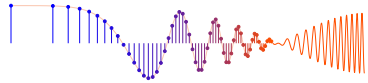


PROBLEM:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] - \beta x[n-1]$$

(a) When the input to this system is

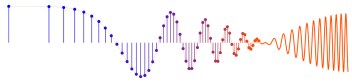
$$x[n] = \begin{cases} 0 & n < 0 \\ \beta^n & n = 0, 1, 2, 3, 4, 5, 6 \\ 0 & n > 6 \end{cases}$$

Use convolution to compute the values of $y[n]$, over the range $0 \leq n \leq 10$. Give a general formula in terms of β, and also show that most of the output values are equal to zero.

(b) Use the results from the previous part and plot both $x[n]$ and $y[n]$ for the case where $\beta = \frac{1}{2}$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



$$x[n] = \sum_{k=0}^{6} \beta^k \delta[n-k]$$

$$y[n] = x[n] - \beta x[n-1]$$

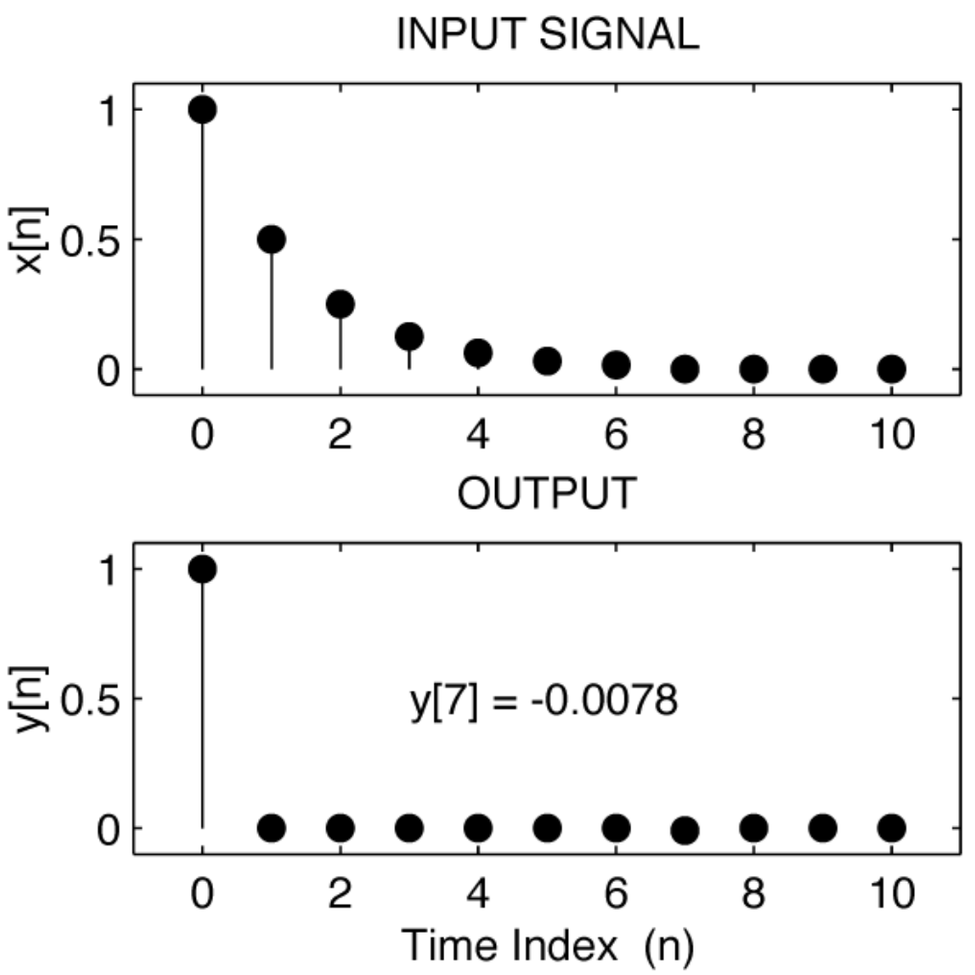
(a) This problem is similar in computation to Problem 5.3.

$$\begin{aligned}
y[n] &= x[n] - \beta x[n-1] \\
&= \sum_{k=0}^{6} \beta^k \delta[n-k] - \beta \sum_{k=0}^{6} \beta^k \delta[n-1-k] \\
&= \sum_{k=0}^{6} \beta^k \delta[n-k] - \beta \sum_{k=1}^{7} \beta^{k-1} \delta[n-k] \\
&= \delta[n] + \left(\sum_{k=1}^{6} \beta^k \delta[n-k] - \sum_{k=1}^{6} \beta\beta^{k-1} \delta[n-k] \right) - \beta^7 \delta[n-7] \\
&= \delta[n] - \beta^7 \delta[n-7]
\end{aligned}$$

Using another approach, we can make a table that contains the values for $x[n]$ and $\beta x[n-1]$ and then subtract the results to get $y[n] = x[n] - \beta x[n-1]$:

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
$x[n]$	1	β	β^2	β^3	β^4	β^5	β^6	0	0	0	0
$\beta x[n]$	0	β	β^2	β^3	β^4	β^5	β^6	β^7	0	0	0
$y[n]$	1	0	0	0	0	0	0	$-\beta^7$	0	0	0

(b) We plot $x[n]$ and $y[n]$ for the case of $\beta = 0.5$, but notice that β^7 is a very small number so it hardly shows up on the plot of the output signal.



Code to generate this plot:

```
n = [0:10];
beta = 0.5;
x = [ beta .^(0:6), zeros(1,4) ];
y = conv(x,[1,-beta]);
y = y(1:length(n));
subplot(4,2,1),stem(n,x,'filled')
axis([-1 11 -0.1 1.1]);
ylabel('x[n]');
title('INPUT SIGNAL')
h2 = subplot(4,2,3),stem(n,y,'filled')
axis([-1 11 -0.1 1.1]);
xlabel('Time Index  (n)'); ylabel('y[n]');
title('OUTPUT')
text(3,0.5,'y[7] = -0.0078')
h2p = get(h2,'position')
h2p(2) = h2p(2) - 0.025;
set(h2,'position',h2p)
```