Introduction to Systems Cost Uncertainty Analysis

An Engineering Systems Perspective

National Institute of Aerospace (NIA)
Systems Analysis & Concepts Directorate
NASA Langley Research Center

Distinguished Lecture Series

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Lecture Objectives

An Appreciation of…

How the government looks at the problem of quantifying uncertainties in the cost of future systems

The problem context and fundamental technical concepts

Example applications and some best practices

The Questions We'll Address…

When the cost of a future system is estimated, decision-makers often ask…

What is the chance the estimate will be exceeded?

How much could cost overrun?

What are the uncertainties and how do they drive cost?

A Motivating Case-in-Point

Boston's "BIG DIG"

Initial 1987 cost estimate: $4B (current year dollars)

Current cost … $14.6B

An increase of 265 percent!!

Cost uncertainties were never factored into estimates

Ref: http://www.csmonitor.com/2003/1219/p02s01-ussc.html

Ref: http://www.masspike.com/bigdig/index.html

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Introduction
Background

Problem Context

This lecture presents methods for quantifying the cost impacts of uncertainty in the engineering of systems; the term “systems” is used to mean physical systems

Physical systems are those that manifest themselves in physical terms and occupy physical space, e.g.,

Radar Systems	Air Traffic Control Systems
Automobiles	Communications Systems

Systems engineering is a process that produces physical systems; it encompasses the scientific and engineering efforts needed to develop, produce, and sustain systems

Systems engineering is a highly complex technical and managerial undertaking!

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Introduction
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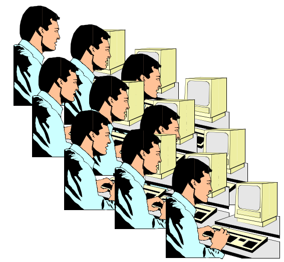
Problem Context

Systems engineering is a “people” activity that can involve

- Integrating custom equipment with commercial products
- Designing external interfaces
- Achieving user requirements
- Meeting aggressive schedules while also keeping within cost



When the cost of a future system is considered, decision-makers often ask

- What is the chance its cost will exceed a particular amount?
- How much could cost overrun?
- What are the uncertainties and how do they drive cost?

Cost uncertainty analysis provides decision-makers insight into these and related questions

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Background

Some Definitions

Definition: Risk: The chance of loss or injury. In a situation that includes favorable and unfavorable events, risk is the probability an unfavorable event occurs

Definition: Uncertainty: The indefiniteness about the outcome of a situation - it includes favorable and unfavorable events. We analyze uncertainty for the purpose of measuring risk!

In systems engineering, analysis might focus on measuring the risk of

{failing to achieve performance objectives}

{overrunning the budgeted cost}

{delivering the system too late to meet user needs}

these are examples of three unfavorable events

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Some Definitions

Definition: Cost Uncertainty Analysis: The process of quantifying the cost impacts of uncertainties associated with an engineering system's technical definition and cost estimation methodologies

Definition: Cost Risk Analysis: The process of quantifying the cost impacts of risks associated with an engineering system's technical definition and cost estimation methodologies

Definition: Cost Risk: A measure of the chance that, due to unfavorable events, the planned or budgeted cost of an engineering systems project will be exceeded

Why Conduct the Analysis?

To produce a defensible assessment of the level of cost to budget such that this cost has an acceptable chance of not being exceeded

Introduction

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Systems Cost Uncertainty Analysis

Systems cost uncertainty analysis had its genesis in a field known as military systems analysis, founded in the 1950's at RAND

Shortly after WWII, military systems analysis evolved as a way to aid defense planners with long-range decisions on force structure, force composition, and future theaters of operations

Cost became (and remains) a critical consideration in military systems analysis models and decision criteria

However, cost estimates of future military systems, particularly in their early planning phases, were often significantly lower than the actual cost or an estimate developed at a later phase

Gene Fisher* (RAND) in *Cost Considerations in Systems Analysis* attributes this difference to the presence of uncertainty; specifically cost estimation uncertainty and requirements uncertainty

* Fisher, G. H., 1971. *Cost Considerations in Systems Analysis*, New York: American Elsevier Publishing Co., Inc.

