretation of solution within the model, and validation to original problem situation.</p>

## Grade Eleven

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Determine what properties hold for matrix addition and matrix multiplication; e.g., use examples to show addition is commutative and when multiplication is not commutative.
2. Determine what properties hold for vector addition and multiplication, and for scalar multiplication.
3. Represent complex numbers on the complex plane.

*Meaning of  
Operations*

4. Use matrices to represent given information in a problem situation.
5. Model, using the coordinate plane, vector addition and scalar multiplication.

*Computation and  
Estimation*

6. Compute sums, differences and products of matrices using paper and pencil calculations for simple cases, and technology for more complicated cases.
7. Compute sums, differences, products and quotients of complex numbers.
8. Use fractional and negative exponents as optional ways of representing and finding solutions for problem situations; e.g.,  $27^{2/3} = (27^{1/3})^2 = 9$ .
9. Use vector addition and scalar multiplication to solve problems.

### Measurement Standard

*Measurement Units*

1. Determine the number of significant digits in a measurement.
2. Use radian and degree angle measures to solve problems and perform conversions as needed.

*Use Measurement  
Techniques and  
Tools*

3. Derive a formula for the surface area of a cone as a function of its slant height and the circumference of its base.
4. Calculate distances, areas, surface areas and volumes of composite three-dimensional objects to a specified number of significant digits.
5. Solve real-world problems involving area, surface area, volume and density to a specified degree of precision.

Grade Eleven

## Geometry and Spatial Sense Standard

- |   |   |
|---|---|
| <i>Spatial Relationships</i>              | 1. Use polar coordinates to specify locations on a plane.   |
| <i>Transformations and Symmetry</i>       | 2. Represent translations using vectors.  |
|   | 3. Describe multiplication of a vector and a scalar graphically and algebraically, and apply to problem situations. |
|   | 4. Use trigonometric relationships to determine lengths and angle measures; i.e., Law of Sines and Law of Cosines.  |
| <i>Visualization and Geometric Models</i> | 5. Identify, sketch and classify the cross sections of three-dimensional objects.                                   |

## Patterns, Functions and Algebra Standard

- |  |  |
|--|--|
| <i>Use Patterns, Relations and Functions</i> | 1. Identify and describe problem situations involving an iterative process that can be represented as a recursive function; e.g., compound interest.   |
|  | 2. Translate a recursive function into a closed form expression or formula for the $n$ th term to solve a problem situation involving an iterative process; e.g., find the value of an annuity after 7 years.  |
|  | 3. Describe and compare the characteristics of the following families of functions: quadratics with complex roots, polynomials of any degree, logarithms, and rational functions; e.g., general shape, number of roots, domain and range, asymptotic behavior. |
|  | 4. Identify the maximum and minimum points of polynomial, rational and trigonometric functions graphically and with technology.  |
|  | 5. Identify families of functions with graphs that have rotation symmetry or reflection symmetry about the $y$ -axis, $x$ -axis or $y = x$ .   |
| <i>Use Algebraic Representations</i>         | 6. Represent the inverse of a function symbolically and graphically as a reflection about $y = x$ .  |
|  | 7. Model and solve problems with matrices and vectors.   |
|  | 8. Solve equations involving radical expressions and complex roots.  |
|  | 9. Solve 3 by 3 systems of linear equations by elimination and using technology, and interpret graphically what the solution means (a point, line, plane, or no solution).   |



10. Describe the characteristics of the graphs of conic sections.
- Analyze Change*      11. Describe how a change in the value of a constant in an exponential, logarithmic or radical equation affects the graph of the equation.

### Data Analysis and Probability Standard

- Data Collection*      1. Design a statistical experiment, survey or study for a problem; collect data for the problem; and interpret the data with appropriate graphical displays, descriptive statistics, concepts of variability, causation, correlation and standard deviation.
2. Describe the role of randomization in a well-designed study, especially as compared to a convenience sample, and the generalization of results from each.
- Statistical Methods*      3. Describe how a linear transformation of univariate data affects range, mean, mode and median.
4. Create a scatterplot of bivariate data, identify trends, and find a function to model the data.
5. Use technology to find the Least Squares Regression Line, the regression coefficient, and the correlation coefficient for bivariate data with a linear trend, and interpret each of these statistics in the context of the problem situation.
6. Use technology to compute the standard deviation for a set of data, and interpret standard deviation in relation to the context or problem situation.
7. Describe the standard normal curve and its general properties, and answer questions dealing with data assumed to be normal.
8. Analyze and interpret univariate and bivariate data to identify patterns, note trends, draw conclusions, and make predictions.
9. Evaluate validity of results of a study based on characteristics of the study design, including sampling method, summary statistics and data analysis techniques.
- Probability*      10. Understand and use the concept of random variable, and compute and interpret the expected value for a random variable in simple cases.
11. Examine statements and decisions involving risk; e.g., insurance rates and medical decisions.



## Grade Twelve

### Number, Number Sense and Operations Standard

*Number and  
Number Systems*

1. Determine what properties (closure, identity, inverse, commutative and associative) hold for operations with complex numbers.

*Computation and  
Estimation*

2. Apply combinations as a method to create coefficients for the Binomial Theorem, and make connections to everyday and workplace problem situations.

### Measurement Standard

*Use Measurement  
Techniques and  
Tools*

1. Solve problems involving derived measurements; e.g., acceleration and pressure.

2. Use radian measures in the solution of problems involving angular velocity and acceleration.

3. Apply informal concepts of successive approximation, upper and lower bounds, and limits in measurement situations; e.g., measurement of some quantities, such as volume of a cone, can be determined by sequences of increasingly accurate approximations.

### Geometry and Spatial Sense Standard

*Transformations and  
Symmetry*

1. Use matrices to represent translations, reflections, rotations, dilations and their compositions.

2. Derive and apply the basic trigonometric identities; i.e., angle addition, angle subtraction and double angle.

*Visualization and  
Geometric Models*

3. Relate graphical and algebraic representations of lines, simple curves and conic sections.

4. Recognize and compare specific shapes and properties in multiple geometries; e.g., plane, spherical and hyperbolic.

## Patterns, Functions and Algebra Standard

- |  |   |
|--|---|
| <i>Use Patterns, Relations and Functions</i> | <ol style="list-style-type: none"> <li>1. Analyze the behavior of arithmetic and geometric sequences and series as the number of terms increases.</li> <li>2. Translate between the numeric and symbolic form of a sequence or series.</li> <li>3. Describe and compare the characteristics of transcendental and periodic functions; e.g., general shape, number of roots, domain and range, asymptotic behavior, extrema, local and global behavior.</li> <li>4. Represent the inverse of a transcendental function symbolically.</li> </ol>  |
| <i>Use Algebraic Representations</i>         | <ol style="list-style-type: none"> <li>5. Set up and solve systems of equations using matrices and graphs, with and without technology.</li> <li>6. Make arguments about mathematical properties using mathematical induction.</li> <li>7. Make mathematical arguments using the concepts of limit.</li> <li>8. Compare estimates of the area under a curve over a bounded interval by partitioning the region with rectangles; e.g., make successive estimates using progressively smaller rectangles.</li> <li>9. Translate freely between polar and Cartesian coordinate systems.</li> </ol> |
| <i>Analyze Change</i>                        | <ol style="list-style-type: none"> <li>10. Use the concept of limit to find instantaneous rate of change for a point on a graph as the slope of a tangent at a point.</li> </ol>  |

## Data Analysis and Probability Standard

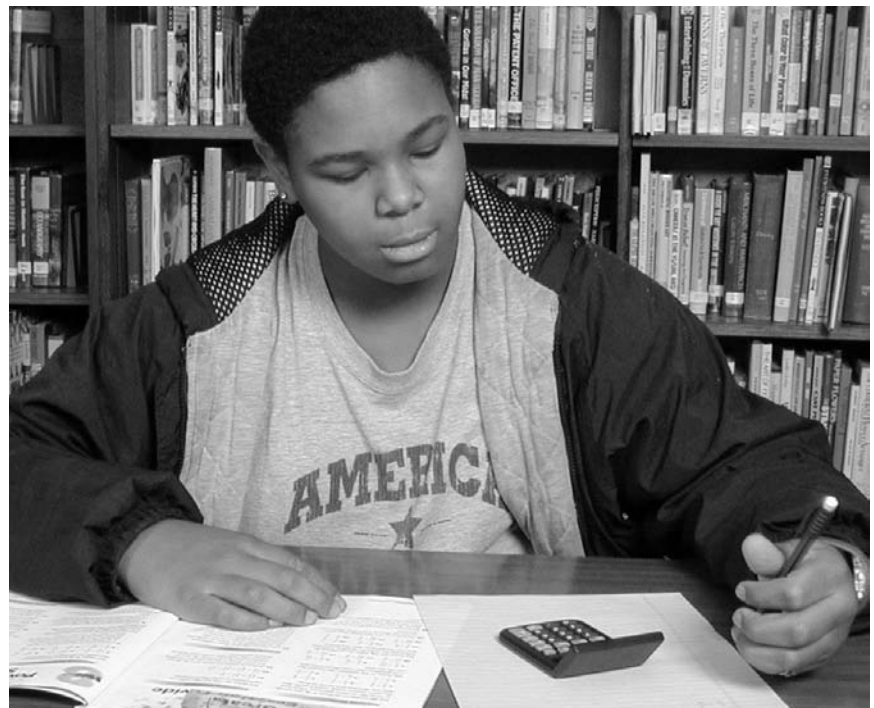
- |                            |  |
|----------------------------|--|
| <i>Data Collection</i>     | <ol style="list-style-type: none"> <li>1. Identify and use various sampling methods (voluntary response, convenience sample, random sample, stratified random sample, census) in a study.</li> </ol>   |
| <i>Statistical Methods</i> | <ol style="list-style-type: none"> <li>2. Transform bivariate data so it can be modeled by a function; e.g., use logarithms to allow nonlinear relationship to be modeled by linear function.</li> <li>3. Describe the shape and find all summary statistics for a set of univariate data, and describe how a linear transformation affects shape, center and spread.</li> </ol> |

- Probability*
4. Apply the concept of a random variable to generate and interpret probability distributions, including binomial, normal and uniform.
  5. Use sampling distributions as the basis for informal inference.
  6. Use theoretical or experimental probability, including simulations, to determine probabilities in real-world problem situations involving uncertainty, such as mutually exclusive events, complementary events, and conditional probability.



## K-12 Mathematics

# Instructional Commentary



# Learning the Basic Skills in Mathematics

*In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures.*

**Principles and Standards for School Mathematics, NCTM, p.5**

Student attainment of basic mathematical skills is the foundation for the academic content standards for Ohio’s students. Often the term “basic skills” is only interpreted as performing computations – adding, subtracting, multiplying and dividing. The goal of the Number, Number Sense and Operations Standard is the development of strong computational skills and also the deep understanding of number concepts. For example, the grade-level indicators and benchmarks in the kindergarten through grade 4 span focus on building conceptual understanding and skills needed to compute with whole numbers efficiently and accurately using mental and paper and pencil methods.

