






# Operations Developmental Map — A Visual Overview

	<b>Phase 1</b> <b>Beginner</b> Focus on counting to solve problems <i>Guide to Using the Developmental Map, page 74</i>	<b>Phase 2</b> <b>Concrete</b> Formal operations with numbers to 20; Concrete operations with numbers to 100 <i>Guide to Using the Developmental Map, page 84</i>	<b>Phase 3</b> <b>Whole Number Comfort</b> Formal operations with whole numbers; Concrete operations with decimals <i>Guide to Using the Developmental Map, page 94</i>	<b>Phase 4</b> <b>More Abstract</b> Fluency with whole number operations; Formal operations with decimals <i>Guide to Using the Developmental Map, page 103</i>	<b>Phase 5</b> <b>Flexible</b> Fluency with whole number and decimal operations; Concrete operations with integers and fractions <i>Guide to Using the Developmental Map, page 115</i>
<b>Concept 1</b> Addition leads to a total and subtraction indicates what's missing. Addition and subtraction are intrinsically related.	This student solves simple concrete adding and subtracting problems. S/he counts to solve the problems, usually counting one item at a time.  <ul style="list-style-type: none"> <li>I am going to put the apples and oranges together. I'll count to see how many there are altogether: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. There are 10 pieces of fruit.</li> <li>I am going to take away 3 green apples. I'll count to see how many apples are left: 1, 2, 3, 4. There are 4 apples left.</li> </ul>	This student relates subtraction to addition, is familiar with several meanings of subtraction, and creates and solves addition and subtraction problems. S/he begins to use strategies to simplify computations. <ul style="list-style-type: none"> <li>To solve <math>11 - 5 = \square</math>, I can add: I think, "What do I add to 5 to get 11 (<math>5 + \square = 11</math>)?" Then I think, "5 more to get to 10 and 1 more to 11. There's 6."</li> <li>When I see <math>11 - 5</math>, sometimes I think, "How much is left if I take 5 away from 11?" and sometimes I think, "How much more is 11 than 5?"</li> <li>I can create, represent and solve problems: Kevin has 18 cards, Jen has 12 cards. How many more does Kevin have? <math>18 - 12 = 6</math></li> <li>I can add <math>53 + 10</math> by changing the 5 to 6, it's just adding 1 ten: <math>53 + 10 = 63</math></li> </ul>	This student creates and solves whole number addition and subtraction problems, knows when to estimate, and uses addition and subtraction principles to calculate. S/he relates addition to subtraction, and adds/subtracts 10 and 100 mentally. <ul style="list-style-type: none"> <li>I can create and solve a problem for <math>137 + \square = 182</math>: I have \$137 and need \$182. Is \$50 more enough? I need to calculate exactly to find out because <math>137 + 50</math> is close to 182. <math>137 + \square = 182 \rightarrow 182 - 137 = 45</math> <math>182 - 100 = 82</math> <math>82 - 30 = 52</math> <math>52 - 7 = 45</math></li> <li>I know the 3 in the hundreds place changes to 4 in <math>325 + 100 = 425</math>, since it's 1 more hundred.</li> <li>To solve <math>0.7 - 0.4</math>, I think, "What do I add to 4 tenths to make 7 tenths?"</li> </ul>	This student uses mathematical principles to simplify calculations with whole numbers and decimals. S/he solves and creates complex problems involving whole numbers and simple addition and subtraction decimal problems. <ul style="list-style-type: none"> <li>I can solve some complicated whole number problems: Abby climbed 598 steps and then another 78 steps of the CN Tower. How many more steps must she climb? Step 1: <math>198 + 78 = 200 + 76 = 276</math> Step 2: <math>1769 - 300 = 1469</math> (<math>300 = 276 + 24</math>) <math>1769 - 276 = 1469 + 24 = 1493</math> steps</li> <li>I can solve simple adding and subtracting problems with decimals: Jenna spent \$1.85 and paid with a \$5 bill. How much change will she get? <math>5 - 1.85 \rightarrow 1.85 + \square = 5</math> <math>1.85 + 0.15 = 2</math> She gets \$3.15 change.</li> </ul>	This student adds simple fractions concretely, solves problems involving addition and subtraction of decimals, and knows when it's appropriate to estimate. <ul style="list-style-type: none"> <li>I can solve this problem: Any doctor predicts I will grow to be 1.76 m tall. I was 1.35 m last year and grew 0.11 m this year. How many more metres might I grow? <math>1.35 + 0.11 = 1.46 \rightarrow 1.76 - 1.46 = 0.3</math> m</li> <li>I can estimate to decide if there's enough wood to cut 2 pieces, 3.2 m and 2.1 m, from a 6 m length, because <math>3.2 + 2.1</math> is just a bit more than 5.</li> </ul>