Introduction

Background

Types of Uncertainties

Cost Estimation Uncertainty: Originates from inaccuracies in cost-schedule estimation models, from the misuse (or misinterpretation) of cost-schedule data, or from misapplied cost-schedule estimation methods; economic uncertainties that influence the cost of technology, the labor force, or geo-political policies further contributes to cost estimation uncertainty

Requirements Uncertainty: Originate from changes in the system's mission objectives, from changes in performance requirements necessary to meet mission objectives, or from changes in the business or political landscapes that affect the need for the system

Requirements uncertainty most often results in changes to the system's specified hardware-software configuration, also referred to as the system's architecture

Uncertainty is also present in elements that define a system's configuration (or architecture); this is referred to as system definition uncertainty

From Garvey, P. R., "*Probability Methods for Cost Uncertainty Analysis-A Systems Engineering Perspective*", published by Marcel Dekker, Inc and CRC Press., 270 Madison Avenue, New York, NY, 10016-0602, 2000

Introduction

Background

Types of Uncertainties

System Definition Uncertainty: Includes uncertainties in the amount of software to develop, the extent that code from another system can be reused, the number of workstations to procure, or the delivered weight of an end item (e.g., a satellite)

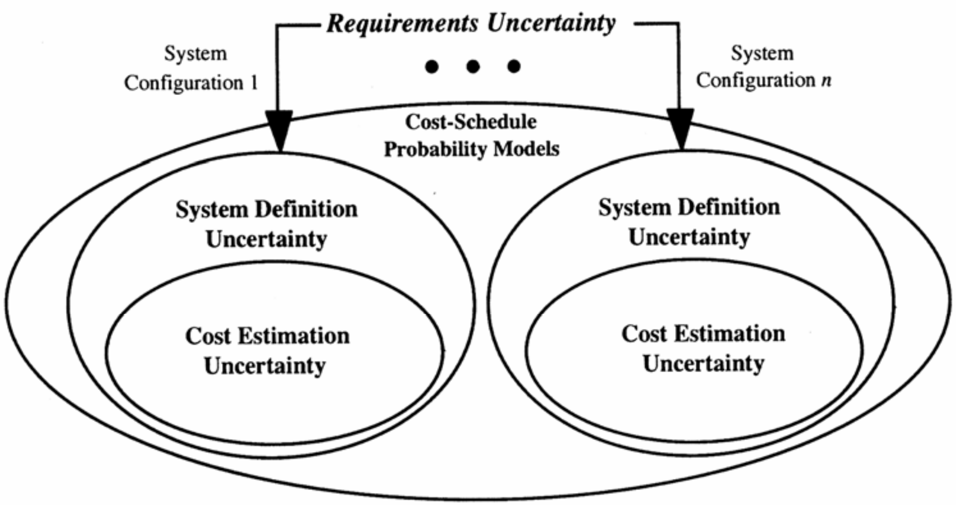


Figure 1-1. Types of Uncertainty Captured by Cost-Schedule Probability Models

From Garvey, P. R., "*Probability Methods for Cost Uncertainty Analysis-A Systems Engineering Perspective*", published by Marcel Dekker, Inc and CRC Press., 270 Madison Avenue, New York, NY, 10016-0602, 2000

Fundamental Technical Concepts

Cost as a Probability Distribution: For A Single Cost Element

Probability Theory

A traditional formalism for quantifying and analyzing uncertainty

Used to represent and model uncertainties in an engineering system project that impacts the project's cost

Probability Distributions

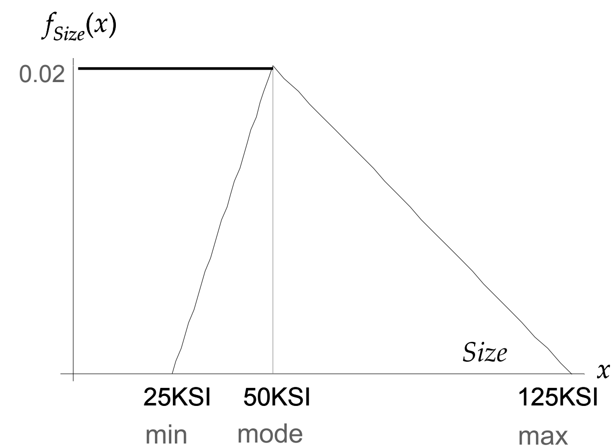
Probability distributions are specified that characterize uncertainties in the input variables of elements that contribute to an engineering system's cost

For example, the cost (effort) to develop software might be estimated as follows:

$$Effort = 2.8(Size)^{1.2}$$

Here, *Size* is the number of thousands of source instructions (KSI) to develop and *Effort* is in Staff Months (s)

The uncertainty in the number of source instructions might be characterized by a triangular probability density function (PDF), as shown to the right

