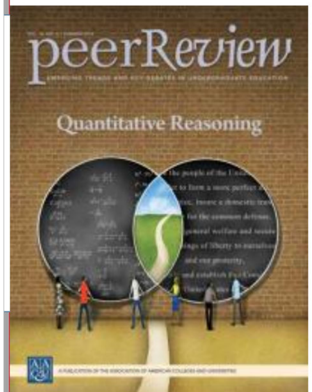

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Peer Review

Beyond Calculation

By: Bernard L. Madison and David Deville

Ten years ago, the first author wrote "Two Mathematics: Ever the Twain Shall Meet?" (Madison 2004) for an issue of *Peer Review* that focused on quantitative literacy (QL). In the ensuing decade QL, now referred to by many as quantitative reasoning, has gained considerable recognition as an effort by colleges and universities to ready their graduates for life in the quantitatively demanding US society and global community. In many ways, the quantitative demands of that society differ from the disciplinary world of academe, adding significant challenges to education for QL. Just what kind of course or program can prepare students for confronting the myriad quantitative issues in their everyday lives, saying nothing about the demands of their chosen professions?

A Decade of Progress

Before we look at some principles to consider and possible course models, we note some progress in QL education over the past decade. Starting in 2000, historian Robert Orrill and mathematician Lynn Steen led an initiative to promote better education for QL in high school and the early years of college. Part of that initiative was the creation of the National Numeracy Network (NNN), initially conceived as a confederation of QL centers but reconstituted as an interdisciplinary membership organization in 2004. Now NNN has hundreds of members and its journal, *Numeracy: Advancing Education in Quantitative Literacy,* is publishing in 2014 its seventh volume of two issues annually. Textbooks aiming at QL and instruments for assessing QL have been written, scores of institutions have added courses or learning centers for QL, some institutions have integrated QL across the curriculum, and the Mathematical Association of America has created a special interest group in QL that is noting its tenth anniversary this year. QL is becoming accepted as an expected learning outcome of college. For example, the Arkansas Department of Higher Education approved the inclusion of a QL course as part of the State Minimum Core of collegiate courses as an alternative to college algebra for students not majoring in science, engineering, or mathematics.

In 2009, AAC&U's Valid Assessment of Learning in Undergraduate Education (VALUE) project included QL as one of the ten intellectual and practical skills and developed a rubric for assessing QL at the institutional level (see page 22). Subsequently modified by Boersma et al. (2011) for assessing individual student work, the rubric identified six criteria for QL: interpretation, representation, calculation, analysis/synthesis, assumptions, and communication. These VALUE rubric core competencies, described below, provide a way to structure students' work toward QL and a guide for developing instructional materials, as well as a framework of an assessment instrument.

- *Interpretation:* Ability to glean and explain mathematical information presented in various forms (e.g., equations, graphs, diagrams, tables, words).

- *Representation:* Ability to convert information from one mathematical form (e.g., equations, graphs, diagrams, tables, words) into another.
- *Calculation:* Ability to perform arithmetical and mathematical calculations.
- *Analysis/Synthesis:* Ability to make and draw conclusions based on quantitative analysis.
- *Assumptions:* Ability to make and evaluate important assumptions in estimation, modeling, and data analysis.
- *Communication:* Ability to explain thoughts and processes in terms of what evidence is used, and how it is organized, presented, and contextualized.

Challenges that Remain

The progress toward better QL education has been significant, driven by the recognition that QL is absolutely necessary for understanding democratic processes and thriving in a rapidly moving, economically volatile US society. Yet, significant educational questions remain. What learning theory best identifies issues in QL? Is it situated learning, since, from our view, all QL learning is situational or contextual? What pedagogy is most effective for QL education? What is/are the community/ies of practice for QL? How should QL fit into higher education? In mathematics? In statistics? Across the curriculum? Elsewhere?

Currently, in K-12, QL depends almost completely on the mathematics strand, and in higher education, many QL courses are housed in mathematical science departments or interdisciplinary learning centers. As of now, there are no established guidelines for QL courses and no accepted, effective measures of long-term retention and transfer. Mathematics and statistics courses are usually described by their content (e.g. calculus, differential equations, probability, or experimental design). And mastery of content is the measure of success. Such a description for a QL course is elusive, as the mathematical and statistical content needed for QL currently does not have a clear description and consequently varies from course to course. In the absence of accepted mathematical content and measures of success, one must look elsewhere for building or evaluating QL courses. The first author (Madison 2014) has described such a process that stems from evaluating a QL course at the University of Arkansas that is housed in the mathematical sciences department.

