PROBLEM:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n-2] + 2x[n] + x[n+2]$

(b) $y[n] = nx[n]$

(c) $y[n] = (x[-n])^2$

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SOLUTION

(a) $y[n] = x[n-2] + 2x[n] + x[n+2]$

This has the same form as an FIR filter with multiplies, adds and delays, so it is also <u>LINEAR</u> and <u>TIME-INV</u>.

It is <u>NOT</u> <u>CAUSAL</u>. Here is a counter-example:

Let $x[n] = \delta[n]$ ← this input starts at $n=0$.

Then $y[n] = \delta[n-2] + 2\delta[n] + \delta[n+2]$

This output starts at $n=-2$, so it starts <u>before</u> the input $\Rightarrow$ <u>NON-CAUSAL</u>

(b) $y[n] = n\,x[n]$

<u>CAUSAL</u>? <u>Yes</u> $y[n_0] = n_0 x[n_0]$ means that $y[n_0]$ depends only on the input at $n=n_0$.

<u>LINEAR</u>? <u>Yes</u>

$$\left.\begin{aligned} y_1[n] &= n x_1[n] \\ y_2[n] &= n x_2[n] \end{aligned}\right\}$$

If $x[n] = \alpha x_1[n] + \beta x_2[n]$ then

$$\begin{aligned} y[n] &= n(\alpha x_1[n] + \beta x_2[n]) \\ &= \alpha(n x_1[n]) + \beta(n x_2[n]) \\ &= \alpha y_1[n] + \beta y_2[n] \end{aligned}$$

<u>TIME-INV</u>? <u>No</u>

When the input is $x[n] = \delta[n]$, the output is $y[n] = n x[n] = n\delta[n] = 0$ for all n.

When the input is a "shifted impulse", e.g. $x_1[n] = \delta[n-1]$ then the output is $y_1[n] = n\,\delta[n-1] = 1\cdot\delta[n-1]$

But $y_1[n]$ is <u>not</u> the shifted version of $y[n]$ because $y_1[n] \neq y[n-1]$

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(c)

$$y[n] = (x[-n])^2 \longleftarrow \text{"FLIP" \& SQUARE}$$

<u>LINEAR</u>? <u>NO</u>

$$\left.\begin{aligned} y_1[n] &= (x_1[-n])^2 \\ y_2[n] &= (x_2[-n])^2 \end{aligned}\right\}$$

If $x[n] = \alpha x_1[n] + \beta x_2[n]$ then

$$y[n] = (x[-n])^2 = (\alpha x_1[-n] + \beta x_2[-n])^2$$

$$= \alpha^2 (x_1[-n])^2 + 2\alpha\beta x_1[-n]\, x_2[-n] + \beta^2 (x_2[-n])^2$$

$$\neq \alpha (x_1[-n])^2 + \beta (x_2[-n])^2$$

<u>TIME-INV</u>? <u>NO</u>

Let $x[n] = \delta[n-1]$, then the output is $y[n] = (\delta[-n-1])^2$

Since the system flips the input and squares

the output is $y[n] = \delta[n+1] \longleftarrow$ SAME

Now shift the input: $x_2[n] = x[n-1] = \delta[n-2]$.

The output is $y_2[n] = (\delta[-n-2])^2 = \delta[n+2] \longleftarrow$ An impulse at $n = -2$

But $y_2[n] \neq y[n-1]$ because $y[n-1] = \delta[n-1+1] = \delta[n]$.

<u>CAUSAL</u>? <u>NO</u>

If $x[n] = \delta[n-7]$, then $y[-7] = (x[-(-7)])^2 = (x[7])^2 = 1$

So the output at $n = -7$, needs a value from the <u>future</u>.

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