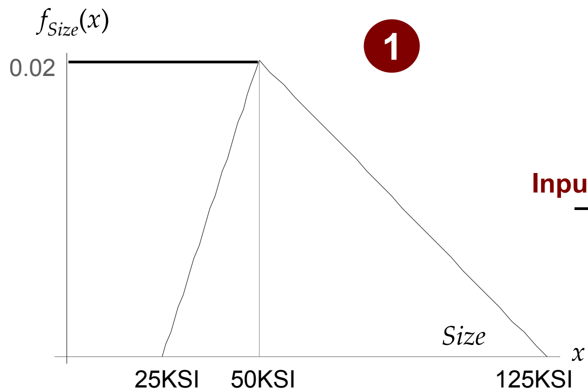


Fundamental Technical Concepts

Cost as a Probability Distribution: For A Single Cost Element

This is an example of an Element level cost probability distribution

1



Input…

Size PDF

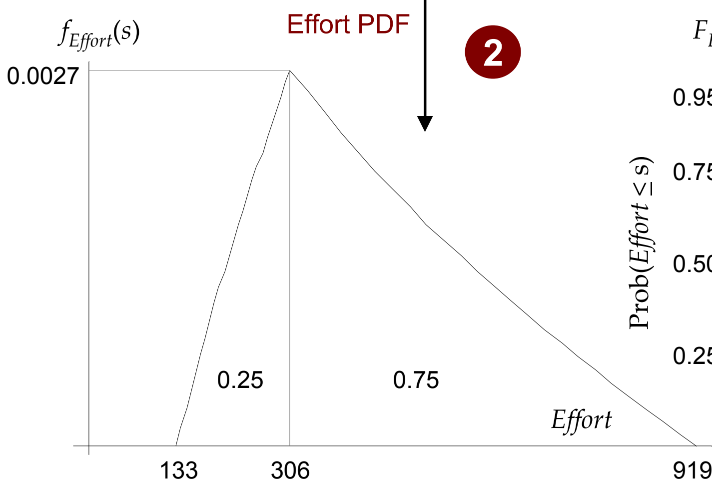
$$Effort = 2.8(Size)^{1.2}$$

Outputs: Probability Calculus Yields…

This can be "tricky" mathematically

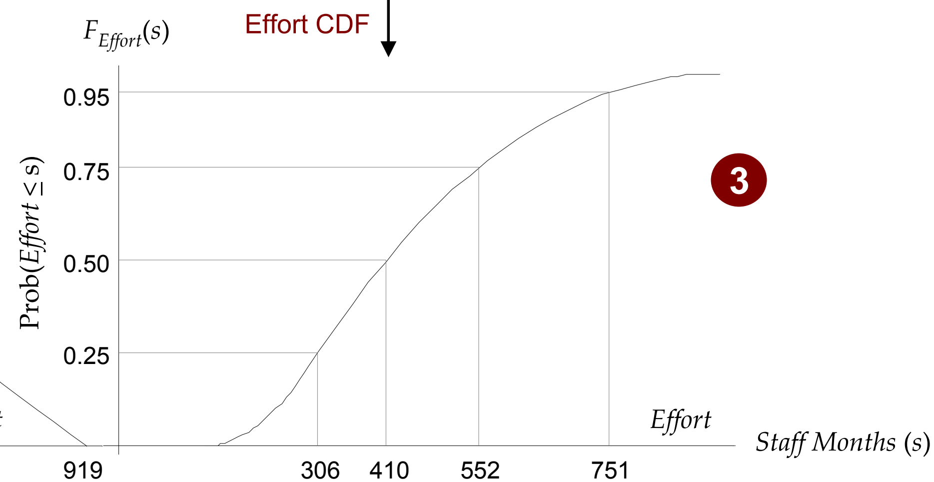
Effort PDF

2



Effort CDF

3



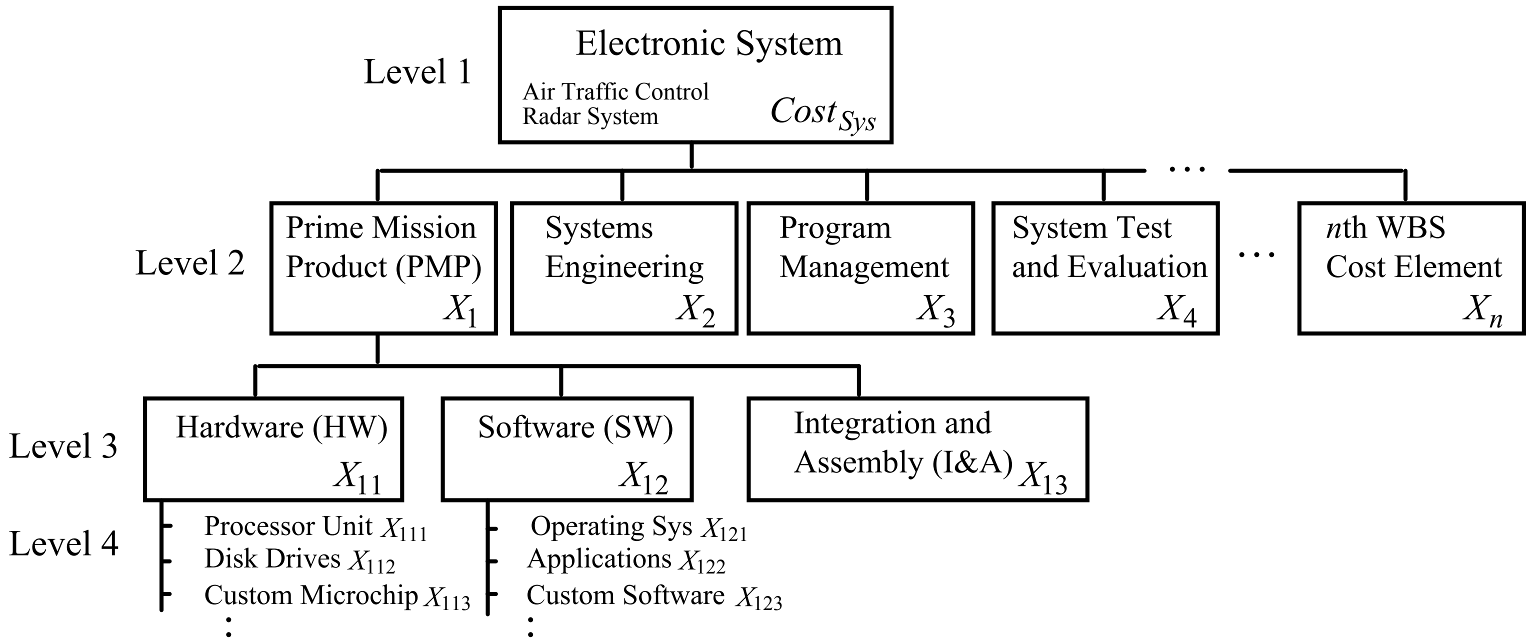
Fundamental Technical Concepts

Cost as a Probability Distribution: For Multiple Cost Elements

The Work Breakdown Structure (WBS)

A representation of an engineering system's complete set of cost element costs, whose sum is the total cost of the system, $Cost_{Sys}$

$$Cost_{Sys} = X_1 + X_2 + X_3 + \ldots + X_n$$

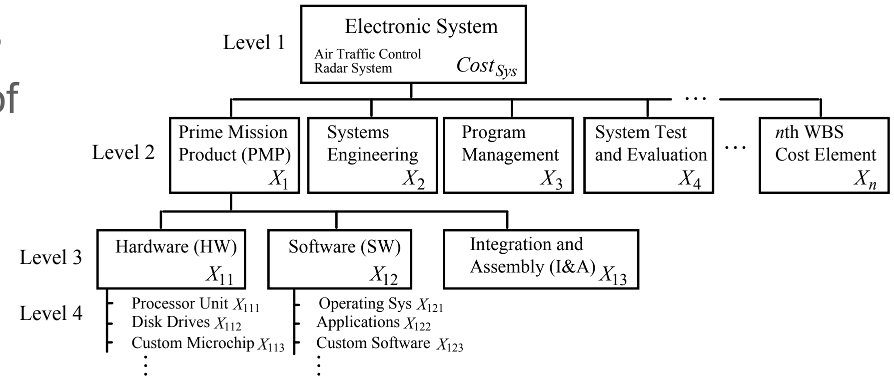


Fundamental Technical Concepts

Cost as a Probability Distribution: For Multiple Cost Elements

Fundamental Statistical Formulas

The WBS is essentially a “math model” of the cost of an engineering system; as such, one can write down statistical formulas for this model



1. $$Cost_{Sys} = X_1 + X_2 + X_3 + \ldots + X_n$$

2. $$E(Cost_{Sys}) = E(X_1) + E(X_2) + E(X_3) + \ldots + E(X_n)$$

3. $$Var(Cost_{Sys}) = \sum_{i=1}^{n} Var(X_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \rho_{X_i, X_j}\sigma_{X_i}\sigma_{X_j}$$

4. $$Cost_{Sys} \sim Normal(E(Cost_{Sys}), Var(Cost_{Sys}))$$ **or sometimes LogNormal**

From Garvey, P. R., “*Probability Methods for Cost Uncertainty Analysis-A Systems Engineering Perspective*”, published by Marcel Dekker, Inc and CRC Press., 270 Madison Avenue, New York, NY, 10016-0602, 2000