The “basics” in mathematics – essential for success in the workplace, post-secondary education, and daily life – also include understanding and applying concepts and skills in measurement, algebra, geometry, data analysis and probability. Mathematical processes provide ways of acquiring and using content knowledge to solve problems and serve as the cornerstone for building a firm foundation in those basics and for Ohio’s academic content standards.

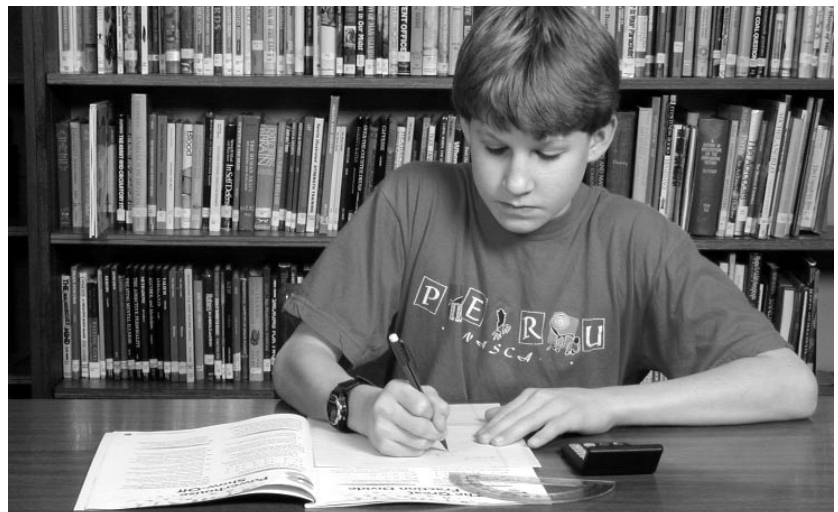
Mathematics skills are required of competent workers. Recent reports articulating the skills required of employees for success in today’s high performance workplace include significant mathematics knowledge and skills beyond those of a traditional skills-based approach to mathematics. Workplace competencies identified in the national SCANS 2000 (Secretary’s Commission on Achieving Necessary Skills) report include mathematics knowledge and know-how among the foundational skills needed by competent workers. And Ohio’s Skills Gap Initiative identifies skill in applying mathematical reasoning to a variety of work-related problems among critical work skills in Ohio’s current and future employment picture.

These reports verify that “basic skills” in mathematics for Ohio’s workers include far more than computational skills. Ohio’s graduates also need to apply “basic skills” in all areas of mathematics, including

algebra, geometry and data analysis. They need to be adept in using mathematical processes – problem solving, reasoning, communication, representation and connections among mathematical ideas and to a variety of contexts and situations outside mathematics - that cut across mathematical topics.

Mathematics skills are also required for the successful transition to post-secondary education. The number of high school graduates seeking post-high school educational experiences in university, community or technical college, and workplace settings continues to grow. And the level of mathematics skills needed to keep options open for continuing education either immediately after high school graduation or some years later requires students to complete a rich and challenging mathematics program.

Mathematics skills are required to make decisions in daily life. “Basic skills” include recognizing and using mathematical ideas in everyday activities, such as reading a newspaper, buying a car or home, and making medical decisions.



The Mathematics Learning Study Committee has defined a view of successful mathematics learning recognizing that no single term completely captures all aspects of mathematics knowledge, understanding, and skill.

“Mathematical proficiency, as we see it, has five strands:

- **conceptual understanding** – comprehension of mathematical concepts, operations, and relations
- **procedural fluency** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **strategic competence** – ability to formulate, represent, and solve mathematical problems
- **adaptive reasoning** – capacity for logical thought, reflection, explanation, and justification
- **productive disposition** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy

The most important observation we make about these five strands is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve this goal.”

**Adding It Up: Helping Children Learn Mathematics**  
**National Research Council, 2001, page 5**

Students bring a variety of basic mathematical knowledge and problem-solving skills to learning, both in school and outside school. These need to be identified, respected, challenged and expanded. There are many different kinds of learning in the mathematics classroom – content, specific techniques, relationships, generalizations, and problem solving to name a few. Each type of learning requires different teaching strategies and lead to different expectations and demonstrations of learning.



Teaching and learning must focus on components for acquiring and using mathematics. Conceptual understanding provides a firm grasp of the ideas, definitions, and relationships in mathematics. Procedural knowledge and skill consist of fluency in the techniques or procedures, including knowing when and how to use them and skill in performing them accurately, efficiently, and flexibly. Problem solving and reasoning refers to the ability to formulate, represent, and solve mathematical problems and to relate mathematics to real-world situations by thinking logically about relationships among concepts and situations. Learning basic skills in mathematics requires understanding and applying concepts, procedures, and processes.

Ohio's mathematics standards, benchmarks, and grade-level indicators provide a strong foundation for student learning by defining the content, or building blocks, and the mathematical processes, or mortar, for acquiring and using mathematics in school, in the workplace, and in daily life.

*In a balanced mathematics program, students become proficient with basic skills, develop conceptual understanding, and become adept at problem solving.*

- *Skills without conceptual understanding are meaningless.*
- *Conceptual understanding without skills is inefficient.*
- *Without problem solving, skills and conceptual understanding have no utility.*

*A rigorous, balanced program requires that students are competent in each area.*

**California Department of Education, Draft of Mathematics Program Advisory, June 1996**



# Understanding Mathematical Processes



*Content and process are “inextricably linked. One cannot solve problems without understanding and using mathematical content.*

*Establishing geometric knowledge calls for reasoning. The concepts of algebra can be examined and communicated through representations.”*

**Principles and Standards  
for School Mathematics,  
NCTM, page 7**

For teachers and curriculum leaders implementing standards-based instruction in Ohio’s classrooms, the first step will be to become familiar with the Ohio academic content standards. A close look at the standards reveals the importance of both content and processes within mathematics. It is important for teachers to understand the role of both mathematical content and mathematical processes when making instructional decisions and fostering success for students in mathematics. Students need to learn the content and skills that form the basis of mathematics, and they will also need to be able to apply these skills using the mathematical processes. Content and processes are not isolated skills in mathematics and should always be taught as interconnected parts of a whole.

The academic content standards articulate the mathematics standards through five content standards and one process standard. Within the standards document, the mathematical processes are specified through benchmark statements only. This is done intentionally as good instruction consists of teaching mathematical content through mathematical processes. This idea is further reinforced within the standards by thoroughly embedding mathematical process statements within the grade-level indicators for each content standard at each grade, in the same way that processes are embedded within effective mathematics instruction. This model for incorporating mathematical processes is not new and is clearly demonstrated within most instructional materials that have been identified as effective.

Indicators like the following show how process skills are embedded within the mathematics content standards:

**Number, Number Sense and Operations Standard**

**Grade-Level Indicator, Grade 1**

*Model and represent multiplication as repeated addition and rectangular arrays in contextual situations; e.g., four people will be at my party and if I want to give 3 balloons to each person, how many balloons will I need to buy?*

**Data Analysis and Probability Standard**

**Grade-Level Indicator, Grade 5**

*Determine appropriate data to be collected to answer questions posed by students or teacher, collect and display data, and clearly communicate findings.*

**Geometry Standard**

**Grade-Level Indicator, Grade 8**

*Make and test conjectures about characteristics and properties (e.g., sides, angles, symmetry) of two-dimensional figures and three-dimensional objects.*

**Patterns, Functions and Algebra Standard**

**Grade-Level Indicator, Grade 10**

*Use algebraic representations and functions to describe and generalize geometric properties and relationships.*

Throughout the five content standards, students will use mathematical processes. Mathematical processes can be categorized into five areas, including problem solving, reasoning, communication, representation and connections. Following is a description and ideas for instruction within the mathematics classroom for each area.

**Problem Solving:** Problem solving is the process of determining a method for arriving at a solution to a problem. Effective mathematical problem solvers can translate words and situations into mathematical terms and representations. They have a range of strategies available to them for solving problems and can effectively select a strategy based upon the problem situation and/or the

**Mathematical Processes Standard:**

*Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques, and communicate mathematical ideas.*

**Ohio's Academic Content Standards**

desired result. They can recognize when results do not make sense, when solutions do not exist, and when results do not apply for particular situations. They realize the importance of and have the capacity to check their solutions given a diverse set of problem situations and circumstances.

Problem solving is integrated into most activities in mathematics classrooms, and can be emphasized through real-world situations connected to the students' lives and interests. Problem solving in the earlier grades may initially focus on learning a variety of strategies. In the later grades focus should include a complete problem-solving process that incorporates selecting and adapting strategies to address more complex problem situations.

*People who reason and think analytically tend to note patterns, structure or regularities in both real-world situations and symbolic objects.*

**Principles and Standards for School Mathematics, NCTM, page 56**

**Reasoning:** The ability to reason is essential in mathematics, other educational disciplines, such as English, history, and science, and in everyday situations. Reasoning involves examining patterns, making conjectures about generalizations, and evaluating those conjectures. Reasoning also includes creating mathematical arguments using inductive and deductive techniques to support or refute mathematical ideas and concepts. Another facet of reasoning is the ability to evaluate reasoning and problem-solving processes of self and others.

Reasoning must be included whenever the opportunity affords itself in mathematics classrooms. In the earlier grades reasoning will primarily consist of pattern recognition and categorizing objects and data. Emphasis should also be placed on justifying answers and using simple deductive reasoning based on established facts. As students' reasoning skills develop they should begin to formulate conjectures and counter examples and apply their reasoning techniques to mathematical ideas, concepts, and relationships. Students begin to evaluate assertions, prove conjectures informally, and move towards more formal inductive and deductive arguments in the middle and later grades. Finally, students will begin to evaluate their own arguments and solutions, and those developed by others, and make decisions based upon those evaluations. The ability to systematically reason and communicate this reasoning will help students in school, in the workplace and in everyday life.

**Communication:** Communication is an essential component of study in any discipline, including mathematics, and for success in life. Communication includes reading, writing and speaking. Reading for mathematical meaning is an essential tool for understanding and using mathematical terms, ideas and concepts. Students who communicate orally and in writing about mathematical concepts and their own problem-solving processes clarify, organize and reflect upon their own understanding in the process. Clearly, communication will lead students to a deeper level of knowledge and enable them to apply that knowledge in new and different situations.

Communication is integrated in the classroom through activities such as having students present their findings to peers, explain their thought processes, or write about how they arrived at the solution to a problem. It is important for students beginning in the early grades to learn that the mathematics classroom is a place where their mathematical thoughts are important and worthy of being read and heard. It is important that students be encouraged and given opportunities to write and speak to teachers, peers and audiences in order to share mathematical ideas and thoughts, to clarify concepts and to articulate results.

**Representation:** There are many ways to represent mathematical ideas, including representations that are symbolic, such as those using letters, numbers or equations, and those that are visual, such as those using charts, graphs or physical objects. Understanding and using representation effectively are essential to the study of mathematics and to the communication of mathematical ideas.

An emphasis on representation in the early grades classroom might involve the use of physical objects to represent mathematical ideas. It will include the use of fractions, decimals, percents, and exponential and scientific notation to show very small and very large numbers in later grades. With the use of technology, graphs can be used more efficiently to allow teachers and students to describe and explain mathematical concepts and to represent data. These skills will enable students to be much better users of information as well as communicators of the more complex information that accompanies today's society.

