Mathematical formulation of features

Since *Tr* creates a monotonic trend for a given period, its simplest form is as follows:

$$Tr(i) = a(i) + b(i) \cdot i \tag{1}$$

In (1), $a(i)$, $b(i) \in \Re$ are two constants, and $i = 0,1,...$. These constants remain the same for a given period and may vary in a systematic manner.

Since *Cy* creates a cyclic behavior, its simplest form is as follows:

$$Cy(i) = c(i) \sin\left(\frac{i}{d(i)}\right) \tag{2}$$

In (2), $c(i)$, $d(i) \in \Re$ are two constants. In particular, $c(i)$ is the amplitude constant and $d(i)$ is the frequency constant. These constants can be varied randomly with the iterations $i = 0,1,...$, if needed.

Since *Ir* contributes an amount of noise to the simulated signal, its simplest form is as follows:

$$Ir(i) = rn(i) \tag{3}$$

In (3), $rn(i) \in \Re$ is a normally distributed random variable with mean μ and standard deviation σ. Alternatively, one can use a pure random number for adding an amount of noise to a signal.

Since *Br* introduces a sudden shift in the signal, its mathematical setting is somewhat complex, requiring at least six parameters, namely, position (P_B), likelihood (L_B), span of ascending shift (p), span of descending shift (q), magnitude of ascending shift (B_A), and magnitude of descending shift (B_D). As such, the following formulation holds:

$$Br(i) = f(P_B, L_B, p, q, B_A, B_D) \quad (4)$$

The nature of the function $f(.)$ defined in (4) depends on the given signal. For the cases shown in the concept map, the following algorithm can be used to create $Br(i)$. Here, r_i is a random number in the interval [0,1], $N(\mu_{(.)},\sigma_{(.)})$ is a normally distributed variable where $\mu_{(.)}$ and $\sigma_{(.)}$ are the mean and standard deviation, respectively.

Burst Algorithm (*Br*):

```
L_B ← [0,1]
p, q, n_1, n_2, n ∈ ℵ, n_1, n_2 < n
P_B ← [n_1, n_2]
For i = 0,...,n
        Br(i) = 0, r_i ← [0,1]
          If (i = P_B) and (r_i < L_B)
                For k = 0,...,p–1
                        j = i + k, μ_j < μ_{j+1}
                        Br(i) = Br(j) ← N(μ_j,σ_j)
                End For
                For l = 0,...,q
                        m = i + p + l, μ_m > μ_{m+1}
                        Br(i) = Br(m) ← N(μ_m,σ_m)
                End For
End For
```

Therefore, adding $Tr(i)$, $Cy(i)$, $Ir(i)$, and $Br(i)$ creates the simulated signal $Sx(P,C)$. As a result, the following relationship holds.

$$Sx_i(P, C) = Tr(i) + Cy(i) + Ir(i) + Br(i) \tag{5}$$

Ask Sharif if you have any questions (ammsharifullah@gmail.com)