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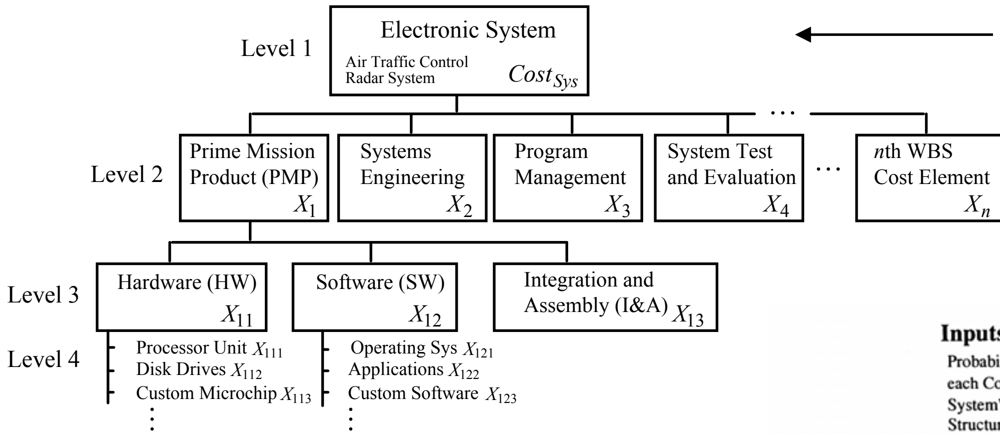
Fundamental Technical Concepts

Cost as a Probability Distribution: For Multiple Cost Elements



Deterministic WBS Model

Which produces a deterministic estimate of the cost of an engineering system project

Probabilistic WBS Model

Which produces a range of possible costs, as a function of the uncertainties inherent the variables that drive the individual Element costs

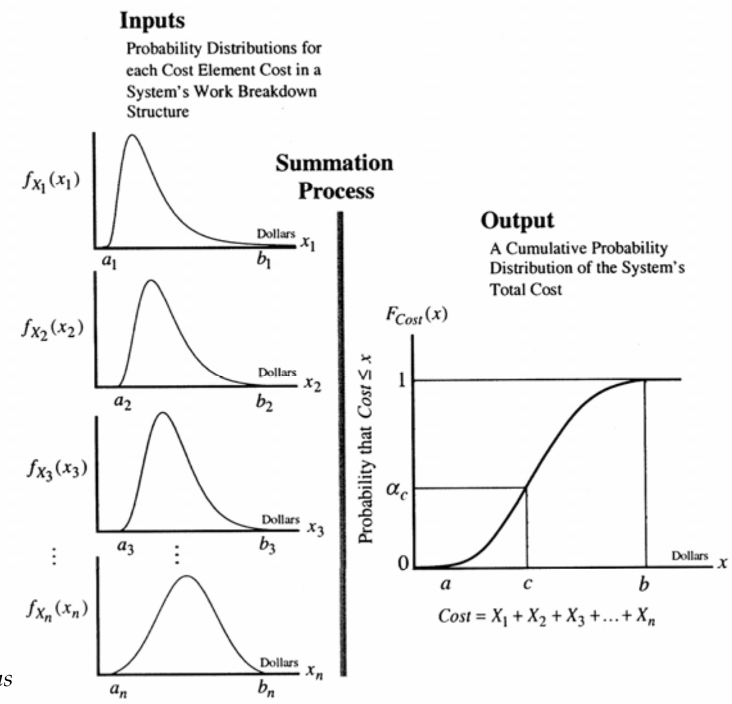


Figure 1-3. Cost Uncertainty Analysis Process

From Garvey, P. R., "*Probability Methods for Cost Uncertainty Analysis-A Systems Engineering Perspective*", published by Marcel Dekker, Inc and CRC Press., 270 Madison Avenue, New York, NY, 10016-0602, 2000

Fundamental Technical Concepts

Cost as a Probability Distribution: For Multiple Cost Elements

Table 6-2. WBS for Case Discussion 6-1

Cost Element Name	Cost Element Cost X_i ($M)	Distribution of X_i or the Applicable Functional Relationship
Prime Mission Product (PMP)	X_1	$N(12.5, 6.6)$
System Engineering and Program Management (SEPM)	X_2	$X_2 = \frac{1}{2} X_1$
System Test & Evaluation (STE)	X_3	$X_3 = \frac{1}{4} X_1 + \frac{1}{8} X_2 + W$, where $W \sim Unif(0.6, 1.0)$
Data and Technical Orders	X_4	$X_4 = \frac{1}{10} X_1$
Site Survey and Activation	X_5	$Trng(5.1, 6.6, 12.1)$
Initial Spares	X_6	$X_6 = \frac{1}{10} X_1$
System Warranty	X_7	$Unif(0.9, 1.3)$
Early Prototype Phase	X_8	$Trng(1.0, 1.5, 2.4)$
Operations Support	X_9	$Trng(0.9, 1.2, 1.6)$
System Training	X_{10}	$X_{10} = \frac{1}{4} X_1$