*Communication is an essential part of mathematics and mathematics education. It is a way of sharing ideas and clarifying understanding.*

**Principles and Standards  
for School Mathematics,  
NCTM, page 60**

*The ways in which mathematical ideas are represented is fundamental to how people can understand and use those ideas.*

**Principles and Standards  
for School Mathematics,  
NCTM, page 67**



*Making connections within mathematics and between mathematics and other disciplines is critical for student success in using mathematics effectively in school, work, and daily life.*

**Ohio's Academic  
Content Standards**

**Connections:** The concept of connections encompasses both the idea of making connections between skills and concepts within mathematics, as well as making connections between mathematical concepts and other disciplines and the outside world.

Students in the classroom need opportunities to recognize and draw upon the connections between and among topics studied. Teachers can facilitate this process by engaging students in multi-layered problem solving situations that cross disciplines and engage students' interests. Students need opportunities to investigate and extend their understanding about connections between mathematical concepts and the workplace. School districts and businesses can facilitate this process through working together to provide internship opportunities and mentoring programs.

When students first begin exploration with simple mathematical concepts and problem-solving situations, students use both mathematical concepts and mathematical processes. As they continue through their education, students become more adept at drawing upon specific process skills and using them in more sophisticated or complex situations. Success at understanding and using mathematical content and processes will enhance students' opportunities in life and in the workplace. Our role as educators in guiding students to this achievement is critical. Because of what we know about implementing research-based and standards-based programs, we are more able than ever to ensure that we reach all children. Ohio's standards take us one step closer toward meeting our vision of high mathematical achievement for all Ohio students.

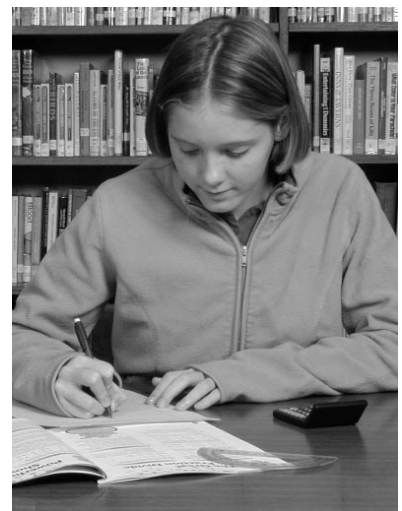


# Planning for Instruction

Ohio's academic content standards provide the foundation for planning integrated instruction and assessment in the classroom. The indicators are presented as separate statements of knowledge, but the intent was not to promote isolated instruction. The indicators should suggest specific content for lessons that will incorporate multiple indicators and cut across the standards and disciplines. Effective instruction will not target individual skills alone, but will integrate those skills. Mathematical content will be taught within the context of mathematical processes. Students will be encouraged to use processes like writing, communication, problem solving and investigation of mathematical topics. Whenever possible, learning will take place within a context that has relevant, real-world connections for the students. Technology and a hands-on approach will be used to enhance student learning.

The first step for teachers implementing standards-based instruction in their classrooms is to develop an understanding of the standards. Educators will develop a sense of the bigger picture of how each indicator fits within the total standards as they familiarize themselves with the standards. We will ask questions through this process, such as the following:

- Which topics from different standard areas could be effectively linked in instruction?
- How do the standards at my grade level build on those taught at the previous grade?
- How are the indicators built on in the next grade?
- How do the indicators culminate in the benchmarks at the end of the grade band?



*Educators will need to have a sense of the big picture in order to see how individual skills can be integrated into one coherent lesson plan, how one lesson fits into a year's program plan, and how one set of grade-level indicators fit into the entire standards.*

*The Geometry and Spatial Sense benchmark (K-2) (“Describe location, using comparative (before, after), directional (above, below) and positional (first, last) words.”) is one example of a benchmark addressed by multiple standard areas.*

**Grade 1 indicators from Geometry and Spatial Sense and Number, Number Sense and Operations build to this benchmark.**

The next step for teachers will be to evaluate their current instructional programs in light of the standards. Some activities may remain the same; some will necessarily change. Teachers will be able to increase the instructional power of existing lessons by developing additional questions or problems that connect to other indicators. We will ask questions through this process, such as the following:

- What standards and indicators are addressed in my current program plan?
- What standards and indicators do I need to address more effectively?
- Where can these fit into my program plan?
- How can I improve the depth and effectiveness of my instruction?

The following vignette shows an example of how multiple standards can be integrated in instruction.

## Indicators and Benchmarks that are evident in this vignette.

### Number, Number Sense and Operations Standard

#### Grade 8, #3

*Apply order of operations to simplify expressions and perform computations involving integer exponents and radicals.*

### Measurement Standard

#### Grade 8, #10

*Use conventional formulas to find the surface area and volume of prisms, pyramids and cylinders and the volume of spheres and cones to a specified level of precision.*

### Patterns, Functions and Algebra Standard

#### Grade 8, #1

*Relate the various representations of a relationship; (i.e., relate a table to graph, description and symbolic form.*



**Grade 8, #3**

*Identify functions as linear or nonlinear based on information given in a table, graph or equation.*

**Grade 8, #14**

*Differentiate and explain types of changes in mathematical relationships such as linear vs. nonlinear, continuous, vs. noncontinuous, direct variation vs. inverse variation.*

**Grade 8, #16**

*Use Graphing calculators of computers to analyze change.*

**Mathematical Processes Standard**

**Grade Band 8–10 C**

*Recognize and use connections between equivalent representations and related procedures for a mathematical concept.*

**Grade Band 8–10 E**

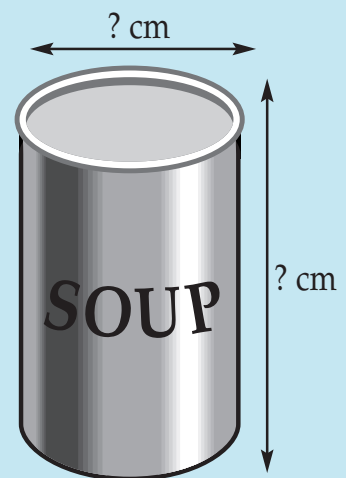
*Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.*


Mrs. Nance started with a simple measurement lesson on volume. She introduced the lesson by bringing in a soup can for each small group. She asked each group to find the volume of the can by finding the diameter of the can and the height.

She extended the lesson by asking each group to determine the volume of the can if the height were changed. Mrs. Nance clarified the instruction for each group by having them increase or decrease the height by increments of 1 centimeter while maintaining the diameter of the soup can. The students recorded their findings as they progressed through the activity.

Next, each group was asked to repeat this process with the diameter of the can. The groups were now told to maintain the height of the can while changing the diameter by increments of 1 centimeter.

*Find the volume of a can of soup by measuring the diameter and height.*





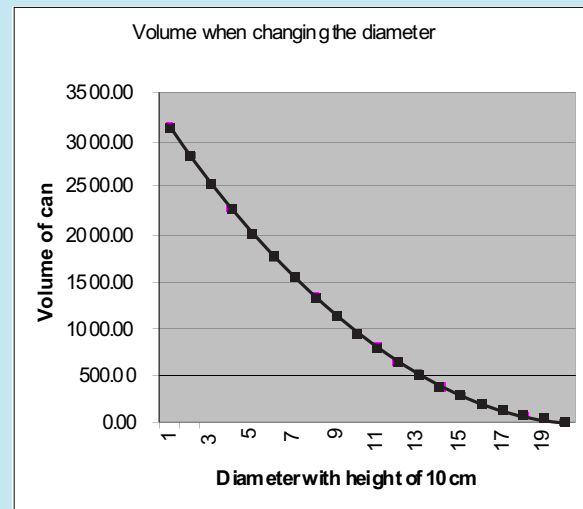
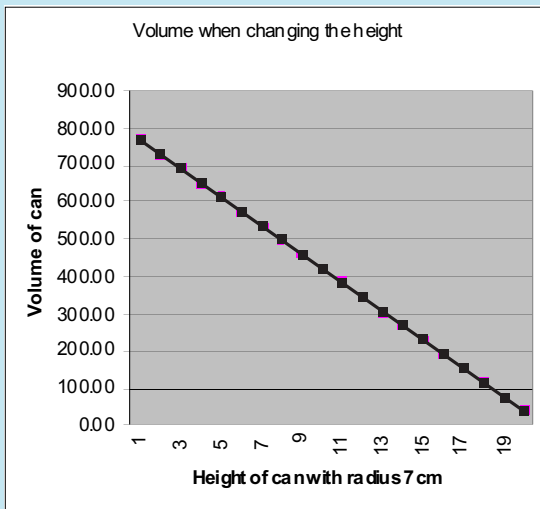
$$\left(\frac{7 \div 2}{2}\right)^2 \pi \times 10 = 510.94 \text{ cm}^3$$

H	D	Vol	H	D	Vol
7	7	357.66	10	4	88.15
8	7	408.78	10	5	177.64
9	7	459.85	10	6	314.89
10	7	510.94	10	7	510.94
11	7	562.04	10	8	777.08
12	7	613.13	10	9	1,124.83
13	7	664.23	10	10	1,565.91
14	7	715.32	10	11	2,112.22

When the diameter is constant and the height changes the volume changes at a constant rate.  
 When the height is constant and as the diameter increases, the volume increases at a significantly faster rate.

Mrs. Nance then asked each group to compare the results by plotting each set of data on separate coordinate planes using a graphing calculator or a spreadsheet on a computer.

As the groups plotted the data, several students raised questions. Natalie wanted to know why changing the diameter affected the volume differently than changing the height, even though you were changing the measurements by the same amount. Mrs. Nance answered Natalie by asking a question: “What are you using the diameter to find?” Several of the students responded that they used the diameter to find the radius, by dividing it by two, then used the radius to find the area of the top of the soup can. Through discussion, students could see that in finding the area, the radius is multiplied by itself or squared in the calculation. Therefore, the effect on the volume is different.



At this point another student, Dillon, asked if that is why the graphs looked different as well. Dillon noted that the graph for the changing height was linear while the graph for the changing diameter was non-linear, or curved. Again Mrs. Nance posed a question: “How could you check to see if this is the case?” To clarify, she asked what was taking place with the variable that represented the height. The students responded that the height was being multiplied by the area of the can’s top, while the variable that represented



the diameter was being divided by two, squared, and then multiplied by the value of pi and the height. Mrs. Nance probed further into the difference. After some discussion, the students made the connection that one is just being multiplied by constants while the other is being squared. Using the graphing calculator, Mrs. Nance had the students experiment with functions that involved the two situations mentioned. She then had them summarize their finding in their mathematics journal.

Mrs. Nance plans to expand upon this lesson by having the students maintain the volume of the soup can and find other dimensions. Each group will also make paper tube representations for several sizes of cans with the same volume. These activities will also allow Mrs. Nance to bring up real-world issues as to the relevance and limitations of the differently sized cans. Questions about why marketers of products might choose one size can rather than another size of the same volume will be discussed to have students think about the implications of their inquiry into their real-world decision making.