<b>Concept 2</b> Multiplication and division are extensions of addition and subtraction. Multiplication and division are intrinsically related.	This student uses repeated addition to count the total of a set of equal groups and repeated subtraction to count the number of equal groups in a total.  <ul style="list-style-type: none"> <li>I can count the bears to find out how many there are: 1, 2, 3, 4, then 5, 6, and then 7, 8. There are 8 bears.</li> <li>I can give 2 animals to each of my friends: 2 for Jessica, 2 for Ko, 2 for Will, and 2 for me.</li> </ul>	This student is beginning to be comfortable with multiplication and division notation to describe concrete or pictorial situations presented in a context. S/he solves and creates simple multiplication and division problems concretely. <ul style="list-style-type: none"> <li>I can figure out how many juice boxes there are in 4 packs of 3 by modelling with counters: <math>4 \times 3 = 12</math></li> <li>I can make up and solve a division problem for the counters: There are 12 cookies and 4 people sharing. How many cookies will each person get? <math>12 \div 4 = 3</math></li> </ul>	This student uses symbolic notation to describe a variety of multiplication and division meanings. S/he uses strategies to multiply, and solves and creates multiplication problems. S/he multiplies by 10 and 100 and uses multiplication to divide. <ul style="list-style-type: none"> <li>I know <math>3 \times 5</math> and <math>15 \div 3</math> can describe different situations: <math>3 \times 5 = 15</math>, <math>15 \div 3 = 5</math></li> <li>I know <math>4 \times 5</math> is twice <math>2 \times 5</math></li> <li>I know <math>3 \times 100 = 3 \times 1</math> hundred = 3 hundreds or 300</li> <li>I can solve <math>20 \div 5 = \square</math> by thinking "<math>5 \times \square = 20</math>"</li> <li>I can create and solve multiplication problems: I can solve <math>3 \times \square = 402</math> using <math>402 \div 3 = \square</math></li> <li>I can calculate <math>123 \times 4 = \square</math> and <math>456 \div 7 = \square</math></li> </ul>	This student solves and creates whole number problems, knows when to estimate and interprets remainders. S/he simplifies multiplication and division of whole numbers and interprets multiplication and division decimal situations. <ul style="list-style-type: none"> <li>I can make up and solve a problem with addition and division: Two classes, one with 26 students and one with 28 students, are forming teams of 5. How many teams are there? <math>26 + 28 = 54</math> and <math>54 \div 5 = 10</math> R 4 (4 left over means 1 team of 4 or 4 teams of 6)</li> <li>I can estimate to decide if \$20 is enough for 3 T-shirts, because <math>3 \times \\$5</math> is only \$15</li> <li>I can multiply <math>17 \times 100 = 1700</math></li> <li>I know <math>0.5 \times 40</math> means half of 40 and I know <math>4 \div 2</math> means <math>\frac{1}{2}</math> of 42 tenths.</li> </ul>	This student knows that a decimal quotient is evidence of a remainder, estimates in different ways, and solves and creates simple multiplication and division problems involving whole numbers and decimals. <ul style="list-style-type: none"> <li>The decimal in the answer below means that, if I divide 127 into 6 equal groups, there would be a remainder: <math>0.166666</math> is a bit less than 0.5 so the remainder would be a bit less than half of 6. <math>127 \div 6 = 21.166666</math></li> <li>Sometimes I underestimate, like when I estimate if I have enough money to buy something. But, sometimes I overestimate, like when I estimate the total cost of things I'm buying.</li> <li>I can make up a decimal multiplication problem: There is 1.48 L of juice in a can. How much space is left in a 6 L pitcher if 3 cans of juice are poured in?</li> </ul>
