Literal Equations

Manipulating Variables and Constants

A *literal equation* is one which is expressed in terms of variable symbols (such as *d*, *v*, and *a*) and constants (such as *R*, *g*, and π). Often in science and mathematics you are given an equation and asked to solve it for a particular variable symbol or letter called the *unknown*.

The symbols which are not the particular variable we are interested in solving for are called *literals*, and may represent variables or constants. Literal equations are solved by isolating the unknown variable on one side of the equation, and all of the remaining literal variables on the other side of the equation. Sometimes the unknown variable is part of another term. A *term* is a combination of symbols such as the products *ma* or πr^2. In this case the unknown (such as *r* in πr^2) must be factored out of the term before we can isolate it.

The following rules, examples, and exercises will help you review and practice solving literal equations from physics and chemistry.

PROCEDURE

In general, we solve a literal equation for a particular variable by following the basic procedure below.

1. Recall the conventional order of operations, that is, the order in which we perform the operations of multiplication, division, addition, subtraction, etc.:
   a. Parentheses
   b. Exponents
   c. Multiplication and Division
   d. Addition and Subtraction

   This means that you should do what is possible within parentheses first, then exponents, then multiplication and division from left to right, then addition and subtraction from left to right. If some parentheses are enclosed within other parentheses, work from the inside out.

2. If the unknown is a part of a grouped expression (such as a sum inside parentheses), use the distributive property to expand the expression.

3. By adding, subtracting, multiplying, or dividing appropriately,

   (a) move all terms containing the unknown variable to one side of the equation, and

   (b) move all other variables and constants to the other side of the equation. Combine like terms when possible.

4. Factor the unknown variable out of its term by appropriately multiplying or dividing both sides of the equation by the other literals in the term.

5. If the unknown variable is raised to an exponent (such as 2, 3, or ½), perform the appropriate operation to raise the unknown variable to the first power, that is, so that it has an exponent of one.

EXAMPLES

1. $F = m\mathbf{a}$. Solve for $\mathbf{a}$.
   $F = ma$

   Divide both sides by *m*:

   $$\frac{F}{m} = \mathbf{a}$$

   Since the unknown variable (in this case *a*) is usually placed on the left side of the equation, we can switch the two sides:

   $$\mathbf{a} = \frac{F}{m}$$

2. $P_1V_1 = P_2\mathbf{V}_2$. Solve for $\mathbf{V}_2$.
   $P_1V_1 = P_2\mathbf{V}_2$

   Divide both sides by P_2:

   $$\frac{P_1V_1}{P_2} = \mathbf{V}_2$$

   $$\mathbf{V}_2 = \frac{P_1V_1}{P_2}$$

3. $v = \frac{d}{\mathbf{t}}$. Solve for $\mathbf{t}$.
   Multiply each side by $\mathbf{t}$:

   $$\mathbf{t}v = d$$

   Divide both sides by *v*:

   $$\mathbf{t} = \frac{d}{v}$$

4. $PV = n\mathbf{R}T$. Solve for $\mathbf{R}$.
$PV = n\mathbf{R}T$

Divide both sides by n:

$$\frac{PV}{n} = \mathbf{R}T$$

Divide both sides by $\mathbf{T}$:

$$\frac{PV}{nT} = \mathbf{R}$$

$$\mathbf{R} = \frac{PV}{nT}$$

5. $R = \frac{\rho \mathbf{L}}{A}$. Solve for $\mathbf{L}$.

$$R = \frac{\rho \mathbf{L}}{A}$$

Multiply both sides by A:

$$RA = \rho \mathbf{L}$$

Divide both sides by ρ:

$$\frac{RA}{\rho} = \mathbf{L}$$

$$\mathbf{L} = \frac{RA}{\rho}$$

6. $A = h(a + \mathbf{b})$. Solve for $\mathbf{b}$.

7. $P = P_0 + \rho \mathbf{g} h$. Solve for **g**.

8. $U = \frac{1}{2}\mathbf{Q}V$. Solve for **Q**.

9. $U = \frac{1}{2}k\mathbf{x}^2$. Solve for **x**.

10. $T = 2\pi\sqrt{\frac{\mathbf{L}}{g}}$. Solve for $\mathbf{L}$.

11. $F = \frac{Gm_1m_2}{\mathbf{r}^2}$. Solve for $\mathbf{r}$.

12. $\frac{h_i}{h_o} = -\frac{s_i}{\mathbf{s}_o}$. Solve for $\mathbf{s}_o$.

13. $\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{\mathbf{R}_3}$. Solve for $\mathbf{R}_3$.

14. $F = qvB\sin\theta$. Solve for $\boldsymbol{\theta}$.

15. $\boldsymbol{\mu} mg\cos\theta = mg\sin\theta$. Solve for $\boldsymbol{\mu}$.

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EXERCISES

Directions: For each of the following equations, solve for the variable in **bold** print. Be sure to show each step you take to solve the equation for the **bold** variable.

1. $v = \mathbf{a}t$

2. $P = \dfrac{F}{\mathbf{A}}$

3. $\lambda = \dfrac{\mathbf{h}}{p}$

4. $F\left(\Delta \mathbf{t}\right) = m\Delta v$

5. $U = -\dfrac{G\boldsymbol{m_1}m_2}{r}$

6. $C = \frac{5}{9}(\mathbf{F} - 32)$

7. $v^2 = v_0{}^2 + 2\mathbf{a}\Delta x$

8. $K_{avg} = \frac{3}{2}k_B\mathbf{T}$

9. $K = \frac{1}{2}m\mathbf{v}^2$

10. $v_{rms} = \sqrt{\frac{3RT}{\mathbf{M}}}$

11. $v_{rms} = \sqrt{\frac{3\mathbf{k}_B T}{\mu}}$

12. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{\mathbf{r}^2}$

13. $\dfrac{1}{s_i} + \dfrac{1}{s_o} = \dfrac{1}{\mathbf{f}}$

14. $\dfrac{1}{C_{EQ}} = \dfrac{1}{\mathbf{C}_1} + \dfrac{1}{C_2}$

15. $V = \dfrac{4}{3}\pi\mathbf{r}^3$

16. $P + \mathbf{D}gy + \dfrac{1}{2}\mathbf{D}v^2 = C$

17. $P + Dgy + \frac{1}{2}D\mathbf{v}^2 = C$

18. $x = x_0 + v_0 t + \frac{1}{2}\mathbf{a}t^2$

19. $n_1 \sin\theta_1 = n_2 \sin \boldsymbol{\theta}_{\mathbf{2}}$

20. $mg \sin \boldsymbol{\theta} = \mu mg \cos \boldsymbol{\theta} \left(\frac{M + m}{m}\right)$