

Uncertainty Classification for the Design and Development of Complex Systems

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Abstract

Uncertainty plays a critical role in the analysis for a wide and diverse set of fields from economics to engineering. The term ‘uncertainty’ has come to encompass a multiplicity of concepts. This paper begins with a literature survey of uncertainty definitions and classifications from various fields. A classification of uncertainty for the design and development of complex systems follows. The various classifications are more practical than theoretical: to make distinct the techniques used to address each type of uncertainty and to demonstrate the effects of each type of uncertainty in each field. The classification for the design and development of complex systems delineates ambiguity, epistemic, aleatory, and interaction uncertainty. Epistemic uncertainty is further subdivided into model-form, phenomenological, and behavioral uncertainty, each of which is described in detail. The uncertainty taxonomy presented is an integral part of ongoing research into propagating and mitigating the effect of all types of uncertainty in the design and development of complex multidisciplinary engineering systems.

Introduction

Ideas and concepts of uncertainty have long been associated with gambling and games. The earliest-known form of gambling was a kind of dice game played with an astragalus (knuckle-bone) in 3500 BC Egypt.¹ Gambling has developed considerably in the centuries that followed but the underlying form of this type of uncertainty is unchanged. Pure games of chance, such as the astragalus, roulette, or craps, deal with aleatory uncertainty, essentially inherent randomness. These games are distinct from games such as poker or horse racing in which skill or knowledge makes a difference. Formally addressing this type of uncertainty in games of chance began in the Renaissance and culminated in the theory of probability during the 17th century.²

The Greeks of the 4th century BC were the first recorded civilization to have considered uncertainty explicitly, primarily in the context of epistemology. The word epistemology is derived from the Greek *episteme*, meaning “knowledge”, and *logos*, which has several meanings, including “theory”. Epistemology deals with the possibilities and limits of human knowledge. Basically it tries to arrive at a knowledge of knowledge itself. Aristotle suggested that people should make decisions on the basis of “desire and reasoning to some end” but offered no guidance to the likelihood of a successful outcome. Despite their explicit consideration of uncertainty, when the Greeks wanted a prediction of what the future might hold they turned to the oracles instead of consulting their wisest philosophers.¹

Refs. 1 and 2 provide an extensive history of uncertainty in the context of risk management and probability theory, respectively. Ideas about aleatory and epistemic uncertainty have developed significantly since the early Egyptians and Greeks but the distinction has persisted almost unchanged until the 20th century and only recently has the impact of uncertainty been analyzed and understood. Uncertainty influences decisions, designs, and behavior in a wide variety of fields from economics to engineering. Reducing uncertainty has been and continues to be a costly business in time and resources. Efforts to classify and define uncertainty, propagate it through an analysis, and devise methods to mitigate its impact have been the objective of research efforts. The remainder of this paper summarizes uncertainty taxonomies and definitions of various fields then provides a new classification for the design and development of complex systems with detailed uncertainty definitions.

Uncertainty Classifications & Definitions

A fundamental definition of uncertainty is “liability to chance or accident”, “doubtfulness or vagueness”, “want of assurance or confidence; hesitation, irresolution”, and “something not definitely known or knowable”.³ This definition has motivated a wide variety of classifications of uncertainty in a variegated set of fields. Many of the uncertainty classifications that follow have similarities and most have an emphasis on one aspect of uncertainty which most impacts that particular field. Hence, these classifications are often more of a practical than theoretical significance. Unfortunately, many of these taxonomies have different definitions for the same words. The references provided are not exhaustive but are representative of the general areas. Research is ongoing into collecting more detailed information from a variety of fields, in particular the field of life sciences which is not discussed in this paper.

Social Sciences

Research into uncertainty in the field of social sciences has a rich history. The following section summarizes this research in the fields of economics; decision making, management, and system analysis; and policy and risk analysis.

Economics

Classical economic theory had no room for uncertainty. The theory assumed that people decide how to consume, produce, and invest with full knowledge of what the outcome of their decisions will be. Uncertainty was either ignored or explicitly “assumed away”. The resulting theory was neither realistic nor useful.⁴ To develop a realistic theory, economists began studying uncertainty extensively starting in the early 20th century. The American economist Frank Knight wrote in 1921, “Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated.”⁵ Knight refers to “risk” as situations where the decision-maker can assign mathematical probabilities to the randomness with which he is faced. In contrast, “uncertainty” refers to situations when this randomness “cannot” be expressed in terms of specific mathematical probabilities. As the English economist, journalist, and financier John Maynard Keynes was later to express it:

“By ‘uncertain’ knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty ... The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence ... About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.”⁶