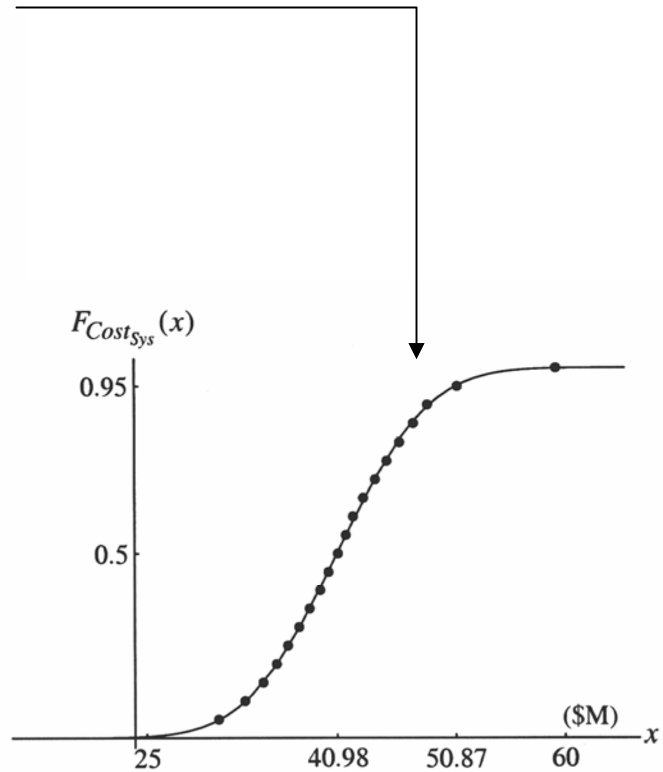


Figure 6-5a. Assumed Normal CDF for $Cost_{Sys}$ (defined by the solid line) vs the Simulated CDF (defined by the points)

$$Cost_{Sys} = X_1 + X_2 + X_3 + \ldots + X_n$$

From Garvey, P. R., "*Probability Methods for Cost Uncertainty Analysis-A Systems Engineering Perspective*", published by Marcel Dekker, Inc and CRC Press., 270 Madison Avenue, New York, NY, 10016-0602, 2000

Fundamental Technical Concepts

Cost as a Probability Distribution

How Many Reserve Dollars to Budget?

Suppose the point estimate of a system is 100 ($M); this value of *Cost* has just over a 30 percent chance of not being exceeded

A reserve of 20 ($M) added to the point estimate is associated with a value of *Cost* that has a 67 percent chance of not being exceeded

A reserve of 40 ($M) added to the point estimate is associated with a value of *Cost* that has just over a 90 percent chance of not being exceeded