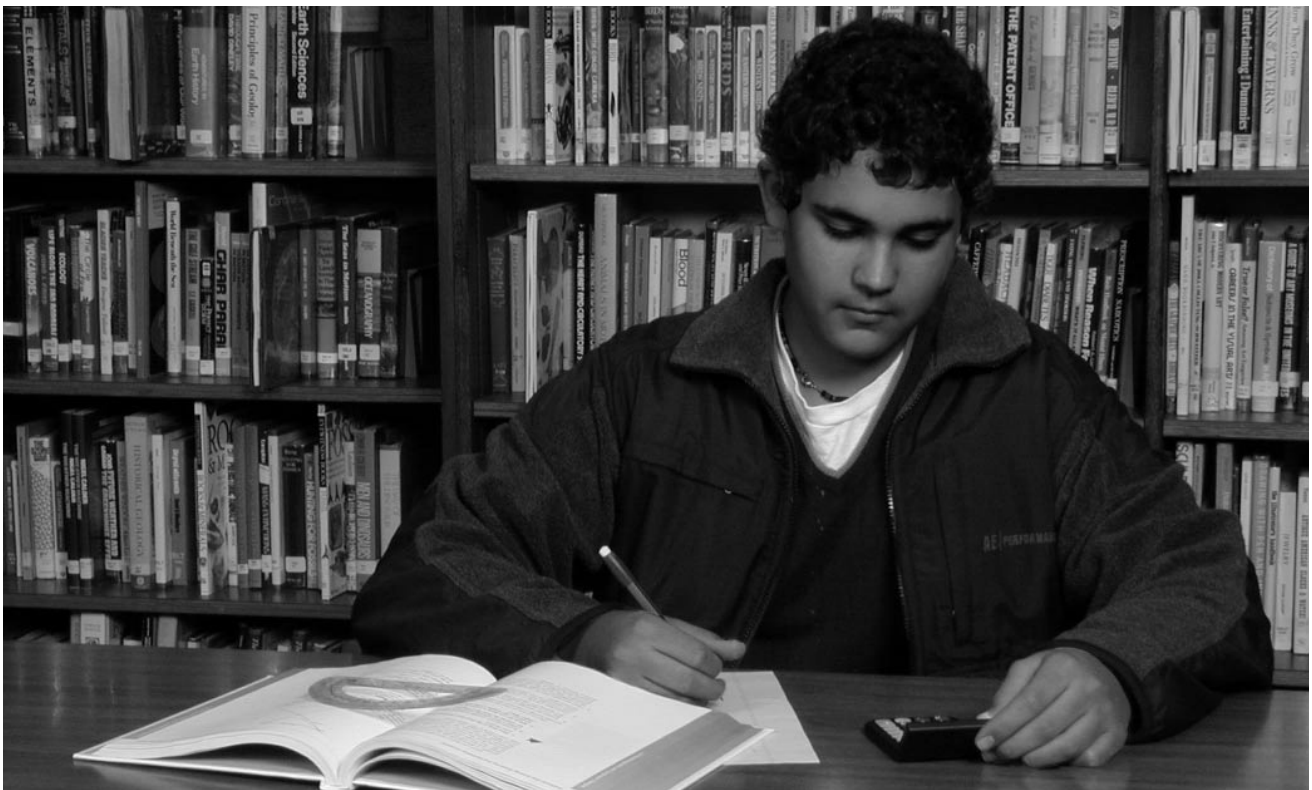
Throughout this lesson we can see how Mrs. Nance incorporated many different indicators and related them to this simple topic of volume of a can. Mrs. Nance also included process skills, the use of technology and real-world applications. This integration across indicators, standards, benchmarks and other disciplines represents the most effective way that teachers will implement standards-based instruction into their classrooms.

In addition to planning these kinds of integrated standards-based lesson plans, teachers and curriculum leaders will consider how each lesson fits into the progression of the year. They will look at what skills are being built upon and how each lesson influences what can be dealt with in the future. They will evaluate the overall plan to ensure that Ohio's academic content standards and local standards

# ACADEMIC CONTENT STANDARDS

are appropriately covered. They will plan and design assessments that will help to reinforce student learning and provide teachers with invaluable information for program planning. This process of aligning classroom, school and district program plans to the standards will require that we think carefully about what we teach and when and how we teach it. This will not be an easy process, but the result will be that all Ohio teachers will have clear instructional plans that focus instruction on helping all students to meet the demands of the 21st century.

*Regular classroom assessment aligned with the standards and instruction will be an important part of implementing a standards-based system.*





# Program Planning

*Teachers should not have to set aside good instruction to implement standards.*

*Instead, good instruction itself should be the best implementation.*

Ohio's mathematics academic content standards provide clear expectations for all students. They form the basis for what every student should know and be able to do in school mathematics programs across the state. While local programs and curriculum may go beyond those expectations, the benchmarks and grade-level indicators provide clarity to Ohio teachers and curriculum leaders of what content and skills should be the focus of teaching and learning at each grade band and level. In order to effectively implement a standards-based mathematics program, teachers and curriculum leaders must determine how instructional programs can be organized, delivered and managed in Ohio's classrooms, schools and districts.

Program planning and implementation is a shared responsibility. Teachers, curriculum leaders, school boards, parents and community members play important roles in making decisions about local mathematics programs. This shared responsibility contributes to a coherent mathematics program in which all components – curriculum, instruction and assessment – fit together well.

Looking at the level of alignment of local mathematics programs with Ohio's mathematics academic content standards is an important first step in program planning. All components of the program should be considered, including the written curriculum or course of study, instructional practices and materials, and classroom and district assessments.

Strategies for evaluating and aligning programs with standards include:

- **Become familiar with the standards, benchmarks and grade-level indicators.** All teachers and curriculum leaders need to be cognizant of the specific content and expectations in the mathematics standards for their grade level and across the K-12 program.
- **Discuss interpretations of the standards, benchmarks and grade-level indicators.** Engaging in discussion of the concepts, skills and applications within the benchmarks and grade-level indicators is critical to examining the level of alignment of an instructional program with content and process standards.
- **Identify required learning experiences based on the standards.** A critical component of alignment with standards is a balanced mathematics program in which students develop conceptual understanding, use procedural skills fluently, and become adept in problem solving. The mathematics course of study and instructional program should be reviewed and modified, as needed, to ensure all students have access to the full mathematics curriculum represented by the benchmarks and grade-level indicators, regardless of past achievement or course options taken.
- **Match instructional strategies and materials to identified student learning experiences.** Instructional practices across the program and within grade levels should be reviewed for alignment and consistency with both standards and how children learn mathematics. Matching instruction with student needs is a complex task requiring a thorough knowledge of mathematics, how students learn, and effective instructional strategies.
- **Develop a “critical eye” for selecting or developing assessment strategies and items.** Multiple assessment strategies for assessing student progress and achievement should be included within the mathematics program. Particular attention should be given to the alignment of assessment with standards and the quality of the inferences that can be made about student progress towards demonstrating those expectations. Key aspects for aligning assessment strategies and items with standards include

*Effective teaching requires understanding of what students know and need to learn and then challenging and supporting them to learn well.*

**Principles and Standards for School Mathematics, NCTM, p.16**

identifying what standard(s) the assessment addresses, what mathematical thinking students use during assessment, and how the assessment and instruction are linked.

Curriculum mapping is one strategy through which teachers and curriculum leaders can familiarize themselves with the standards and identify alignment needs. There are a number of models that can be used to map curriculum to standards and to evaluate the level of alignment of current programs with standards. A key component, whether the process is done by an individual teacher, by a grade level team, or by a school or district curriculum committee, is to carefully examine the standards and identify supporting evidence for those standards within existing programs, lessons and assessments.



Teachers and curriculum leaders will find that some of their current lessons, projects and units fulfill the expectations of the standards. Because the mathematics standards incorporate new research in teaching and learning mathematics and because they are more rigorous, there will likely be some areas in which existing programs and lessons require adjustments in order to align with the academic content standards.

Ohio's mathematics standards include specific areas with implications for teachers, schools and districts and for programs. Those areas may vary from teacher-to-teacher, building-to-building, and district-to-district based on the current status of practices and programs. Some of the most critical areas that may require strengthening through program planning are:

**Stronger integration of mathematical processes.** Mathematical processes – problem solving, connections, reasoning, communication and representation – enable students to acquire and use mathematics. Mathematical processes should be routinely incorporated within instruction and assessment to meet this standard. Planning and implementation should focus on integrating mathematical processes into the program as a means for learning mathematics content and to support the continuing development and strengthening of those key processes.

**Increased emphasis on data analysis and probability.** Interpreting and using data to make predictions and decisions is an essential skill. The increased availability of data and of technologies for displaying



and analyzing data require greater understanding of data analysis and related aspects of probability. Representing data in tables and graphs, using statistical methods to analyze data, and applying concepts of probability are used to interpret information and make decisions in all content areas. An increased emphasis on data analysis and probability spans all grade levels within Ohio’s standards.

**Greater level of rigor.** The standards set high expectations for all students. The need to understand and use mathematics has never been greater. Making standards a reality for all students is an essential goal and significant challenge. Programs must give each student opportunities to reach the expectations set in the standards and the necessary support to do so to meet this challenge. Students in other countries spend significant time in grades 7 and 8 studying topics in algebra and geometry. It should be noted that mathematics programs internationally do not include courses that focus on a single area or topic in mathematics, such as algebra or geometry. For example, students in Japan do not take an “algebra course” at grade 8 or any other grade, as all areas of mathematics are included in every course.

**Articulation of content within the high school program.** The benchmarks and grade-level indicators within Ohio’s standards represent a well-articulated curriculum that provides guidance for developing local courses of study. The local curriculum should provide a road map for guiding teaching and learning. The content and expectations within the curriculum should lead to increasingly deep and sophisticated knowledge and use of mathematics content and processes as students progress through the program. The depth and breadth of the content and instructional experiences are key in determining the contribution of a specific grade level or course within the program.

Course “titles” alone are not sufficient for determining if all students have access to the full curriculum and instruction needed to achieve the expectations in the standards. For example, specific courses, such as a traditional algebra I and geometry sequence, are not required to attain the benchmarks for end-of-grade 10. Careful examination and planning may be needed to ensure that course options, particularly traditional topic-center courses, fully addresses

*Excellence in mathematics education requires equity – high expectations and strong support for all students.*

**Principles and Standards for School Mathematics, NCTM, p. 12**

*Implications of Standards for Local Curriculum Teachers and curriculum leaders need to consider:*

- *when topics are introduced;*
- *how long they remain in the curriculum;*
- *when (if ever) they receive special attention;*
- *what emphasis a particular topic is given;*
- *how many other topics are competing for attention; and*
- *what students are expected to be able to do.*

the expectations data analysis and probability within this grade band. The expectations for grades 9 through 12 can be attained through a variety of course organizations, such as topic-centered or integrated courses. Care must also be taken and strategies put into place to ensure that all students are provided access to the full mathematics curriculum through whatever options are available within the program.