A distinction in this classification arrived in the mid-20th century, influenced by pioneering work in the creation and development of Game Theory by von Neumann and Morgenstern; Nash; and others.⁷ Uncertainty and information about the environment was viewed as distinct from that of uncertainty and information about others’ behavior or the outcome of as yet unperformed computations.⁸ Building on the mid-20th century work, economists have recently gone a step further arguing that Knightian risk and uncertainty are one and the same thing. In Knightian uncertainty the problem is not that the agent cannot assign probabilities but in fact that the agent does not assign probabilities. That is to say, that uncertainty is really an epistemological and not an ontological problem, a problem of “knowledge” of the relevant probabilities and not of their “existence”. Uncertainty has recently been classified as fundamental uncertainty or ambiguity.⁹ Fundamental uncertainty is not merely that there is not enough information to reliably attach probabilities to a given number of events but that in fact, an event which cannot be imagined may occur in the future. This implies that some relevant information cannot be known, not even in principle, and that something unimaginable may happen.⁹ Ambiguity is defined as “uncertainty about probability, created by missing information that is relevant and could be known.”¹⁰ It should be noted that some economists argue in the opposite direction: that there are actually no probabilities out there to be “known” because probabilities are really only “beliefs”. In other words, probabilities are merely subjectively-assigned expressions of beliefs and have no necessary connection to the true randomness of the world (if it is random at all).¹¹ The evolution in economic uncertainty belief is illustrated in Fig. 1.

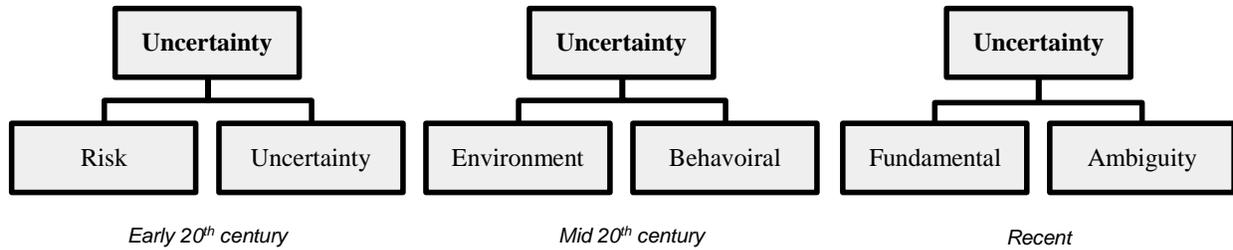


Fig. 1. Uncertainty Classifications in Economics.

Decision Making, Management, and Systems Analysis

Most decision making and management texts follow an early influential text and simplify uncertainty into risk and uncertainty (ignorance).¹² Risk is if each action leads to one of a set of possible specific outcomes with each outcome occurring with a known probability. Risk implies that all possible acts are known, all possible outcomes arising from each act are known, and it is possible to assign probabilities to each act. Uncertainty is if either action or both action and outcome have as its consequence a set of possible specific outcomes where the probabilities of these outcomes are unknown or are not meaningful. Uncertainty implies either that all possible acts and outcomes are not known or it makes no sense to assign probabilities to them. This simple classification is an idealization and philosophical controversial.^{12,13}

The field of management follows a somewhat similar tack. Management stresses the need to not only theorize possible eventualities but also their consequences.¹⁴ Although Ref. 14 does not explicitly define or classify uncertainty, it does allude to uncertainty in consequences, modeling, people’s actions, and information available to various parties (so called asymmetric uncertainty). Another management reference does classify risk (uncertainty) as performance, schedule, development cost, technology, market, and business in the context of product development.¹⁵ The definitions for each are provided in Table 1.

Table 1 Risk Definitions for Product Development¹⁵

Risk	Uncertainty in ...
Performance	the ability of a design to meet desired quality criteria (along any one or more dimensions of merit, including price and timing) and the consequences thereof
Schedule	the ability of a project to develop an acceptable design (i.e., to sufficiently reduce performance risk) within a span of time and the consequences thereof
Development cost	the ability of a project to develop an acceptable design (i.e., to sufficiently reduce performance risk) within a given budget and the consequences thereof
Technology	capability of technology to provide performance benefits (within cost and/or schedule expectations) and the consequences thereof [a subset of performance risk]
Market	the anticipated utility or value to the market of the chosen “design to” specifications (including price and timing) and the consequences thereof
Business	political, economic, labor, societal, or other factors in the business environment and the consequences thereof

Systems analysis follows closely the decision making definition.¹⁶ In short, the classification of uncertainty in decision making, management, and systems analysis builds on the Knightian (Keynesian) concept of risk and uncertainty of economics that was previously discussed.

Policy & Risk Analysis

The policy and risk analysis community has classified uncertainty into quantity and model form uncertainty.¹⁷ Fig. 2 illustrates this classification.