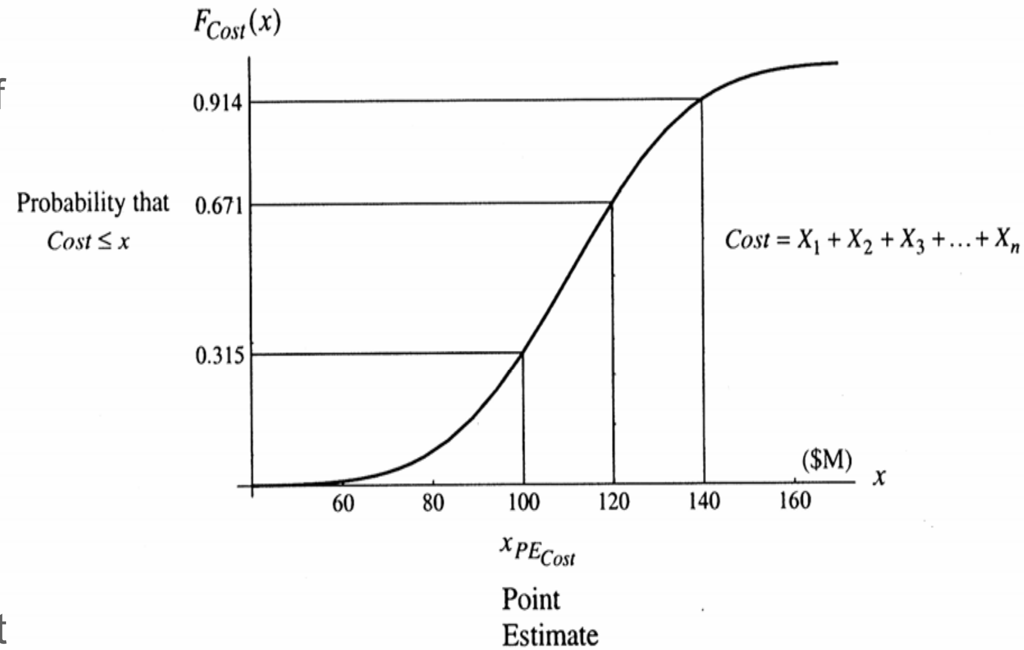


Figure 1-6. A Cumulative Probability Distribution of System Cost

Point Estimate: Is the estimate of an engineering system's cost without adjustments for uncertainty.

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Summary

Some Best Practices

Can Cost be Treated as a Random Variable?

The mathematical vehicle for working with a range of possible costs is the probability distribution, with cost itself viewed as a "random variable". Such terminology does not imply that costs are "random" (though well they may be!) but rather they are composed of a large number of very small pieces, whose individual contributions to the whole we do not have the ability to investigate in a degree of detail sufficient to calculate the total cost precisely

It is much more efficient for us to recognize that virtually all components of cost are simply "uncertain" and to find some way to assign probabilities to various possible ranges of costs

Coin Toss Analogue … S. A. Book

An analogue is the situation in coin tossing where, in theory, if we knew all the physics involved and solved all the differential equations, we could predict with certainty whether a coin would fall "heads" or "tails". However, the combination of influences acting on the coin are too complicated to understand in sufficient detail to calculate the physical parameters of the coin's motion. So we do the next best thing: we bet that the uncertainties will probably average out in such a way that the coin will fall "heads" half the time and "tails" the other half. It is much more efficient to consider the deterministic physical process of coin tossing to be a "random" statistical process and to assign probabilities of 0.50 to each of the two possible outcomes, heads or tails."

From Garvey, P. R., "*Probability Methods for Cost Uncertainty Analysis-A Systems Engineering Perspective*", published by Marcel Dekker, Inc and CRC Press., 270 Madison Avenue, New York, NY, 10016-0602, 2000

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Summary
Some Best Practices

Capturing Cost-Schedule Uncertainties

Decision-makers require understandin how uncertainties between a system's cost and schedule interact

A decision-maker might bet on a "high risk" schedule in hopes of keeping the system's cost within requirements

On the other hand, the decision-maker may be willing to assume "more cost" for a schedule with a small chance of being exceeded

This is a common tradeoff faced by decision-makers on engineering systems projects

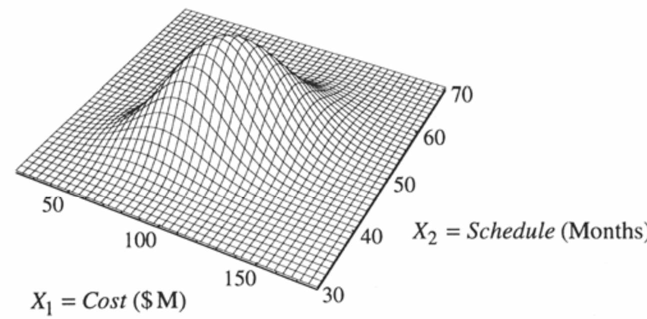

Figure 7-5. A Bivariate Normal-LogNormal Density

$(X_1, X_2) \sim Bivariate\ NLogN((100, 3.86345), (625, 0.0155042, 0.501944))$

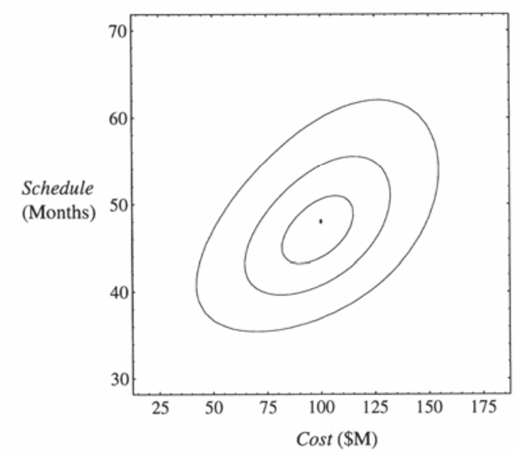

Figure 7-6. Contours of a Bivariate Normal-LogNormal Density

$(X_1, X_2) \sim Bivariate\ NLogN((100, 3.86345), (625, 0.0155042, 0.501944))$

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Summary

Some Best Practices

Capturing Cost-Schedule Uncertainties

The difference between the conditional median cost (107.8 ($M)) given a schedule of 53 months and the conditional median cost (87.4 ($M)) given a “high-risk” schedule of 43 months is 20.4 ($M)