A set of coordinated procedures for monitoring the quality of the mathematics program and its implementation is a key component of program planning. Procedures should include ways to provide ongoing monitoring and periodic evaluation to assure that student achievement goals are being met. Examples of key aspects include alignment of policy decisions with program goals, impact of professional development strategies, and measures of student performance. Conducting periodic reviews, reporting findings publicly, and using results to continually improve the quality of the mathematics program are essential for assuring that all students meet the Ohio's mathematics academic content standards.

## Components of Program Planning

### Building Awareness and Acquiring Commitment

- Develop an awareness of the standards and need for program changes.
- Identify issues related to program planning.
- Empower a group of persons representing all stakeholders to work together over an extended period of time to strengthen the mathematics program.
- Identify goals and key strategies that include continuous reflection and revision along the way.

### Planning and Implementing Improvement Strategies

- Establish leadership teams to guide and monitor program and implementation plans.
  - Identify resources needed for and potential barriers related to implementing goals and key strategies.
  - Phase in strategies and changes over time and monitor their impact.
  - Maintain communication and keep all stakeholders informed of progress.
  - Develop and sustain support structures for assisting students and teachers who encounter difficulties.

### Monitoring Progress

- Establish clear and meaningful criteria for measuring the impact of changes and ongoing program effectiveness.
- Align evaluation methods with program goals and key improvement strategies.
- Use results to make decisions about which goals and strategies have been met and which may need to be modified.
- Identify new goals and strategies that may be needed to sustain and accelerate improvement.

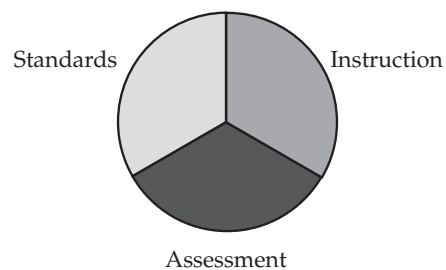
# The Role of Assessment

A strong, effective, aligned educational system has three parts. Standards are one important part. Curriculum and instruction is the second, and assessment aligned with the standards is the third part of an integrated system.



*Ohio is currently engaged in the important process of aligning standards, curriculum and assessments.*

**The Three Parts of an Aligned Educational System**



Ohio has developed and adopted clear and rigorous academic content standards for its students. Educators and the public need to know if students meet these standards. Assessment represents a student's demonstration of understanding, it provides evidence of what students know and are able to do. A comprehensive and thoughtful assessment system also provides needed information for instructional planning and decision-making.

Ohio's comprehensive assessment system includes several types of assessment:

- achievement tests;
- diagnostic assessments;
- classroom assessments; and
- national and international assessments.

Each type of assessment provides invaluable information to Ohio's educators, parents, students and communities. While each piece supports the others, each also serves its own unique purpose.

## Ohio's Comprehensive Assessment System

Assessment Types	Basis for Content	Purpose
Achievement Tests, including the Ohio Graduation Test	Ohio's Academic Content Standards	<ul style="list-style-type: none"> <li>• Measure student achievement.</li> <li>• Demonstrate evidence of continuous improvement at the state and local level.</li> <li>• Provide data for Ohio's accountability system.</li> </ul>
Diagnostic Assessments	Ohio's Academic Content Standards	<ul style="list-style-type: none"> <li>• Monitoring student progress.</li> <li>• Make instructional decisions (e.g., intervention, enrichment).</li> <li>• Provide information to students, parents and teachers.</li> </ul>
Classroom Assessments	Local Courses of Study and Standards	<ul style="list-style-type: none"> <li>• Measure process as well as product of student understanding and knowledge.</li> <li>• Inform teachers and students about progress.</li> <li>• Provide information for instructional planning.</li> </ul>
National and International Assessments	National and International Standards	<ul style="list-style-type: none"> <li>• Compare Ohio achievement against that of other states and nations.</li> </ul>

*Ohio's assessment system includes both diagnostic and achievement assessments. These assessments are being created with substantial input from Ohio's classroom teachers, parents, administrators, higher education faculty, and business representatives.*



*Teachers have the clearest picture of student performance. Through observations of student performance, they can see immediately which students can explain, interpret, apply and*

### **Achievement Tests**

Achievement tests provide the broadest picture of student performance. Ohio's achievement assessments, including the Ohio Graduation Test (OGT), are administered at specified grades and are based on the Ohio academic content standards benchmarks. State-wide assessments measure student achievement and provide guidance for making program decisions and for decisions related to the allocation of resources at the state and local level.

### **Diagnostic Assessments**

Diagnostic assessments are administered annually and are designed to give teachers and parents detailed information as to the strengths and weaknesses of individual students. They provide teachers with important information for instructional planning.

### **Classroom Assessments**

One of the most important components in implementing an aligned standards-based system is ongoing classroom assessment. Teachers constantly assess student performance on an ongoing basis, using both informal and formal measures. Listening to and questioning students are forms of classroom assessment, as are performance assessments, such as writing a research report or solving mathematical problems. Teachers use classroom assessments to evaluate students' performance and progress and to plan instruction that is tailored to students' needs. Classroom assessments provide a rich picture of student performance.

One benefit of classroom assessment is that the feedback is frequent and immediate. Another benefit is that teachers can plan and use assessment in the way that best suits their students' needs. Teachers can design and administer entry-level assessments to determine students' prerequisite skills. They can monitor students' progress frequently to adjust the pace of instruction appropriately and develop and use summative assessments to assess their instructional methods and their students' achievement.

## National and International Assessments

Through participation in national and international assessment opportunities, such as NAEP and TIMSS, Ohio is able to compare the achievement of its students against that of students in other states and other nations. In this way, Ohio ensures that its standards are sufficiently rigorous and world-class.

## The Best Preparation for All Types of Assessment

In Ohio's aligned system, teachers who develop classroom assessments based on the academic content standards grade-level indicators will know that they are evaluating students against a common reference point shared by all Ohio teachers. When teachers design instructional plans based on the grade-level indicators, they will be preparing students for the statewide diagnostic and achievement tests. Teachers will not have to take time out from instruction to prepare students for assessments. Standards-based instruction will prepare students for the assessments. In this way, Ohio's aligned system will support schools, teachers, and parents in ensuring that all students meet the rigorous demands of the new century.

### *Samples of Classroom Assessment:*

- *Projects and investigations*
- *Portfolios*
- *Tests, quizzes and short-answer questions*
- *Extended response and essay questions*
- *Group tests*
- *On-demand assessment*
- *Self-assessment, student reflection*
- *Teacher observations*





*“Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.”*

**Principles and Standards for School Mathematics, NCTM, page 24**

# The Role of Technology

Learning and applying mathematics requires students to become adept in using a variety of techniques and tools for computing, measuring, analyzing data and solving problems. Computers, calculators, physical models, and measuring devices are examples of the wide variety of technologies, or tools, used to teach, learn, and do mathematics. These tools complement, rather than replace, more traditional ways of doing mathematics, such as using symbols and hand-drawn diagrams.

Rapid developments in the design and uses of technology, particularly in electronic tools, have changed how we work, play and learn. For example, graphing calculators and computer-based tools provide powerful mechanisms for communicating, applying, and learning mathematics in the workplace, in everyday tasks, and in school mathematics. Appropriate uses of those and other technologies in the mathematics classroom enhance learning, support effective instruction, and impact the levels of emphasis and ways certain mathematics concepts and skills are learned.

Technology plays a critical role in Ohio’s mathematics academic content standards and their successful implementation. Expectations reflecting the appropriate use of technology are woven into the standards, benchmarks and grade-level indicators. For example, the standards include expectations for students to compute fluently using paper and pencil, technology-supported and mental methods and to use graphing calculators or computers to graph and analyze mathematical relationships. These expectations are intended to support a curriculum rich in the use of technology rather than limit the use of technology to specific skills or grade levels. The benchmarks and indicators were written to enable teachers and programs to use whatever technology deemed most appropriate. Many references to technology within the standards are purposefully non-specific to allow flexibility and incorporation of new technologies, as they become available.



Technology, used appropriately, helps students learn mathematics. Electronic tools, such as spreadsheets and dynamic geometry software, extend the range of problems and develop understanding of key mathematical relationships. A strong foundation in number and operation concepts and skills is required to use calculators effectively as a tool for solving problems involving computations. Graphing calculators allow students to quickly and easily produce multiple graphs for a set of data, determine appropriate ways to display and interpret the data, and test conjectures about the impact of changes in the data.

Technology makes mathematics accessible to all students, including those with special needs. Options for assisting students to maximize their strengths and progress in a standards-based curriculum are expanded through the use of technology-based support and interventions. For example, specialized technologies enhance opportunities for students with physical challenges to develop and demonstrate mathematics concepts and skills.

Technology supports effective mathematics teaching. Technology is a tool for learning and doing mathematics rather than an end in itself. As with any instructional tool or aid, it is only effective when used well. Teachers must make critical decisions about when and how to use technology to focus instruction on learning mathematics. Technology engages students in higher-order thinking, builds strong problem-solving skills, and develops deep understanding of concepts and procedures when used appropriately. The model curriculum, available in June 2003, provides guidance for the appropriate uses of technology in instruction.

The importance of various components of the mathematics curriculum increase as new and enhanced technologies become available. Some examples of mathematics concepts and skills that are more important as a result of advances in technology include:

- Mental arithmetic and estimation skills;
- Concept of variable and conventions such as order of operations;
- Systems for representing figures and the effects of transformations;

*What We Learned about Using Calculators in Mathematics Teaching*

- *Change can occur if we put the potential for change in the hands of everyone.*
- *It takes practiced teachers to change the practice of teaching.*
- *Calculators cause changes in the mathematics we teach.*
- *Calculators cause changes in the way we teach and in the way students learn.*

**Bert Waits and Franklin Demana, "Calculators in Mathematics in Teaching and Learning: Past, Present, and Future" Learning Mathematics for a New Century: 2000 Yearbook, NCTM, pages 52–56**

- Characteristics of functions and their graphs; and
- Statistical operations, such as fitting curves to data.

Students who have learned to use technology effectively throughout their mathematics program will be at an advantage in many situations in school and in the workplace. Mathematics programs that emphasize familiarity and flexibility with technological tools in curriculum, instruction and assessment — firmly grounded in conceptual understanding of mathematics and problem-solving — better equip students for success in a world in which the technology for the next decade, or even the next year, are unknown.



*“Research has shown that instruction that makes productive use of computer and calculator technology has beneficial effects on understanding and learning algebraic representations.”*

**Adding It Up: Helping Children Learn Mathematics, National Research Council, page 420**

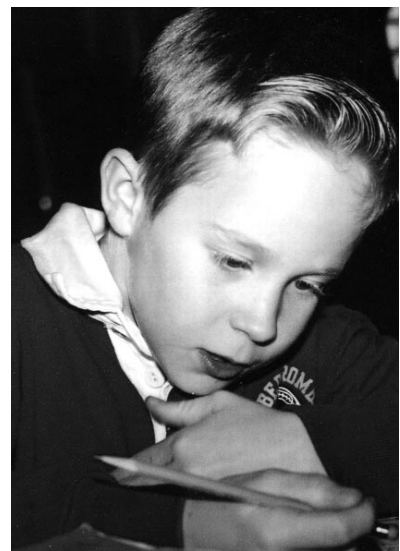
# Making Real-World Connections

Mathematics is an integral part of society and the work place. Thoughtfully planned instruction can help students to realize the importance of mathematics in their lives and to learn the mathematical skills they need in their lives as students, citizens and workers. We know that the more connections a student makes between the “real world” and the mathematical content that is being taught, the greater the students’ motivation, understanding and retention of that content. Our challenge is to find the situations that students face or will face at home, school or within the work environment that will make our instruction relevant.

