There are 3 sample problems that follow, using this process.

Example 1. Liquid ethanol (C_2H_6O) has a density of 0.789 g/cm^3. If 10^4 mm^3 of ethanol were evaporated, what is the mass of gas that evaporated?

1. key words and concepts

key words: density, mass , volume

Concepts: This question requires an understanding of how the density of a liquid is related to the mass and volume of a gas. Some unit conversions will also be needed.

2. Connections: **Understand the problem**

What is given? The density (d) of liquid ethanol (= 0.789 g/cm^3) and the volume (V) of ethanol that evaporates (= 10^4 mm^3).

What do we want to know? the mass (in g) of this volume of ethanol.

The process occurring can be written as follows:

$C_2H_6O(\ell) \rightarrow C_2H_6O(g)$

The volume of gas is not given, but it is also not needed: the mass of gas does not change upon evaporation.

The relationship between density, volume and mass is given by the equation that defines density:

d = m/V so mass = m =d × V

Note: The molecular weight and formula of ethanol are not needed to solve this problem.
The density of the gas is not needed either.

Problem: the units of volume given are different from the units of density. Before solving for mass, the units of volume must be converted to cm^3 or the density needs to be converted to g/mm^3. Either way should work, but we will convert the volume to cm^3

- Dimensional analysis
  - There are 10 mm in 1 cm

3. **Solution**

1. Convert the units of the volume evaporated:

$$10^4\ mm^3 \times \left(\frac{1\ cm}{10\ mm}\right)^3 = 10^4\ mm^3 \times \frac{1^3 cm^3}{10^3 mm^3} = 10^4\ mm^3 \times \frac{1\ cm^3}{1000\ mm^3} = 10cm^3$$

2. Insert into the density equation:

$$density = \frac{mass}{volume}$$

$$0.789\frac{g}{cm^3} = \frac{mass}{10\ cm^3} \qquad mass = 7.89g\ Ethanol$$ answer: 7.89 g

3. For 10^4 mm^3 of ethanol (density 0.789 g/ cm^3) to evaporate, 7.89 g of ethanol must have been present prior to evaporation.

4. Check: Does the answer make sense?

10^4 mm^3 is 10 cm^3: this is 3 orders of magnitude smaller: the mm is smaller than a cm so, it is correct that the number of cm^3 should be smaller than the amount of mm^3. Recall that the volume is a length cubed so the relationship between units is 10^3 in metric not just a factor of 10.
Finally, since 10^4 mm^3 is 10 cm^3 of ethanol, expect mass to be 10 times larger than the density.

Example 2. The average atomic mass of copper, which has only two naturally occurring isotopes is 63.5 amu. One of the isotopes, ^{63}Cu, has an atomic mass of 62.9 amu and 69.1% abundance. What is the atomic mass of the second isotope?

1. **key words and concepts**

   Key Words: Isotopes, % abundance, average atomic mass

   Concepts: This question requires an understanding of average atomic mass. We also need to know what an isotope is and how the mass of each isotope and its abundance in nature is related to the average atomic mass.

2. Connections: **Understand the problem**

   What is given in the problem?

   – Average atomic mass = 63.5 amu (could also be obtained from the periodic table)
   – Cu has two naturally occurring isotopes
   – ^{63}Cu, atomic mass = 62.9 amu, abundance = 69.1%

   What do I want to find: the atomic mass of the second isotope of Cu

   The equation needed to find average atomic mass:

$$\text{Average atomic mass} = \sum_{n=1}^{m} (\text{Isotope atomic mass})_n \ (\text{Isotope relative abundance})_n$$

   where m is equal to the number of isotopes

$$\text{Average atomic mass} = \sum_{n=0}^{m} \left(\text{Isotope atomic mass}\right)_n \left(\text{Isotope relative abundance}\right)_n$$

$$m = \#\text{ of isotopes}$$

   We already know the average atomic mass, the atomic mass and relative abundance of ^{63}Cu, but: don't know the relative abundance of the second isotope.

   How do you find the relative abundance of the second Cu isotope?

   Since there are only two naturally occurring isotopes of Cu, and they both must add up to give a natural abundance of 100%, the relative abundance for the second isotope can be calculated.

