PROBLEM:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n]\cos(.3\pi n)$

(b) $y[n] = |x[-n]|$

McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $y[n] = x[n]\cos(0.3\pi n)$

Linear: because scaling $x[n]$ by α will give the same scaling of the output:

$$y[n] = (\alpha x[n])\cos(0.3\pi n) = \alpha(x[n]\cos(0.3\pi n))$$

Also, the *superposition* property holds:

$$y[n] = (x_1[n] + x_2[n])\cos(0.3\pi n) = x_1[n]\cos(0.3\pi n) + x_2[n]\cos(0.3\pi n)$$

Causal: because $y[n]$ only depends on the current value of $x[n]$.

Not Time-Invariant: because we can make the following counter-example. Let $x[n] = \delta[n]$ so that the output is

$$y[n] = x[n]\cos(0.3\pi n) = \delta[n]\cos(0.3\pi n) = \delta[n]\cos(0.3\pi(0)) = \delta[n]$$

Now change the input to $x[n] = \delta[n-1]$, so that we expect the output to shift by 1 time index. However, the output is actually

$$y[n] = x[n]\cos(0.3\pi n) = \delta[n-1]\cos(0.3\pi n) = \delta[n-1]\cos(0.3\pi(1)) = 0.588\delta[n-1]$$

(b) $y[n] = |x[-n]|$

Not causal: $y[-1] = x[1]$, so $y[n]$ at $n = -1$ depends on a future value of $x[n]$ at $n = +1$.

Not linear: because when we multiply the input by -3, the output does not get multiplied by -3. Here is the equation for the output $y[n] = |-3(x[-n])|$ which equals $y[n] = 3|x[-n]|$.

Not Time-Invariant: because we can show the following counter-example: when $x[n] = \delta[n]$, the output is $y[n] = |x[-n]| = |\delta[-n]| = \delta[n]$. However, when we shift the input to $n = 1$ by using the input $\delta[n-1]$, the output does *not* shift by the same amount: $y[n] = |x[-n]| = |\delta[-n-1]| = \delta[n+1]$.

McClellan, Schafer, and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.