As we develop lessons and assessments, we must consider what is relevant to our students. For example, there are specific situations that a student from a rural setting would relate to that a student from an urban setting might not; the reverse of this is also true. We need to pay close attention to our students’ backgrounds and interests, as well as to the content of our mathematical instruction, in order to plan relevant instruction.

There are two possible approaches for teachers planning instruction that is based in real-world situations. The first is to begin with a real-world situation, or context, and move to the mathematical content. The second is to start with the content and move to the context of the situation.

In the first approach, educators start with a situation that is relevant to the students who are going to participate in the activity and then identify the mathematics that will take place within the situations. This process is most appropriate for lessons that are developed in advance. It is well suited to in-depth, on-going units and can allow us to link multiple indicators and benchmarks within one lesson and



*For years students have been asking the question “Where am I ever going to use this mathematics (stuff)?”*

*This question shows the need for relevant situations that connect the mathematics to the world.*

to make connections between mathematics and other disciplines. A lesson of this type takes time for preparation and coordination with other discipline teachers. The sample lesson below shows one example of a lesson that is closely tied with its real-world context:

The following indicators and benchmarks are addressed in this activity:

*Analyze and solve multi-step problems involving addition, subtraction, multiplication and division using an organized approach....*

**Grade 4 Indicator  
Number, Number Sense  
and Operations**

*Specify location and plot order pairs on a coordinate plane, using first quadrant points.*

**Grade 4 Indicator  
Geometry**

*Use mathematical strategies to solve problems that relate to other curriculum areas and the real world....*

**Grade 3-4 Benchmark  
Mathematical Processes**

## Sample Real-World Mathematics Lesson

### Lesson One: The Ohio State Map

Mrs. Eldridge plans a lesson to focus on travel and the use of maps. She starts with a map of the state of Ohio and talks to students about where they can find themselves on this map. She continues with a discussion of map coordinates, asking students to identify the coordinates of specific places of interest on the map or challenging their peers to name a point of interest that they have identified only by its coordinates. In this way, the lesson plan can cross disciplines, asking students to identify key landmarks and historical sites around the state that they have learned about in English and social studies.

Mrs. Eldridge extends this lesson with students using the map's key to determine the distance between specific places in the state and determining the time that it would take to travel that distance given an average speed, or to determine possible destinations given the origin, average speed, and time traveled.

Mrs. Eldridge connects this to instruction even further, making it even more relevant to students' own lives, by inviting students to make their own maps of a place of interest to them. Students are then asked to plot these maps onto coordinate a plane, like the one on the state map of Ohio. This integrates students' graphing skills into the lesson plan.

The second approach is to determine an appropriate context based upon the content that is currently being taught in the class. This approach is much more focused on a specific topic than the previous method where the intent was to incorporate as many of the standards as possible. The goal is the same, however, to allow mathematical content to come to life for the students. These lessons are appropriate for use in a short timeframe and typically require minimal materials or coordination with other disciplines. However, one must be careful not to force a context onto the mathematical content, making the situation contrived or irrelevant to the students or mathematically inaccurate. In dealing with the topic of fractions an example of a problematic situation might be like the one that follows:

*Martina's mother fixed meatloaf, potatoes, beans, and pudding for dinner. What portion of the dinner was left if Martina ate only the meatloaf and the beans?*

In this example an answer of one half may be the desired response however the size of the portions must be considered within this situation.

The following example, the lesson on access codes, is related to the topic of combinations within the probability standard and serves as a more appropriate example for this quick approach to real-world instruction. This example could be quickly developed and would be relevant to most students.

*Relevant real-world activities are beneficial, but unless the student makes the connection to the mathematics the activity is in vain. The context is important but it does not take the place of explicit math instruction.*



**This activity addresses the following indicator:**

*Use counting techniques and the Fundamental Counting principle to determine the total number of possible outcomes for mathematical situations.*

**Grade 9 Indicator  
Data Analysis and  
Probability**

## Sample Real-World Mathematics Lesson

### Lesson Two: Access Codes

Mr. Cochran wants to address the topic of combinations from the probability standard. He invites his students to investigate the possible combinations of a one-digit numeric code, a two-digit numeric code, and a three-digit numeric code. He then asks students to investigate how the possible number of combinations changes if the digits are letters only or a combination of numbers or letters. Once a pattern has been determined, the students investigate access codes with larger numbers and compare this to the number of people living in the world, the types of people that may have access to the system or the types of devices where a code of this nature may be used. Students are asked to think about and discuss why banks and other companies use these access codes and what the trade-offs are between longer codes (more safety) and shorter codes (easier to use and remember).

Educators have been given a charge to prepare students to become productive members of society and the working community. This task carries with it the challenge that the direction each individual will pursue is unknown and likely subject to change. Whatever students' goals are for the future, however, we know that we can motivate students and enhance their learning of content by making our instruction relevant. In this way, we can effectively prepare our students for the opportunities and challenges they face as citizens in the 21st century.

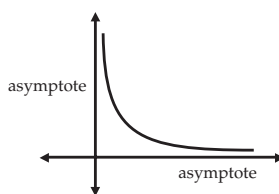


## K-12 Mathematics

# Glossary



- absolute error** The absolute value of the difference between the measured value of a quantity and its true value.
- acute** An angle whose measure is greater than  $0^\circ$  and less than  $90^\circ$ .
- algorithm** A procedure or series of steps used to solve a problem.
- associative property** The result of an operation on real numbers will be unchanged due to grouping; e.g., for addition,  $(a + b) + c = a + (b + c)$  or for multiplication,  $a(bc) = (ab)c$ .
- asymptote** A straight line that a curve approaches but never touches. For example,



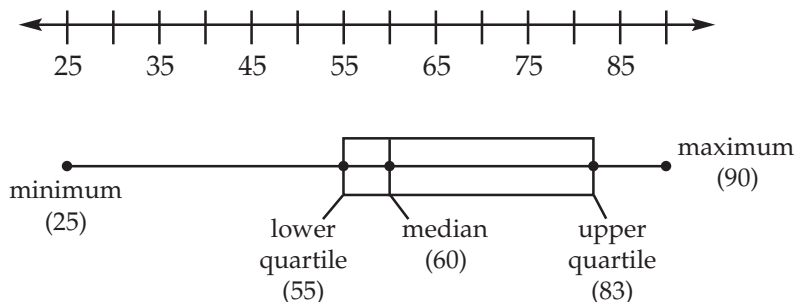
- biased sampling** A *sample* that overrepresents or underrepresents part of the population.
- bivariate data** Data or events described using two *variables*.
- box-and-whisker plot** A diagram that shows pictorially the *median* and *measures of spread* (upper and lower *interquartile ranges* and the *range*) for one set of data. For example,

**Box-and-whisker plot data:**

35	25	90	60	45
40	58	90	90	55
60	55	80	90	60
55	60	85	75	60
56	55	75	80	90

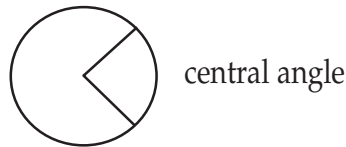
The number of days students in Mr. Jones' homeroom spent studying for the ACT exam.

**box-and-whisker plot:**





- causation** The relationship between two *variables* where a change in one *variable* affects the outcome of the other *variable*.
- categorical data** Data that can be classified by type; e.g., color, types of dogs. These types of data are typically represented using bar chart, pie charts or pictographs.
- central angle** An angle whose vertex is the center of a circle and is in the same plane as the circle.



- coefficient** The numeric factor in a *term*; e.g., the number 3 in the term  $3x^2y$  is the coefficient or in the term  $a^3b$ , 1 is the coefficient.
- combination** A selection of a group of items or events from a set without regard to order; e.g., the number of 3-piece outfits from the set of clothes in the closet.
- common factor** A number, *polynomial* or quantity that evenly divides into two or more mathematical expressions.
- common referents** Something that is familiar that can be used to relate to another thing that is not familiar; e.g., the width of a finger is a centimeter.
- commutative property** The order of the objects in an operation can be changed with out affecting the results; e.g., for addition,  $a + b = b + a$  or for multiplication,  $ab = ba$ .
- compatible numbers** Numbers that go together easily, usually related by pairing in the basic facts; use of compatible numbers generally gives an approximate result; e.g.,  $473 \div 6 \approx 480 \div 6 = 80$ .
- compensatory numbers** Compensatory numbers are used to adjust numbers in a computation after use of *compatible numbers*; e.g.,  $23 + 18 \approx 23 + 20 = 43$ . Since two was added to increase 18 to 20 as compatible numbers, two will be subtracted from 43 to compensate for the change. Therefore, two is the compensatory number.
- complementary events** Two or more *mutually exclusive events* that together cover all possible outcomes. The sum of the probabilities of complementary events is 1.
- compound events** Combining two or more separate events or outcomes and considering it as one single event or outcome.

<b>conditional probability</b>	The probability of an event occurring given that another event has already occurred. For example, What is the probability that the total of two dice will be greater than 8 given that the first die is a 6?
<b>congruent</b>	Having exactly the same size and shape.
<b>continuous data</b>	Data that can be assigned an infinite number of values between whole numbers, the assigned values are approximated; e.g., the size of the apples on an apple tree is continuous data. See <i>discrete data</i> for a counterexample.
<b>coordinate plane</b>	A plane determined by the intersection of two <i>perpendicular</i> number lines in which any point can be located.
<b>correlation</b>	The relation between two sets of data, a positive or direct correlation exists when both sets vary in the same direction (both sets decrease); a negative or inverse correlation exists when one set of data increases as the other decreases.
<b>correlation coefficient</b>	A measure of the <i>correlation</i> between two <i>variables</i> or sets of data. The value of the correlation coefficient, $r$ , is always $-1 \leq r \leq 1$ , where 1 is a perfect positive correlation, 0 is no correlation, and -1 is a perfect negative correlation.
<b>covariants</b>	Varying with another <i>variable</i> quantity in a manner that leaves a specified relationship unchanged.
<b>decomposing</b>	The process of breaking a number into smaller units to simplify problem solving; e.g., 15 can be $10 + 5$ or 10 can be $6 + 4$ .
<b>deductive reasoning</b>	Use logic to arrive at a conclusion from a given premise.
<b>dependent events</b>	A statement or <i>probability</i> for one event affects a statement or <i>probability</i> for another event.
<b>descriptive statistics</b>	To gather and describe data using <i>probability</i> , statistical methods and concepts like graphs and <i>measures of center</i> .
<b>dilation</b>	See <i>transformation</i> .
<b>direct variation</b>	When the values of two <i>variables</i> maintain a constant ratio. This relationship can be expressed as an equation of the form $y = kx$ .
<b>discrete data</b>	Data that can be counted; e.g., the number of people in a town is discrete (there is no such thing as a fractional person). See <i>continuous data</i> for a counterexample.
<b>disjoint events</b>	Two events that have no outcomes in common.