Mathematics as Sensible, Useful, and Worthwhile

At about the time "Two Mathematics" was published in *Peer Review* ten years ago, a QL course was introduced at the University of Arkansas and has been evolving since. Dingman and Madison (2011) wrote for this journal that teaching this course altered their perspectives on several things, including the role of the instructor, relevant mathematical content, use of technology, and sense-making in the messy world of realism. In the past three years our QL faculty members have begun to understand better the structure of QL courses and some guiding principles that seem necessary.

At the current time, there are two Arkansas QL courses: one with college algebra as a prerequisite and another that is an alternative to that course, which in this article will be referred to as QL1. The QL1 course was developed in 2012 using design principles derived from eight years of experience with the other QL course with a college algebra prerequisite. These design principles are supported by research findings about student learning (National Research Council 2000, 2001; Halpern and Hakel 2003). QL1 has two primary goals: (1) encourage students to develop habits of mind to analyze the quantitative content of everyday occurrences, and (2) increase students' productive disposition—that is, the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. Productive disposition is one of the strands of mathematical proficiency from the *Adding It Up* report of the National Research Council (2001) and its absence is a major barrier for QL in many math-phobic students.

Relevant Design Principles for a QL Course
Briefly, the design principles for a QL course are as follows:

Provide a venue for continued practice beyond the course (and beyond school). Quantitative reasoning is a habit of mind, and habits are developed by practice. One or two courses or four years of school can only prepare one for practicing QL. The venue for continued practice in the Arkansas course is media articles.

Keep the material relevant to students' everyday contemporary world. According to John Dewey, "School should be less about preparation for life and more about life itself." Connecting classroom learning to the everyday contemporary world not only can enhance learning in the classroom but can also lead students to adapt their classroom learning to the changing environment of everyday life. Relevance promotes productive disposition, noted above as a primary goal, and keeps material fresh.

Use multiple contexts to practice quantitative reasoning. According to Halpern and Hakel (2003), "The purpose of formal education is transfer" (38). Halpern and Hakel go on to identify retrieval in multiple contexts as one of the most basic principles for enhancing long-term retention and transfer of learning and indicate that periodically spaced, not massed, practice at retrieval is best.

Promote appreciation of arithmetical precision and the power of mathematical concepts and processes. This principle is often difficult to apply in a course where the main goal is to understand contextual situations with quantitative content. Nevertheless, when opportunities arise to make use of mathematical power by developing some algebra, doing so, when needed, shows students the power and utility of mathematics, getting at half of the dual nature of productive disposition.

Help students to structure their quantitative reasoning in resolving problematic situations, including ample doses of critical reading and writing. One way to do this is by using the QL core competencies of interpretation, representation, calculation, analysis/synthesis, assumptions, and communication (Carey 2009; Boersma et al. 2011). Critical reading is the foundation of interpretation, and writing promotes reflection and clear understanding.

Encourage on-the-fly calculations and estimations. If students are able to assess quickly the validity of a quantitative assertion or mentally compute a numerical result, then they will be better able to practice QR in their daily lives. Practice should become reflexive and habitual.

Increase students' supplies of quantitative benchmarks. Personal quantitative benchmarks are quantities that a student understands. For example, a student may understand a speed of 60 miles per hour (MPH) but not 1200 MPH. Such benchmarks are critical for understanding quantities (e.g., 1200 MPH is 20 times 60 MPH) and being able to determine reasonableness of quantitative assertions or numerical answers to questions. Providing multiple contexts for the use of benchmarks increases the chances that students retain the benchmarks and recognize their utility.

Encourage students to use technology to enhance and expedite understanding. Technology, including personal devices, is omnipresent in students' everyday lives, so it should be leveraged in service of understanding QL.

Allow student interests to emerge. As reported in How People Learn, "Students are motivated to spend time needed to learn complex problems that they find interesting. Opportunities to use knowledge to create products and benefits for others are particularly motivating for students" (National Resource Council 2000, 77). Again, this promotes productive disposition.

Provide an interactive classroom environment. Interactive classrooms engage students in sense-making activities and promote personal accountability. Successful QL students are able to step outside of their comfort zones and assume responsibility for their work. Further, if we intend for students to use QL outside of the classroom, possibly in discussions of public issues, then the classroom experience should provide preparation for this practice.

Calculations and Beyond

At the 2007 Wingspread interdisciplinary QL workshop, considerable discussion focused on the role of mathematical methods and calculations in QL. Sociologist Joel Best (2008A) considered calculation "to encompass all of the practices by which mathematical problems are framed and then solved" (125), or what mathematics classes center on. Best went on to argue that QL courses should go beyond calculation to include issues surrounding constructions—more specifically, social constructions.