3. **Solution**

   - 100% = Abundance of ^{63}Cu + abundance of second isotope
     100% = 69.1% + x   x = 30.9% = abundance of isotope 2.
   - Use the equation for the average atomic mass (above),

   63.5 amu = (62.9 amu)(0.691) + (2^{nd} isotope's atomic mass)(0.309)

$$2^{nd}\ \text{isotope's atomic mass} = \frac{63.5\ \text{amu} - (62.9\ \text{amu})(0.691)}{0.309} = 64.8\ \text{amu}$$

$$63.5\ \text{amu} = \left(62.9\ \text{amu}\right)\left(0.691\right) + \left(2^{nd}\ \text{isotope's atomic mass}\right)\left(0.309\right)$$

$$2^{nd}\ \text{isotope's atomic mass} = \frac{63.5\ \text{amu} - \left(62.9\ \text{amu}\right)\left(0.691\right)}{0.309} = 64.8\ \text{amu}$$

$$\text{Answer: D}$$

ANS: 64.8 amu

4. Check: Does the answer make sense?

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- ✓ We calculated the relative abundance of the second isotope, making sure the total percent relative abundance for the element was 100%
- ✓ Since the atomic mass of ^{63}Cu is less than the average atomic mass (63.5 amu), the atomic mass of the second isotope must be greater than the average atomic mass in order to have the average be greater than the atomic mass of ^{63}Cu
- ✓ Solving the equation for the atomic mass of the second isotope can only lead to a value of 64.8 amu (with 3 significant figures)

Example 3. A titanium-sapphire laser is tuned to λ = 900 nm and used to image the distribution of a dye in cultured cells. How many photons can be administered to the cells before they absorb 12 J of heat?

1. key words and concepts

Key Words: Wavelength (λ); Photon energy, number of photons

Concepts: To answer this question we need to know how to find the energy of a photon and realize that the total energy in a beam of light will be the sum of the energy of the individual photons: i.e., $E = n\,h\nu$ where n is the number of photons and v is frequency.

2. Connections: **Understand the problem**

What is given? The wavelength for a specific laser is λ = 900 nm and a total of 12 J of heat (energy) is absorbed.

What do we want to know? the number of photons (n) delivered in a single pulse.

First we need to determine the energy of a single photon. Each photon contains a certain amount of energy that is characteristic of its wavelength.

$E_{single\ photon} = E = h\nu$ This equation (without the subscripts) is on the data sheet.

$E_{single\ photon} = hc/\lambda$ (relationship between energy and wavelength)

This is the energy of ONE photon.

h = Planck's constant (6.62×10^{-34} Js), c = speed of light = 3.0×10^{8} m/s

(These constants are given on the datasheet.)

Several photons are then combined to give a total energy.

$E_{total} = 12\text{ J} = n\,hc/\lambda$ where n = the number of photons.

3. **Solution**

1. Find the energy of a single photon from the wavelength of the photon that is given.

$E_{single\ photon} = hc/\lambda$

$E = (6.62 \times 10^{-34}\text{ Js})(3.0 \times 10^{8}\text{ m/s})/(900 \times 10^{-9}\text{ m}) = 2.21 \times 10^{-19}\text{ J}$

2. Then use E_{total} (that was given) to get n.

$E_{total} = 12\text{ J} = n_{photons}\,E = n_{photons} \times (2.21 \times 10^{-19}\text{ J})$

$n_{photons} = 12\text{ J}/(2.21 \times 10^{-19}\text{ J}) = 5.44 \times 10^{19}$

4. Check: Does the answer make sense?

- The value for $E_{single\ photon}$ comes out to be on the order of 10^{-19} J. This is a reasonable value because visible light tends to be on this order, and the wavelength that was given was 900 nm, which is just a slightly higher wavelength than visible light (in the infrared).

- The value for $n_{photons}$ is also reasonable because the energy value of 12 J is divided by $E_{single\ photon}$. The inverse of the exponent 10^{-19} is $\sim 10^{19}$. The units of meters and seconds cancel with h and c to give Joules as an energy unit. In the second calculation, the units of Joules cancel to give a unit-less value, or a number of photons.