<b>Concept 3</b> There are many algorithms for performing a given operation with multi-digit numbers.	Concept 1 does not apply to this phase.	This student invents personal approaches to add and subtract 1-digit and 2-digit whole numbers. <ul style="list-style-type: none"> <li>To add <math>28 + 14</math>, I'd add 10 (1 rod) to get 38, then 2 more (2 small cubes) to get to 40 and then 2 more (2 small cubes) to get to 42. <math>28 + 14 = 42</math></li> <li>To subtract <math>23 - 5</math> in my head, I would subtract 3 to get to 20 and then 2 more to get to 18. <math>23 - 5 = 23 - 3 - 2 = 18</math></li> </ul>	This student explains a variety of procedures for adding, subtracting, multiplying, and dividing whole numbers and mental procedures for adding and subtracting 1- and 2-digit numbers. <ul style="list-style-type: none"> <li>To add <math>36 + 28</math>, I'd add 30 and 20 to get 50 and 6 and 8 to get 14. Then, I'd add 50 and 14 to get 64.</li> <li>To subtract <math>148 - 57</math>, I'd subtract 48 and then 10 more (that's 58) and then I'd add 1 (because I subtracted 58 instead of 57).</li> <li>To multiply <math>3 \times 26</math>, I made 3 groups of 26, counted the ones, regrouped and then counted the tens. <math>3 \times 26 = 78</math></li> <li>To add <math>38 + 9</math> in my head, I'd add 10 (because it's easy to add 10) to get 48 and then subtract 1 (because adding 10 was 1 too many) to get to 47.</li> </ul>	This student explains, with understanding, procedures for adding and subtracting simple decimals and for multiplying and dividing whole numbers by 2-digit whole numbers. S/he also adds, subtracts, and multiplies whole numbers mentally. <ul style="list-style-type: none"> <li>I can explain how to add <math>1.4 + 2.65</math>: I'd change 1.4 to 1.40. Then, I'd add up each place value starting with the hundredths.</li> <li>I can multiply <math>12 \times 13</math> using a base ten block rectangle: <math>13</math> (1 ten, 3 ones)</li> <li>I can divide <math>247 \div 6</math>: I think, "<math>40 \times 6</math> is 240. That's 40 groups of 6. So, it's 1 more group of 6 and a remainder of 7."</li> <li>I can do lots of adding, subtracting and multiplying in my head, even <math>6 \times 48 = 6 \times 40 + 6 \times 8 = 240 + 48 = 288</math></li> </ul>	This student selects methods to add, subtract, multiply, and divide. S/he performs mental addition and subtraction with some decimals and mental multiplication and division with suitable whole numbers. <ul style="list-style-type: none"> <li>I can calculate decimals in my head: To add <math>0.99 + 0.99</math> I think of money: I would add <math>\\$1 + \\$1</math> and take away 2 cents.</li> <li>To multiply <math>4.8 \times 5</math>, I would multiply 4.8 by 10 and then take half. But, to multiply <math>4.8 \times 4</math>, I would double 4.8 twice.</li> <li>To add 9.89, I would add 10 then take away 0.1 and then take away 0.01.</li> <li>To subtract 2.78, I would subtract 3, then add 0.2 and then add 0.02.</li> </ul>
<b>Skill 1</b> Recalls facts.	This student recalls addition facts with sums to 10 and related subtraction facts.  <ul style="list-style-type: none"> <li>I can add the numbers: I know 4 and 6 is 10</li> <li>I can subtract the numbers: I know 6 take away 4 is 2</li> </ul>	This student recalls addition facts to 9 + 9 and related subtraction facts and multiplication facts to 5 x 5 and related division facts. <ul style="list-style-type: none"> <li>I know my addition facts to <math>9 + 9 = 18</math> and some subtraction facts to <math>18 - 9 = 9</math>. Sometimes, I use addition facts to help with subtraction facts. To figure out <math>17 - 9</math>, I think "<math>8 + 9 = 17</math> so <math>17 - 9 = 8</math>."</li> <li>I know my multiplication facts to <math>5 \times 5 = 25</math> and some division facts to <math>25 \div 5 = 5</math>. Sometimes, I use multiplication facts to help with division facts. To figure out <math>12 \div 4</math>, I think "<math>3 \times 4 = 12</math> so <math>12 \div 4 = 3</math>."</li> </ul>	This student recalls all the multiplication facts to 9 x 9 and most related division facts. <ul style="list-style-type: none"> <li>I know all my multiplication facts to <math>9 \times 9 = 81</math> and most of my division facts to <math>81 \div 9 = 9</math>. Sometimes, I use multiplication facts to help me remember division facts. <math>5 \times 7 = 35</math> so <math>35 \div 5 = 7</math></li> </ul>	Skill 1 does not apply to this phase.	Skill 1 does not apply to this phase.