<b>dispersion</b>	How data is spread out around some central point.
<b>distribution</b>	The distribution of a set of data is a graph or table showing how many pieces of data there are in each class, or of each type.
<b>distributive property</b>	The product of a number and the sum (or difference) of two numbers is equal to the sum (or difference) of the two products; e.g., $7(30 + 5) = (7 \cdot 30) + (7 \cdot 5)$ or $a(b-c) = ab - ac$ .
<b>equation</b>	A statement that shows two mathematical expressions that are equal to each other.
<b>equiangular</b>	In a given shape, all angles have the same measure.
<b>equilateral</b>	In a given shape, all sides have the same length.
<b>equivalent</b>	Two items that have the same value.
<b>experimental probability</b>	The probability based on a series of trials. The experimental probability, $P$ , can be found using the following equation: $P(\text{event}) = \frac{\text{\# of trials w/ favorable outcomes}}{\text{number of trials in experiment}}$
<b>experimental results</b>	The outcome as a result of a probability experiment or test. These outcomes are sometimes called actual results.
<b>expressions</b>	Any combination of <i>variables</i> , numbers, and symbols (excluding the equality and inequality symbols).
<b>extrema</b>	A term that refers to maximum and minimum values.
<b>factoring</b>	Rewriting a mathematical expression as a product of factors.
<b>frequency distribution</b>	A collection of data that represents the number of times a set of numbers, items or events have occurred.
<b>frequency table</b>	A table that shows how often each item, number, or range of numbers occurs in a set of data.
<b>front-end estimation</b>	Using the leading, or left-most, digits to make an estimate quickly and easily. After making an initial estimate using front-end digits, an adjustment can be made to refine the estimate; e.g., Using front-end estimation to estimate the sum of 594, 32, and 221, an initial estimate would be 5 + 0 + 2 hundreds or 700. An adjustment can be made by grouping the tens and ones (about 100 + 50 or 150 more) and adding to get an adjusted estimate of 850.

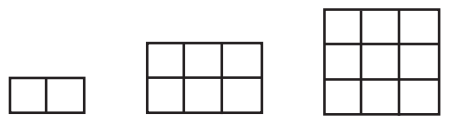
**function** A mathematical relationship between two *variables*, an independent *variable* and a dependent *variable*, where every value of the independent *variable* corresponds to exactly one value of the dependent value.

**Fundamental Counting Principle** The principle which states that all possible outcomes in a *sample space* can be found by multiplying the number of ways each event can occur.

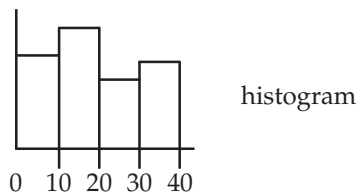
**geometric patterns** A sequence or series, where each term can be found by multiplying the previous term by a constant factor, sometimes referred to as a common ratio.

**geometric probability** The probability that a random point is located in a particular part, or subregion, or a larger region.

**growing patterns** Patterns that involve a progression. For example,



**histogram** A graph that uses bars to show the frequency of data within equal intervals.



**identity property** Of addition: Adding zero to a number gives a sum identical to the given number.

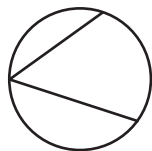
Of multiplication: Multiplying a number by 1 gives a product identical to the given number.

**independent event** Two events in which the outcome of the first event does not affect the outcome of the second event.

**inductive reasoning** Using logic to make generalizations based on observation of specific cases and consideration of patterns.

**inequality** A mathematical sentence that includes one of the inequality symbols,  $<$ ,  $>$ ,  $\leq$ ,  $\geq$ , or  $\neq$  to compare unequal expressions.

**inscribed angle**     An angle whose vertex is on a circle and whose sides are chords of the circle.



inscribed angle

**intercepts**     The value of  $y$  on the *coordinate plane* where  $x = 0$ , called the  $y$ -intercept. The value of  $x$  on the *coordinate plane* where  $y = 0$ , called the  $x$ -intercept.

**intersecting lines**     Two lines that cross at exactly one point.

**inverse operations**     An operation that will undo another operation; e.g., addition and subtraction.

**inverse property**     The result of two real numbers that combine will give the identity elements of zero or one. When a number is added to its additive inverse, the sum is always zero; e.g.,  $8 + -8 = 0$ . When a number is multiplied by its multiplicative inverse, the product is always one; e.g.,  $\frac{2}{3} \cdot \frac{3}{2} = 1$ .

**inverse variation**     The *variables*  $x$  and  $y$  vary inversely if, for a constant  $k$ ,  $yx = k$  or  $y = \frac{k}{x}$ .

**irrational numbers**     Numbers that cannot be written as a ratio of two integers. The decimal form of the number never terminates and never repeats.

**isosceles triangle**     A triangle with at least two congruent sides.

**iterative process**     A process or set of instructions that are repeated.

**Least Squares Regression Line**     A statistical method used to find the *line of best fit* for a set of data points.

**line of best fit**     A line drawn in the midst of the points on a scatter plot in an attempt to estimate the mathematical relationship between the *variables* used to generate the plot.

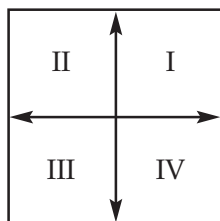
**linear equation**     An equation whose graph on a coordinate grid is a straight line.

**major arc**     On a circle, an arc that is larger than a semicircle and its measure is greater than  $180^\circ$ .

**mean**     See *measures of center*.

<b>measures of center</b>	Numbers that provide information about cluster and average of a collection of data.
<b>mean</b>	The sum of a set of numbers divided by the number of elements in the set.
<b>mode</b>	The number or object that appears most frequently in a set of numbers or objects.
<b>median</b>	The middle number or item in a set of numbers or objects arranged from least to greatest, or the <i>mean</i> of the two middle numbers when the set has two middle numbers.
<b>measures of spread or variability</b>	A term used to refer to how much numbers are spread, varied or dispersed in a set of data.
<b>range</b>	The difference between the greatest and the least numbers in a set of data.
<b>quartile</b>	In conjunction with the <i>median</i> , the quartiles divide the set of data into four groups of equal size.
<b>interquartile range</b>	The difference between the upper quartile range and the lower quartile.
<b>median</b>	See <i>measures of center</i> .
<b>minor arc</b>	An arc that is less than a semicircle or $180^\circ$ .
<b>mode</b>	See <i>measures of center</i> .
<b>monomials</b>	An algebraic expression which is a product of constants and <i>variables</i> .
<b>multiplicative patterns</b>	Number patterns with relationships between consecutive numbers involving multiplication.
<b>mutually exclusive events</b>	Two events that cannot occur at the same time.
<b>nonlinear progressions</b>	A sequence of values that increase in a manner other than linear.
<b>obtuse</b>	An angle measure greater than $90^\circ$ and less than $180^\circ$ .
<b>odds of an event</b>	The ratio of favorable outcomes to unfavorable outcomes.
<b>ordered pairs</b>	A pair of numbers that gives the coordinates of a point on a grid in this order (horizontal coordinate, vertical coordinate).
<b>ordinal position</b>	Identifies the position of an object in a sequence; e.g., first, second, third.

<b>outlier</b>	A data point in a sample widely separated from the main cluster of points in the sample.
<b>parallel lines</b>	Lines in the same plane that do not cross, the distance between the lines is constant.
<b>permutations</b>	Possible orders or arrangements of a set of events or items.
<b>perpendicular lines</b>	Lines that intersect at one point forming $90^\circ$ .
<b>polygon</b>	A closed figure formed from line segments that meet only at their endpoints.
<b>polynomials</b>	The sum of monomials; e.g., $2a^2 + 4a - 5$ .
<b>precision</b>	To determine the size of the unit to be used.
<b>prime factorization</b>	The expression of a number as the product of prime factors; e.g., the prime factorization of 18 is $2 \cdot 3 \cdot 3$ .
<b>probability</b>	The chance of an event occurring. The probability of an event is equal to the number of favorable outcomes divided by the number of possible outcomes.
<b>probability distribution</b>	The set of random data and the probabilities associated with that data.
<b>proportion</b>	An equation showing that two ratios are equal.
<b>quadrants</b>	The two axes of a coordinate system divide the plane into four separate sections known as quadrants. These are identified as the first, second, third, and fourth quadrants.



<b>qualitative data</b>	Data that can be assigned qualities or categories. They are non-numerical data.
<b>quantitative data</b>	Data that are numerical. The data can be <i>discrete</i> or <i>continuous</i> .
<b>random sample</b>	A <i>sample</i> in which every event has an equal chance of selection and each event is chosen by a random process.

<b>random sampling</b>	A random <i>sample</i> is a sample that has been chosen by a process of random selection so that it models the characteristics of the population it is supposed to represent as closely as possible.
<b>random variable</b>	A <i>variable</i> that takes any of a range of values that cannot be predicted with certainty.
<b>rate of change</b>	A relationship such as distance over time, often described by using a slope.
<b>rational expressions</b>	Fractions whose numerators and denominators are polynomials; e.g., $\frac{n^2 - 3n}{2}$ .
<b>rational numbers</b>	Any number that can be written in the form $\frac{a}{b}$ , where $a$ and $b$ are integers and $b \neq 0$ .
<b>rectangular arrays</b>	An arrangement of things or data in rows and columns.
<b>recursive function</b>	A <i>function</i> defined in terms of the repeated application of a number of simpler <i>functions</i> to their own values.
<b>reflection</b>	See <i>transformation</i> .
<b>relative error</b>	The error or uncertainty in a measurement expressed as a fraction of the true value.
<b>right</b>	Relating to $90^\circ$ ; e.g., a right angle measures $90^\circ$ , a right triangle has only one right angle.
<b>roots of equations</b>	A value that will satisfy the equation which has been formed by putting an expression, containing one <i>variable</i> , equal to zero.
<b>rotation</b>	See <i>transformation</i> .
<b>sample</b>	A set of data taken from a larger set used to create or test theories about the data as a whole.
<b>sample space</b>	A list of all possible outcomes of an activity.
<b>sampling method</b>	The process used to collect data; e.g., see <i>random sampling</i> .
<b>scalene triangle</b>	A triangle that has no congruent sides.
<b>scatterplot</b>	A graph with one point for each item being measured. The coordinates of a point represent the measures of two attributes of each item.



