Operations Developmental Map — Phases and Indicators

PRIME

	Phase 1 Beginner Focus on counting to solve problems *Guide to Using the Developmental Map, page 77*	Phase 2 Concrete Formal operations with numbers to 20; Concrete operations with numbers to 100 *Guide to Using the Developmental Map, page 82*	Phase 3 Whole Number Comfort Formal operations with whole numbers; Concrete operations with decimals *Guide to Using the Developmental Map, page 92*	Phase 4 More Abstract Fluency with whole number operations; Formal operations with decimals *Guide to Using the Developmental Map, page 104*	Phase 5 Flexible Fluency with whole number and decimal operations; Concrete operations with integers and fractions *Guide to Using the Developmental Map, page 115*
Concept 1 Addition leads to a total and subtraction indicates what's missing. Addition and subtraction are intrinsically related.	**Interpreting addition and subtraction of whole numbers** 1 Reports the results of combining or joining (for adding) and taking away or separating (for subtracting) presented concretely and in a context.	1 Uses concrete materials to relate various meanings of subtraction (e.g., knows $12 - 7 = 5$ can mean how much more is 12 than 7 and 12 take away 7). 2 Relates addition to subtraction concretely. 3 Uses appropriate mathematical symbols, including the equals symbol, to describe concrete addition / subtraction contexts. 4 Solves simple open sentences of the forms $a + b = \square$, $c - a = \square$, and $a + \square = c$ abstractly. 5 Solves and creates addition and subtraction problems concretely by modelling and with 2-digit and 3-digit numbers using a calculator.	1 Relates addition to subtraction of whole numbers symbolically. 2 Uses appropriate mathematical symbols to describe abstract addition / subtraction contexts involving whole numbers. 3 Solves open addition and subtraction sentences involving whole numbers. 4 Solves and creates addition and subtraction problems involving whole numbers. 5 Distinguishes between situations involving whole numbers that require exact answers and those where estimates are sufficient because of the numbers involved (e.g., comparing the total points of two competitors when the scores are close as compared to when they are not close).	1 Solves complex addition and subtraction problems with whole numbers using a variety of methods.	
	Using addition and subtraction strategies with whole numbers 2 Uses counting all as a strategy for adding and subtracting concretely.	6 Uses computational strategies based on mathematical principles to learn addition / subtraction facts (e.g., if $6 + 6 = 12$, then $6 + 7 = 6 + 6 + 1 = 12 + 1 = 13$). 7 Relates addition and subtraction of 10 to place value concepts concretely.	6 Uses computational strategies based on mathematical principles to perform some additions and subtractions (e.g., adds in parts). 7 Relates addition and subtraction of whole numbers and 10 or 100 to place value concepts symbolically.	2 Uses mathematical principles (such as commutative and / or associative) to rename multi-digit whole numbers in order to calculate (e.g., renames $38 + 12$ as $40 + 10$).	
			Adding and subtracting decimals and fractions 8 Relates addition to subtraction of decimals concretely.	3 Extends the relationship between addition and subtraction of whole numbers to decimal values. 4 Uses mathematical principles (such as commutative and / or associative) to rename decimals in order to calculate (e.g., renames $3.8 + 5.2$ as $4 + 5$). 5 Solves and creates addition and subtraction problems involving simple decimals using formal procedures.	1 Adds simple fractions concretely. 2 Solves problems involving addition and subtraction with decimals. 3 Distinguishes between situations involving decimals that require exact answers and those for which estimates are sufficient because of the numbers involved (e.g., deciding if you have enough money to pay for two items when you have just about the right amount of money compared to having lots more than you need).
Concept 2 Multiplication and division are extensions of addition and subtraction. Multiplication and division are intrinsically related.	**Interpreting multiplication and division of whole numbers** 3 Adds repeatedly in "multiplication situations," but does not use the formal symbolism of $\times$. 4 Subtracts repeatedly in "division situations," but does not use the formal symbolism of $\div$.	8 Represents a single meaning of multiplication in concrete contexts (repeated addition, equivalent sets, or arrays) using multiplication language orally and symbolically (e.g., 2 times 3 is 6 or $2 \times 3 = 6$). 9 Represents a single meaning of division in concrete contexts (sharing or grouping) using division language orally and symbolically (e.g., there are eight 4s in 32 or $32 \div 4 = 8$). 10 Uses appropriate mathematical symbols to describe concrete multiplication / division contexts. 11 Solves simple open sentences of the forms $a \times b = \square$ and $c \div a = \square$ where c is a multiple of a, concretely or pictorially. 12 Solves and creates simple multiplication and division problems by modelling concretely.	9 Uses multiple meanings of multiplication (repeated addition, equivalent sets, arrays, area of a rectangle, and multiplicative comparisons) in concrete and abstract contexts involving whole numbers. 10 Uses multiple meanings of division (grouping and sharing) involving whole numbers. 11 Uses appropriate mathematical symbols to describe abstract multiplication and division contexts involving whole numbers. 12 Solves open sentences of the forms $a \times b = \square$ and $c \div a = \square$ where one factor is a 1-digit number and the product is less than 1000. 13 Solves and creates simple multiplication problems involving whole numbers (3-digit × 1-digit).	6 Solves open sentences of the forms $a \times b = \square$, $c \div a = \square$, and $a \times \square = c$. 7 Solves and creates problems involving all four operations with whole numbers. 8 Treats remainders that result from dividing whole numbers appropriately depending on the context. 9 Distinguishes between situations involving whole numbers that require exact answers and those for which estimates are sufficient because of the numbers involved.	4 Recognizes that any digits after the decimal point in a calculator quotient indicate that there is a remainder. 5 Recognizes that there are different estimates that might be appropriate, depending on the context and / or the numbers involved (e.g., the average score could be described as 84 or 85, depending on whether the scores are reported to the nearest whole number or the nearest multiple of 5).