<b>Skill 2</b> Uses standard mental math and estimation procedures with multi-digit numbers.	Skill 2 does not apply to this phase.	This student rounds 2-digit numbers to the nearest 10 and applies mental addition and subtraction with these rounded numbers to estimate sums and differences.  <ul style="list-style-type: none"> <li>I can round <math>43 + 25</math> to <math>40 + 30</math>. Then, I can add <math>40 + 30</math> in my head using adding facts: <math>4 \text{ tens} + 3 \text{ tens} = 7 \text{ tens}</math> or 70</li> <li>I can round <math>43 - 25</math> to <math>40 - 20</math>. Then, I can subtract <math>40 - 20</math> in my head using subtracting facts: <math>4 \text{ tens} - 2 \text{ tens} = 2 \text{ tens}</math> or 20</li> <li>I can solve <math>26 + 10</math> and <math>54 - 10</math> in my head by changing the tens place digit: <math>26 + 10 = 36</math> and <math>54 - 10 = 44</math></li> </ul>	This student rounds whole numbers to the nearest 10 and 100 to estimate sums and differences, and mentally adds, subtracts, and multiplies with 10 and 100. <ul style="list-style-type: none"> <li><math>382 + 217</math> is about <math>400 + 200 = 600</math> <math>382 - 217</math> is about <math>400 - 200 = 200</math></li> <li><math>587 + 10 = 597</math>, <math>587 + 100 = 687</math> <math>587 - 10 = 577</math>, <math>587 - 100 = 487</math> <math>3 \times 10 = 30</math>, <math>3 \times 100 = 300</math>, <math>32 \times 10 = 320</math></li> </ul>	This student rounds whole numbers to estimate products, rounds decimals to the nearest whole or half to estimate sums and differences, and mentally multiplies whole numbers by powers of 10. <ul style="list-style-type: none"> <li>I can multiply by 100 and 1000 in my head: <math>42 \times 100 = 42 \times 1</math> hundred = 42 hundreds = 4200</li> <li><math>57 \times 49</math> is about <math>60 \times 50 = 3000</math></li> <li><math>13.64 \times 23.8</math> is about <math>13.5 \times 24 = 324</math></li> </ul>	This student uses benchmarks to round whole numbers to estimate and rounds decimals to estimate products. S/he mentally adds, subtracts and multiplies decimals and whole numbers by powers of 10. <ul style="list-style-type: none"> <li>I can calculate decimals in my head: <math>35.6 + 1000 = 1035.60</math> <math>35.6 - 18 = 25.6</math> <math>35.6 \times 100 = 3560</math> <math>35.6 \div 10 = 3.56</math></li> <li>I know <math>2356 \div 8</math> is between <math>2900 \div 8 = 362.5</math> and <math>2400 \div 8 = 300</math></li> <li>I know <math>378 \times 9</math> is about <math>4 \times 10 = 40</math></li> </ul>
<b>Skill 3</b> Computes with multi-digit whole numbers and decimals using pencil and paper without the aid of a calculator.	Skill 3 does not apply to this phase.	This student adds three single-digit numbers. <ul style="list-style-type: none"> <li>I can add <math>2 + 3 + 2</math> in my head: <math>2 + 3</math> is 5 and then <math>5 + 2 = 7</math>.</li> <li>Sometimes I need a model for adding bigger numbers like <math>4 + 7 = 8</math>.</li> </ul>	This student adds and subtracts 3-digit numbers symbolically. S/he sometimes uses concrete materials to multiply 2- and 3-digit numbers by 1-digit numbers and divide 2-digit numbers by 1-digit numbers. <ul style="list-style-type: none"> <li>I can add and subtract: <math>342 + 232 = 574</math>, <math>342 - 158 = 184</math></li> <li>I can multiply and divide: <math>25 \times 3 = 75</math>, <math>48 \div 3 = 16</math></li> </ul>	This student multiplies and divides by single digit whole numbers and multiplies 2-digit numbers symbolically. S/he adds and subtracts decimal tenths and hundredths and multiplies decimals by 1-digit whole numbers. <ul style="list-style-type: none"> <li>I can multiply and divide whole numbers: <math>146 \times 5 = 730</math>, <math>31357 \div 3 = 10452.333</math></li> <li>I can add, subtract, and multiply decimals: Sometimes I use manipulatives to help me. <math>1.38 + 1.92 = 3.30</math>, <math>1.11 - 0.3 = 0.81</math>, <math>12 - 3.4 = 8.6</math></li> </ul>	This student adds and subtracts decimals as well as whole numbers beyond 10 000 and divides decimals by 1-digit whole numbers. <ul style="list-style-type: none"> <li>I can add and subtract decimals and numbers beyond 100 000: <math>1130 + 111 = 1241</math>, <math>123893 - 387 = 123506</math>, <math>853 + 181029 = 181882</math></li> <li>I can divide decimals by 1-digit whole numbers: <math>1.82 \div 4 = 0.455</math></li> </ul>