PROBLEM:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

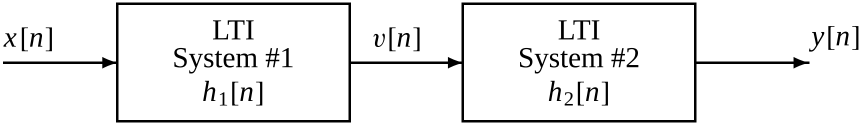

Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 has impulse response,

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ -1 & n = 1 \\ 0 & n > 1 \end{cases}$$

and System #2 is described by the difference equation

$$y[n] = 0.25v[n] + 0.25v[n-1] + 0.25v[n-2] + 0.25v[n-3] \tag{1}$$

(a) Determine the difference equation of System #1; i.e., the equation that relates $v[n]$ to $x[n]$.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $v[n]$ and make a plot. Show that the resulting output is the given impulse response $h_1[n]$.

(c) From the difference equation in (1), determine $h_2[n]$, the impulse response of System #2.

(d) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$.

(e) From the impulse response of the overall cascade system as obtained in part (d), obtain a single difference equation that relates $y[n]$ directly to $x[n]$ in Fig. 1.

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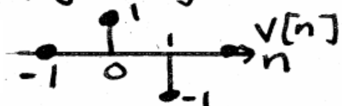
SOLUTION

a) $h_1[n] = \delta[n] - \delta[n-1]$

$v[n] = x[n] - x[n-1]$

b) Let $x[n] = \delta[n]$. Then $v[n] = \delta[n] - \delta[n-1]$



c) Let $v[n] = \delta[n]$ then $y[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$

d) The impulse response associated with the cascade implies that $x[n] = \delta[n]$, which results in $v[n] = \delta[n] - \delta[n-1]$. Using this $v[n]$ as input to system 2, we obtain

$$y[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3] - \frac{1}{4}\delta[n-1] - \frac{1}{4}\delta[n-2] - \frac{1}{4}\delta[n-3] - \frac{1}{4}\delta[n-4]$$

$\therefore$ $y[n] = \frac{1}{4}\delta[n] - \frac{1}{4}\delta[n-4]$

e) $y[n] = \frac{1}{4}x[n] - \frac{1}{4}x[n-4]$

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