PROBLEM:

Evaluate the “running” average:

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k]$$

when the input signal is a geometric sequence. As before, compute the numerical values of y[n] over the range $-5 \le n \le 10$; let $L = 4$ and $a = 0.8$. Then derive a general formula for $y[n]$ that will apply for any value of the parameters L and a, when $n \ge 0$.

$$x[n] = \begin{cases} 0 & \text{for } n < 0 \\ a^n & \text{for } n \ge 0 \end{cases}$$

If you cannot visualize the output signal, use MATLAB to create a plot of the output for $a = 0.9$ and $a = 0.8$ over the range $0 \le n \le 20$. Take L to be equal to 7.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

$x[n] = a^n u[n]$ IS INPUT

4-POINT RUNNING AVG.

n	$x[n]$
0	1
1	0.8
2	.64
3	.512
4	.4096
5	.328

$y[0] = \frac{1}{4}(1+0+0+0) = 1/4 = 0.25$

$y[1] = \frac{1}{4}(0.8+1+0+0) = 0.45$

$y[2] = \frac{1}{4}(.64+.8+1+0) = \frac{2.44}{4} = .61$

$y[3] = \frac{1}{4}(.512+.64+.8+1.0) = .738$

$y[4] = \frac{1}{4}(.4096+.512+.64+.8) = .5904$

PROBLEM values
for the $\boxed{\text{4-POINT}}$ RUNNING AVERAGE

```
y =
  Columns 1 through 7
        0         0         0         0         0    0.2500    0.4500
  Columns 8 through 14
   0.6100    0.7380    0.5904    0.4723    0.3779    0.3023    0.2418
  Columns 15 through 21
   0.1935    0.1548    0.1238    0.0991    0.0792    0.0634    0.0507
```

for the $\boxed{\text{7-POINT}}$ RUNNING AVERAGE

```
y =
  Columns 1 through 7
        0         0         0         0         0    0.1429    0.2571
  Columns 8 through 14
   0.3486    0.4217    0.4802    0.5270    0.5645    0.4516    0.3613
  Columns 15 through 21
   0.2890    0.2312    0.1850    0.1480    0.1184    0.0947    0.0758
```

In order to do a <u>general</u> derivation we need the following formula:

$$\sum_{k=N_1}^{N_2} r^k = \frac{r^{N_1} - r^{N_2+1}}{1-r} \quad \text{if } r \neq 1$$

or a simplified version:

$$\sum_{k=0}^{N} r^k = \frac{1-r^{N+1}}{1-r}$$

GENERAL DERIVATION

#1) For $n<0$ $y[n]=0$

This is OBVIOUS because $x[n]=0,\ n<0$

#2) When $0 \le n < L$, the average takes in some points that are zero.

$$y[n]=\frac{1}{L}\sum_{k=0}^{n} x[n-k]$$

$$=\frac{1}{L}\sum_{k=0}^{n} a^{n-k} = \frac{a^n}{L}\sum_{k=0}^{n}(a^{-1})^k = \frac{a^n}{L}\,\frac{1-(a^{-1})^{n+1}}{1-a^{-1}}$$

$$=\frac{1}{L}\,\frac{a^n-a^{-1}}{1-a^{-1}} = \frac{1}{L}\,\frac{a^{n+1}-1}{a-1} = \frac{1}{L}\,\frac{1-a^{n+1}}{1-a}$$

#3) When $n \ge L$

$$y[n]=\frac{1}{L}\sum_{k=0}^{L-1} x[n-k] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k}$$

$$=\frac{a^n}{L}\sum_{k=0}^{L-1}(a^{-1})^k = \frac{a^n}{L}\,\frac{1-(a^{-1})^L}{1-a^{-1}}$$

$$= a^n\left(\frac{1}{L}\,\frac{1-a^{-L}}{1-a^{-1}}\right)$$

this is a constant

$\therefore$ in region #3, the output $y[n]$ will decay like a^n.

in region #2, $y[n]$ is rising.