We broaden Best's view a bit and consider a model of a QL course that consists of two components: our model includes calculation, in the sense described by Best, but also goes beyond calculation to encompass the many issues and dispositions involved in twenty-first-century QL in the United States. By "beyond calculation" we mean to include contributions from the arts, humanities, social sciences, natural sciences, public media, and entertainment—any area of human activity. As a mathematics course, the calculation component is unusual in the sense that it is fragmented, without any obvious unifying concepts. Further, the mathematical concepts and methods in the Arkansas QL1 course do not necessarily include concepts and methods of other QL courses that may be directed at different audiences. The Arkansas QL1 course is directed toward the general education of students in majors other than science, engineering, or business.

Because the mathematics of the QL1 course is largely from the

K–12 curriculum, it could be seen as developmental. However, the sophistication of the course is in the "beyond calculation" component—echoing Lynn Steen's characterization of QL as sophisticated uses of elementary mathematics and statistics.

In table 1, the top rectangle represents the resolution of a canonical QL situation using the AAC&U LEAP Essential Learning Outcomes. One encounters a QL situation—say in a media article about economics—interprets the quantitative content, and produces a mathematical representation—say a linear equation. Then the problem becomes one of calculation. After the calculation and the results are analyzed and assumptions evaluated or noted, the results are communicated. This illustrates the habit of mind we want our students to develop. One of the major obstacles to developing this habit is a low level of productive disposition—the ability to see calculation as useful and have the confidence and skill to use it to understand the situation. Also, observing and critiquing this process can be difficult, especially since many students are comfortable with their traditionally passive roles in the mathematics classroom.

Table 1. The Arkansas QL1 Course—Calculation and Beyond

CALCULATION A LA BEST		BEYOND CALCULATION (QL)
	VALUE Rubric Criterion	
• Calculation[1]	← Representation Analysis/Synthesis →	• Interpretation • Assumptions • Communication
	Arkansas QL1 Course	
• Rational numbers • Ratio • Linear change • Exponential change • Statistical averages • Statistical spreads • Rates • Counting • Probability • Mental arithmetic • Measurement & units	↔ Representation	• Habit of mind • Productive disposition • Writing • Critical reading • Argument & evidence • Quantitative benchmarks • Contextual accuracy • Social constructions • Economic constructions • Comparative analysis • Political constructions

[1] *Calculation here is one of the QL core competencies and differs somewhat from the component of the same name as articulated by Joel Best (2008A).*

Some of the items in the beyond calculation component are familiar, but some are not. Neil Lutsky and colleagues (2008) have written extensively about the use of quantitative evidence in argument in student writing at Carleton College. Joel Best (2008B) has noted the importance of statistical benchmarks —broadened here to go beyond statistics—in understanding US social statistics. Economic constructions, such as the various stock indices, appear frequently in the media, and yet they are mysterious to most people. Political constructions that require QL often arise in disagreements about or differing views of budgetary situations—for example, the expression of the annual federal budget deficit in nominal dollars or as a percent of the gross domestic product. And graduates will frequently encounter comparative analyses, which usually require ad hoc methods. For example, comparing two credit card offers often comes down to individual preferences or expected uses, as the specifics of the offers are not directly comparable.

Final Thoughts

For quite some time, "quantitative reasoning" has been an accepted learning outcome of college, often without exemplification or clarification and frequently without authentic assessment. As US society has become immersed in quantification and quantitative analyses, specific and intentional QL education efforts have become essential. Over the past decade these efforts have been taking shape, but they remain varied and largely unevaluated. Communities of practice are forming, and research on QL learning is deepening. Professional societies, most notably AAC&U, the National Numeracy Network, and the Mathematical Association of America, lead many of these efforts. Most colleges and universities have created educational responses—courses, cross-curricular programs, or learning centers. Frameworks for QL courses are emerging, presaging standards for evaluation and some coherence, transferability, and expansion of learning. Many of the developments in QL education are described in the NNN journal Numeracy, available at http://scholarcommons.usf.edu/numeracy/. Significant progress has marked the decade since the 2004 QL issue of Peer Review, but the educational ground is fertile for better understanding of QL and ways to help students achieve it. Informed participation in US society and individual prosperity depend greatly on those outcomes.

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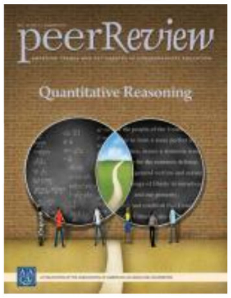

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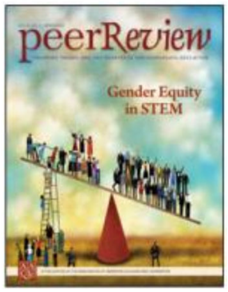

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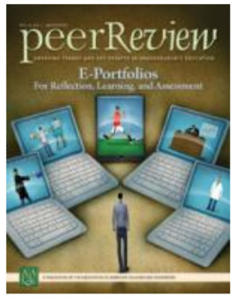

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