- scientific notation** A form of writing numbers as the product of a power of 10 and a decimal number greater than or equal to 1 and less than 10; e.g., 8,924,000 is written as  $8.924 \cdot 10^6$ .
- sequence** An ordered set of objects or numbers.
- series** Sum of a finite or infinite sequence of terms.
- simple event** A subset of the *sample space* that contains only one outcome that cannot be broken down into a simpler, more basic outcome.
- standard deviation** The measure of the *dispersion* of a distribution is equal to the square root of the *variance*.
- stem-and-leaf plot** A frequency diagram which displays the actual data together with its frequency, by using a part of the value of each piece of data to fix the class or group (the stem), while the remainder of the value is actually listed (the leaves). For example,

**Stem-and-leaf plot data:** Coach Smith’s last 30 basketball game scores for the 7th grade Wildcats.

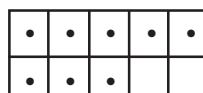
50	65	70	35	40	57	66	65	70	35
29	33	44	56	66	60	44	50	58	46
67	78	79	47	35	35	44	57	60	57

**stem-and-leaf plot**

Stem	Leaves
2	9
3	3 5 5 5 6
4	0 4 4 4 6 7
5	0 0 6 7 7 7 8
6	0 0 5 5 6 6 7
7	0 0 8 9

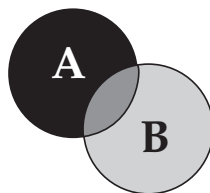
Key: 4|6 represents a score of 46.

- successive approximation** To find the approximate value of a quantity by starting from a first estimate and then deriving from each approximation another that is more accurate.
- symbolic form** To represent something using numbers and symbols.
- target population** The set from which a sample will be selected.
- tens frame** A physical model that represents the structure of the number system’s place value; e.g., the following diagram represents the number eight using a tens frame.



tens frame

<b>terms</b>	The quantities in an algebraic equation that are linked to each other by means of + or - signs.
<b>theoretical probability</b>	Identifying, using mathematical expectations, the number of ways an event could happen compared to all the events that could happen.
<b>theoretical results</b>	The expected results given the <i>theoretical probability</i> of an event.
<b>transcendental function</b>	<i>Functions</i> that are not algebraic; e.g., trigonometric functions.
<b>transformation</b>	An operation that creates an image from an original figure, or preimage.
<b>reflection</b>	A <i>transformation</i> that results in a mirror image of the original shape.
<b>rotation</b>	A rotation is a <i>transformation</i> about a fixed point such that every point in the object turns through the same angle relative to that fixed point.
<b>translation</b>	A <i>transformation</i> in which an image is formed by moving every point on a figure the same distance in the same direction.
<b>dilation</b>	A <i>transformation</i> that preserves the shape of a figure, but allows the size to change.
<b>translation</b>	See <i>transformation</i> .
<b>two-dimensional figures</b>	A shape that has two dimensions, usually described in terms of length and breadth, or length and height.
<b>univariate data</b>	Having one <i>variable</i> .
<b>variable</b>	A changing quantity, usually a letter in an algebraic equation or expression, that might have one of a range of possible values.
<b>variance</b>	A measure of the <i>dispersion</i> of the <i>distribution</i> of a <i>random variable</i> .
<b>variants</b>	<i>Variables</i> .
<b>Venn Diagrams</b>	A diagram that is used to show relationships between sets.



<b>zeros of a function</b>	The solutions of a <i>function</i> or the <i>x</i> -intercepts.
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## K-12 Mathematics

# Resources



These sample resources can be used to aid in the understanding of academic content standards. In addition, these resources can be used to begin the process of implementing standards-based instruction and assessment. The model curriculum will provide a greater opportunity to explore best practices, research-based instruction, and effective lessons and strategies for all children.

### **Instructional Resources**

Resources listed in this section provide information for educators seeking practical and creative ways to implement standards-based instruction.

#### *Instructional Resources on the Internet*

- [ERIC Clearinghouse for Science, Mathematics and Environmental Education](http://www.ericse.org) – the Educational Resources Information Center is a collection sponsored by the U.S. Department of Education. Their goal is to provide access to the best information about teaching and learning in the areas of science, mathematics, and the environment for educators, students, and others. Use the icons to access the different areas within ERIC; find research papers and journal articles, use the AskERIC database to get questions answered, access the National Education Library to find information and answer questions.  
<http://www.ericse.org>
- [Eisenhower National Clearinghouse for Mathematics and Science Education \(ENC\)](http://www.enc.org) – is an Internet site full of useful information. The site is organized into four areas; curriculum resources, web links, professional resources and topics. Within the professional resources under “timesavers” there are lesson plans organized by discipline and standards. This is only a starting point for an educator implementing Standards using this site.  
<http://www.enc.org>
- [Illuminations](http://illuminations.nctm.org/) – a collection of Internet resources for improving the teaching and learning of mathematics using the national standards as a foundation.  
<http://illuminations.nctm.org/>
- [K-12 Mathematics Curriculum Center](http://www.edc.org/mcc) – funded by the National Science Foundation this center supports school districts to build effective mathematics education programs using the National mathematics standards. This web site offers curriculum analysis for specific programs of study, resources to assist educators, and much more.  
<http://www.edc.org/mcc>

- [The Math Forum](#) – is an Internet staple for many mathematics teachers. The Internet address has changed (old address: forum.swarthmore.edu). The information, however, is still good. Educators can communicate with one another, find lesson plans, access the Internet Math Library, ask questions of Dr. Math, and access many other useful and interesting mathematical things.

<http://mathforum.org>

- [NCTM's Principles and Standards for School Mathematics](#) – provides guidelines for excellence in mathematics education and issues a call for all students to engage in more challenging mathematics. These guidelines are often referred to as the national mathematics standards. The PSSM content is extended online through the E-Standards web site through resources, Internet links, and more.

<http://standards.nctm.org>

### Instructional Publications

- Barnett, C., Goldstein, D., & Jackson, B. (Eds.). (1994). *Fractions, Decimals, Ratios, & Percents Hard to Teach and Hard to Learn?*. Portsmouth, NH: Heinemann
- Driscoll, Mark (1999). *Fostering Algebraic Thinking: A Guide for Teachers Grades 6-10*. Portsmouth, NH: Heinemann.
- House, Peggy A. (Ed.). (2001). *NCTM Principles and Standards for School Mathematics Navigations Series, Navigating through Algebra in Prekindergarten - Grade 2*, (also in grades 3-5, 6-8 and 9-12). Reston, VA: The National Council of Teachers of Mathematics, Inc .
- National Council of Teachers of Mathematics (2001). *Mathematics Assessment – Cases and Discussion Questions for Grades K-5*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

## Professional Resources

Resources listed in this section provide access to professional organizations and public institutions to afford educators opportunities to stay informed within their field.

### *Professional Organizations*

- [National Council of Teachers of Mathematics](#) – NCTM is an organization dedicated to providing the vision and leadership necessary to ensure a mathematics education of the highest quality for all students. They have provided leadership for the creation and development of the national mathematics standards. This web site offers access to resources, grants, research and much more.  
[www.nctm.org](http://www.nctm.org)
- [National Council of Supervisors of Mathematics](#) – NCSM is an organization for anyone interested in leadership in mathematics education. Find articles from past issues of the Journal of Mathematics Education Leadership, access professional development opportunities, exchange ideas with leaders within the field of mathematics, and more.  
[www.mathforum.org/ncsm](http://www.mathforum.org/ncsm)
- [Ohio Mathematics Education Leadership Council](#) – OMELC is an organization focused on the enhancement and extension of leadership in mathematics education in and for Ohio schools. This site offers current news and information about the organization.  
[www.omelc.educ.kent.edu](http://www.omelc.educ.kent.edu)
- [Ohio Council of Teachers of Mathematics](#) – OCTM is the state affiliate for Ohio of NCTM. The web site offers resources by grade level, Ohio Journal of School Mathematics article abstracts, OCTM calendar, and more.  
[www.ohioctm.org](http://www.ohioctm.org)

### Departments of Education

- [Ohio Department of Education](#) [www.ode.state.oh.us](http://www.ode.state.oh.us)  
 Office of Curriculum and Instruction (Standards info)  
[www.ode.state.oh.us/curriculum-assessment/ci/](http://www.ode.state.oh.us/curriculum-assessment/ci/)  
 Office of Assessment [www.ode.state.oh.us/curriculum-assessment/assessment/](http://www.ode.state.oh.us/curriculum-assessment/assessment/)  
 Career-Technical and Adult Education  
[www.ode.state.oh.us/ctae/](http://www.ode.state.oh.us/ctae/)

- Other state Departments of Education (via CCSSO)  
<http://www.ccsso.org/seamenu.html>
- U.S. Department of Education  
<http://www.ed.gov>

### Research Resources

Resources listed in this section provide theory, skills and strategies to build knowledge and understanding of standards and other related topics.

#### *Research Resources on the Internet*

- **International Association for the Evaluation of Educational Achievement (IEA)** – is an independent, international cooperative of national research institutions and governmental research agencies. Its primary purpose is to conduct large-scale comparative studies of educational achievement, with the aim of gaining a more in-depth understanding of the effects of policies and practices within and across systems of education. To find information about Trends in Mathematics and Science Study (TIMSS) go to this site.  
<http://www.iea.nl/Home/home.html>
- **International Study Center** – is the principle site for IEA’s Trends in Mathematics and Science Study (TIMSS) which is an ongoing study scheduled to end in 2003. This site gives status of the current TIMSS study, elaborates on its framework, provides information for the previous TIMSS study, and details the specifics for other educational studies past and current.  
<http://timss.bc.edu>
- **National Assessment of Educational Progress (NAEP)** – also known as “the Nation’s Report Card,” is the only nationally representative and continuing assessment of what America’s students know and can do in various subject areas. NAEP offers results regarding subject-matter achievement, instructional experiences, and school environment for populations of students (e.g., fourth-graders) and subgroups of those populations (e.g., female students, Hispanic students). From this site you can look at the data and use the data analysis tool to analyze the data.  
<http://nces.ed.gov/nationsreportcard>



- [National Center for Education Statistics](http://nces.ed.gov) – is a federal agency responsible for collecting and analyzing statistical data for education in the US and other countries. One very interesting thing on this web site is the Encyclopedia of Ed Stats which houses a compendium of statistical data from program areas within and outside the US. The encyclopedia also includes statistical projections for the future.  
<http://nces.ed.gov>
  
- [Project 2061](http://www.project2061.org) – is the long-term initiative of the American Association for the Advancement of Science (AAAS) working to reform K-12 science, mathematics, and technology education nationwide. This Internet site offers Standards for All Americans, which includes mathematics components; a proposed school's framework through the Blueprint; professional development opportunities; and much more. This web site also offers methodology and analysis of middle school textbooks for mathematics and science based on the Project 2061 Standards.  
<http://www.project2061.org>
  
- [US National Research Center](http://ustimss.msu.edu) – is a site funded by the National Science Foundation and the National Center for Education Statistics. There are several things of interest from this site related to TIMSS: Internet links and related resources; presentation materials; research and publications; and assessment items used for the study.  
<http://ustimss.msu.edu>

### Research Publications

- Barnett, C., Goldstein, D., & Jackson, B. (Eds.). (1994). *Fractions, Decimals, Ratios, & Percents: Hard to Teach and Hard to Learn? Facilitator's Discussion Guide*. Portsmouth, NH: Heinemann.
- Leinwand, Steven (2000). *Sensible Mathematics: A Guide for School Leaders*. Portsmouth, NH: Heinemann.
- Loucks-Horsley, Susan, Hewson, Peter W., Love, Nancy, & Stiles, Katherine E. (1998). *Designing Professional Development for Teachers of Science and Mathematics*. Thousand Oaks, CA: Corwin Press, Inc..