Operations Developmental Map — A Visual Overview

PROFESSIONAL RESOURCES AND INSTRUCTION FOR MATHEMATICS EDUCATORS
PRIME

	Phase 1 Beginner Focus on counting to solve problems *Guide to Using the Developmental Map, page 74*	Phase 2 Concrete Formal operations with numbers to 20; Concrete operations with numbers to 100 *Guide to Using the Developmental Map, page 84*	Phase 3 Whole Number Comfort Formal operations with whole numbers; Concrete operations with decimals *Guide to Using the Developmental Map, page 94*	Phase 4 More Abstract Fluency with whole number operations; Formal operations with decimals *Guide to Using the Developmental Map, page 104*	Phase 5 Flexible Fluency with whole number and decimal operations; Concrete operations with integers and fractions *Guide to Using the Developmental Map, page 115*
Concept 1 Addition leads to a total and subtraction indicates what's missing. Addition and subtraction are intrinsically related.	This student solves simple concrete adding and subtracting problems. S/he counts to solve the problems, usually counting one item at a time.	This student relates subtraction to addition, is familiar with several meanings of subtraction, and creates and solves addition and subtraction problems. S/he begins to use strategies to simplify computations.	This student creates and solves whole number addition and subtraction problems, knows when to estimate, and uses addition and subtraction principles to calculate. S/he relates addition to subtraction, and adds/subtracts 10 and 100 mentally.	This student uses mathematical principles to simplify calculations with whole numbers and decimals. S/he solves and creates complex problems involving whole numbers and simple addition and subtraction decimal problems.	This student adds simple fractions concretely, solves problems involving addition and subtraction of decimals, and knows when it's appropriate to estimate.
Concept 2 Multiplication and division are extensions of addition and subtraction. Multiplication and division are intrinsically related.	This student uses repeated addition to count the total of a set of equal groups and repeated subtraction to count the number of equal groups in a total.	This student is beginning to be comfortable with multiplication and division notation to describe concrete or pictorial situations presented in a context. S/he solves and creates simple multiplication and division problems concretely.	This student uses symbolic notation to describe a variety of multiplication and division meanings. S/he uses strategies to multiply, and solves and creates multiplication problems. S/he multiplies by 10 and 100 and uses multiplication to divide.	This student solves and creates whole number problems, knows when to estimate and interprets remainders. S/he simplifies multiplication and division of whole numbers and interprets multiplication and division decimal situations.	This student knows that a decimal quotient is evidence of a remainder, estimates in different ways, and solves and creates simple multiplication and division problems involving whole numbers and decimals.
Concept 3 There are many algorithms for performing a given operation with multi-digit numbers.	Concept 1 does not apply to this phase.	This student invents personal approaches to add and subtract 1-digit and 2-digit whole numbers.	This student explains a variety of procedures for adding, subtracting, multiplying, and dividing whole numbers and mental procedures for adding and subtracting 1- and 2-digit numbers.	This student explains, with understanding, procedures for adding and subtracting simple decimals and for multiplying and dividing whole numbers by 2-digit whole numbers. S/he also adds, subtracts, and multiplies whole numbers mentally.	This student selects methods to add, subtract, multiply, and divide. S/he performs mental addition and subtraction with some decimals and mental multiplication and division with suitable whole numbers.
Skill 1 Recalls facts.	This student recalls addition facts with sums to 10 and related subtraction facts.	This student recalls addition facts to 9 + 9 and related subtraction facts and multiplication facts to 5 × 5 and related division facts.	This student recalls all the multiplication facts to 9 × 9 and most related division facts.	Skill 1 does not apply to this phase.	Skill 1 does not apply to this phase.
Skill 2 Uses standard mental math and estimation procedures with multi-digit numbers.	Skill 2 does not apply to this phase.	This student rounds 2-digit numbers to the nearest 10 and applies mental addition and subtraction with these rounded numbers to estimate sums and differences.	This student rounds whole numbers to the nearest 10 and 100 to estimate sums and differences, and mentally adds, subtracts, and multiplies with 10 and 100.	This student rounds whole numbers to estimate products, rounds decimals to the nearest whole or half to estimate sums and differences, and mentally multiplies whole numbers by powers of 10.	This student uses benchmarks to round whole numbers to estimate and rounds decimals to estimate products. S/he mentally adds, subtracts and multiplies decimals and whole numbers by powers of 10.
Skill 3 Computes with multi-digit whole numbers and decimals using pencil and paper without the aid of a calculator.	Skill 3 does not apply to this phase.	This student adds three single-digit numbers.	This student adds and subtracts 3-digit numbers symbolically. S/he sometimes uses concrete materials to multiply 2- and 3-digit numbers by 1-digit numbers and divide 2-digit numbers by 1-digit numbers.	This student multiplies and divides by single digit whole numbers and multiplies 2-digit numbers symbolically. S/he adds and subtracts decimal tenths and hundredths and multiplies decimals by 1-digit whole numbers.	This student adds and subtracts decimals as well as whole numbers beyond 10 000 and divides decimals by 1-digit whole numbers.