In the context of figure 7-1, this difference in cost is certainly significant for any cost-schedule tradeoffs under consideration

This highlights how joint probability models can be used to analyze cost-schedule interactions and reveal important tradeoffs between them

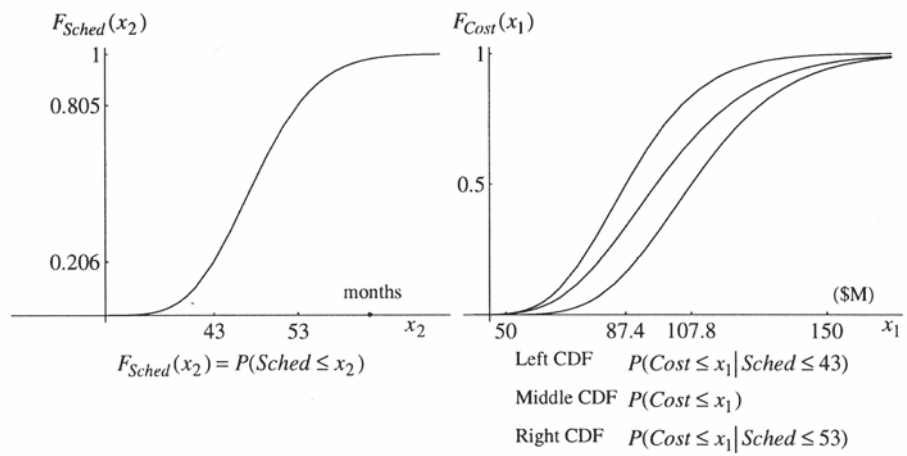

Figure 7-1. Illustrative Distributions for a System’s Cost and Schedule

From Garvey, P. R., “*Probability Methods for Cost Uncertainty Analysis-A Systems Engineering Perspective*”, published by Marcel Dekker, Inc and CRC Press., 270 Madison Avenue, New York, NY, 10016-0602, 2000

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Summary

Benefits of Cost Uncertainty Analysis

Determining Cost Reserve

Cost uncertainty analysis provides a basis for determining cost reserve as a function of the uncertainties specific to a system

The analysis provides the direct link between the amount of cost reserve to recommend and the probability that a system's cost will not exceed a prescribed (or desired) magnitude

An analysis should be conducted to verify the recommended cost reserve covers fortuitous events (e.g., unplanned code growth, unplanned schedule delays) deemed possible by the system's engineering team

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Summary

Benefits of Cost Uncertainty Analysis

Conducting Risk Reduction Tradeoff Analyses

Cost uncertainty analyses can be conducted to study the payoff of implementing risk reduction initiatives (e.g., rapid prototyping) on lessening a system’s cost and schedule risks

Furthermore, families of probability distribution functions can be generated to compare the cost and cost risk impacts of alternative system requirements, schedule uncertainties, and competing system configurations or acquisition strategies

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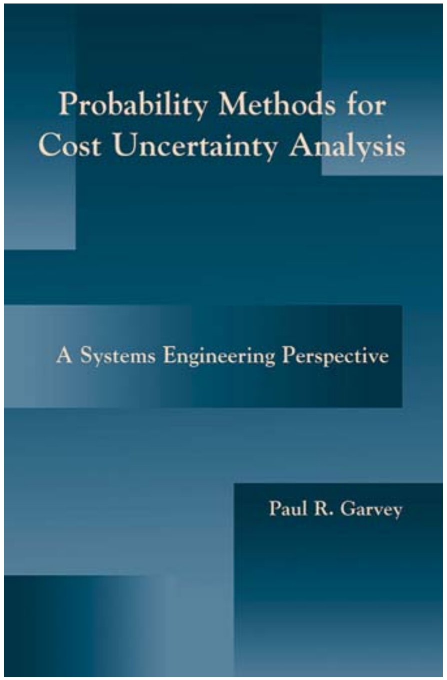
To Learn More…



A Textbook

A student and reference text on this topic in the systems cost analysis and cost engineering communities

Published in January 2000 by Marcel Dekker, Inc. New York, NY; CRC Press, Taylor & Francis Group

URLs:…lots of places…

http://www.amazon.com

http://www.crcpress.com

A Manuscript

Analytical Methods for Risk Management: A Systems Engineering Perspective

Will be published in 2007 by CRC Press, Taylor & Francis Group