			Using computational strategies 14 Uses computational strategies based on mathematical principles to learn multiplication facts (e.g., if $2 \times 8 = 16$, then 4×8 is $16 + 16 = 32$). 15 Interprets divisibility in terms of a multiplicative relationship (e.g., finds the number of 3s in 24 by deciding by what to multiply 3 to get 24). 16 Relates multiplication by 10 and 100 to place value concepts concretely.	10 Uses the relationship between multiplication and division to solve problems involving whole numbers. 11 Relates multiplication by powers of 10 to place value concepts symbolically.	
				Multiplying and dividing with decimals 12 Uses multiple meanings of multiplication and division of decimals.	6 Solves and creates simple problems involving whole numbers and decimals.
Concept 3 There are many algorithms for performing a given operation with multi-digit numbers.	Concept 3 does not apply to this phase.	**Adding and subtracting in different ways** 13 Invents "personal" procedures for adding and subtracting numbers, with and without the support of concrete materials.	17 Explains procedures for multi-digit whole number addition and subtraction, using language that demonstrates understanding of the operations (e.g., for $50 - 37$, regroup 50 as $40 + 10$, and then subtract 7 from 10). 18 Performs mental addition and subtraction with any 2-digit and 1-digit whole numbers.	13 Explains procedures for addition and subtraction of simple decimals, using language that demonstrates understanding of the operations. 14 Performs mental addition and subtraction with appropriate whole numbers.	7 Chooses an appropriate method for adding and subtracting whole numbers and decimals, depending on the numbers involved. 8 Performs mental addition and subtraction with some decimals (e.g., 0.9 or 0.99).
			Multiplying and dividing in different ways 19 Explains procedures for multiplication of whole numbers (2-digit by 1-digit) and division of whole numbers (3-digit by 1 digit and 2-digit by 1-digit), with and without concrete materials.	15 Explains procedures for multiplying and dividing by 2-digit or multi-digit numbers, with and without concrete materials. 16 Performs some multiplications with whole numbers mentally (e.g., relates multiplication by 10, 100, or 1000 to place value concepts, or calculates 3×35 mentally by adding 3×25 to 3×10).	9 Chooses an appropriate method for multiplying and dividing whole numbers and decimals, depending on the numbers involved. 10 Performs mental multiplication and division with whole numbers when the numbers are suitable (e.g., 20×15 or $424 \div 4$). 11 Performs multiplication of whole numbers and decimals, and division of whole numbers using place value concepts (e.g., multiplies by 0.1 or 0.01, and divides by 10, 100, or 1000).
Skill 1 Recalls facts.	**Recalling addition facts** 5 Recalls addition facts with sums to 10 and related subtraction facts.	14 Recalls addition facts with sums to 18 and related subtraction facts.		Skill 1 does not apply to this phase.	Skill 1 does not apply to this phase.
		Recalling multiplication facts 15 Recalls multiplication facts to 5×5 and related division facts.	20 Recalls multiplication facts to 9×9 and most related division facts.		
Skill 2 Uses standard mental math and estimation procedures with multi-digit numbers.	Skill 2 does not apply to this phase.	**Rounding to estimate** 16 Rounds numbers to a multiple of 10, with concrete or pictorial support, in order to estimate the sum or difference of two 2-digit numbers.	21 Rounds whole numbers to multiples of powers of 10 in order to estimate a sum or difference.	17 Rounds whole numbers to multiples of powers of 10 in order to estimate a product. 18 Rounds decimals to the nearest whole or half (0.5) to estimate a sum or difference.	12 Rounds whole numbers to multiples of powers of 10 in order to estimate a quotient. 13 Uses multiple benchmark numbers to round whole numbers to find sums, differences, products, and quotients (e.g., 32×15 is between 300 and 600). 14 Rounds decimals to wholes to estimate a product.
		Mentally calculating 17 Mentally adds and subtracts rounded numbers when only one fact is required (e.g., $20 + 40$). 18 Mentally adds and subtracts 10.	22 Mentally adds and subtracts whole numbers and 10 and 100. 23 Mentally multiplies whole numbers by 10 and 100.	19 Mentally multiplies whole numbers by whole number powers of 10.	15 Mentally adds, subtracts, and multiplies decimals with whole number powers of 10. 16 Mentally multiplies whole numbers and decimals by decimal powers of 10 (e.g., 0.1, 0.01, ...).
Skill 3 Computes with multi-digit whole numbers and decimals using pencil and paper without the aid of a calculator.	Skill 3 does not apply to this phase.	**Computing with whole numbers** 19 Adds three 1-digit numbers mentally or supported by the use of concrete materials.	24 Adds and subtracts whole numbers with up to 3 digits symbolically. 25 Multiplies and divides 2-digit and 3-digit whole numbers by 1-digit whole numbers with and without the use of concrete materials.	20 Multiplies and divides by 1-digit whole numbers symbolically. 21 Multiplies 2-digit by 2-digit whole numbers symbolically.	
				Computing with decimals 22 Adds and subtracts decimal tenths and hundredths, supported by the use of concrete materials. 23 Multiplies decimals by 1-digit whole numbers symbolically.	17 Adds and subtracts decimal tenths and hundredths, and whole numbers beyond 10 000 symbolically. 18 Divides decimals by 1-digit whole numbers.

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