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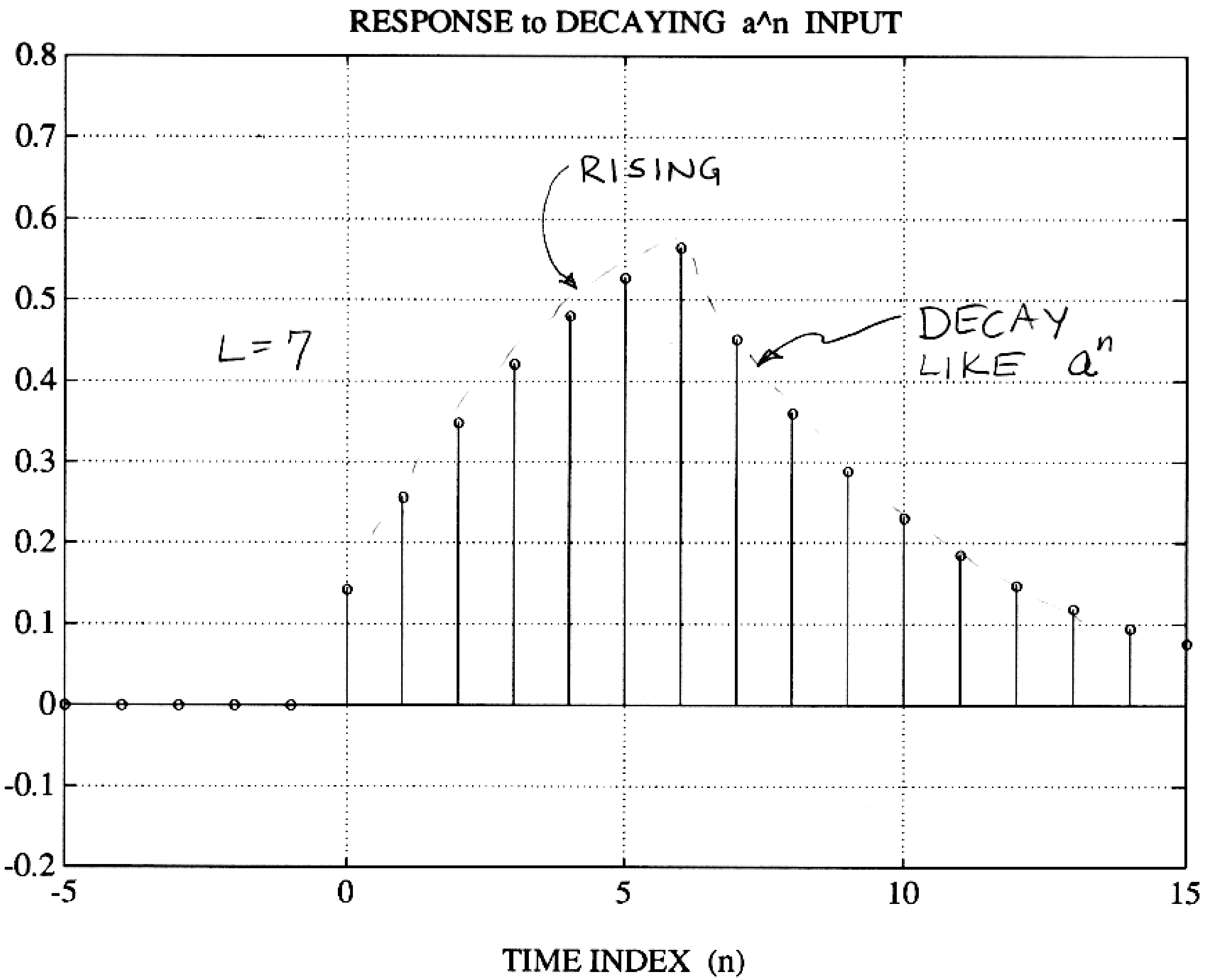
RESPONSE to DECAYING a^n INPUT
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-5
0
5
10
15
TIME INDEX (n)
RISING
L = 7
DECAY
LIKE a^n