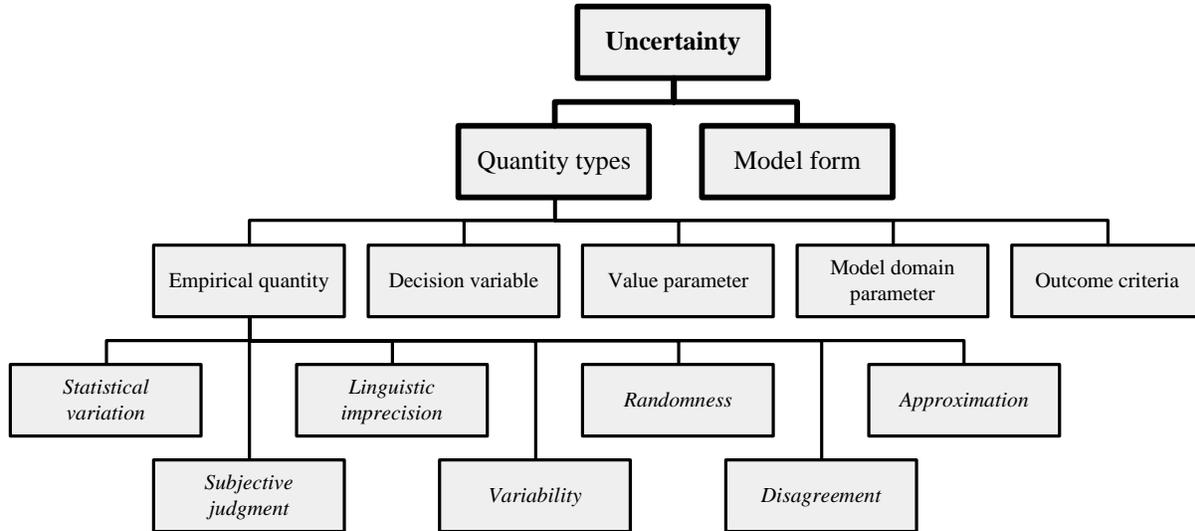


Fig. 2. Uncertainty Classification for Policy & Risk Analysis.¹⁷

Quantity type uncertainty is defined in Table 2.

Table 2 Quantity Type Uncertainty Definitions for Policy & Risk Analysis¹⁷

Uncertainty	Subclassification	Definition/Explanation
Empirical quantity	Statistical variation	arises from random error in direct measurements of a quantity
	Subjective judgment	teamed with systematic error as the difference between the true value of a quantity of interest and the value to which the mean of the measurements converges as more measurements are taken
	Linguistic imprecision	refers to quantities that are not well-specified and could not be empirically measured in principle
	Variability	refers to quantities that are variable over time and space
	Randomness	uncertainty that is irreducible in even principle
	Disagreement	refers to differences of opinion between informed experts about a quantity
	Approximation	difference between the assumed quantity value and the real-world value
Decision variable	n/a	quantity over which the decision maker exercises direct control
Value parameter	n/a	parameter that represents aspects of the preferences of the decision makers or the people they represent
Model domain parameter	n/a	specifies the domain or scope of the system being modeled
Outcome criteria	n/a	variable used to rank or measure the desirability of possible outcomes

Model form uncertainty refers to the approximations that a model provides to a real-world system. Model form uncertainty is differentiated here from (quantity type) model domain parameter uncertainty by referring to the actual model itself as opposed to the quantities assumed in the model. Any model is unavoidably a simplification of reality. A real-world system contains phenomena or behaviors that cannot be produced by even the most detailed model. The difference between the real-world system and such a model is the model form uncertainty. It should be noted that Ref. 17 stresses that defined constants (such as the number of days in December, the number of joules in a kilowatt-hour, etc.) and index variables (used to identify a location or cell in the spatial or temporal domain of a model) are not uncertainties.

Physical Sciences

Uncertainty in the physical sciences has primarily concentrated on error analysis and quantum physics. Error analysis uncertainty often goes by the name measurement uncertainty and represents the difference between a measured value and the actual value. This uncertainty impacts a wide range of fields in the physical sciences and engineering. Much has been made of Werner Heisenberg's uncertainty principle that was first proposed in 1927. Heisenberg introduced the notion that it is impossible to determine simultaneously with unlimited precision the position and movement of a particle. Heisenberg was careful to point out that the inescapable uncertainties in momentum and position do not arise from imperfections in practical measuring instruments but rather from the quantum structure of matter itself.¹⁸ This uncertainty in quantum physics is analogous to the inherent randomness in Policy & Risk Analysis described by Ref. 17. It has been argued that this indeterminacy is not a matter of principle but simply a result of the limited (current human) understanding of the world (an epistemological issue). There may be hidden variables and causal mechanisms that, if discovered and understood, would resolve the apparent inherent randomness. This difference of opinion is similar to the notion of risk and uncertainty discussed in economics and decision making.

Engineering

Research into uncertainty in the field of engineering has been significant, particularly in the last two decades. This section briefly summarizes uncertainty research that has been completed in the engineering fields of control and dynamical systems; systems; civil, structural, and environmental, management science; computational methods and simulation; mechanical; and aerospace.

Control & Dynamical Systems

Control and dynamical systems define uncertainty as the difference or errors between models and reality.¹⁹ The field classifies uncertainty as structured or unstructured. Structured uncertainty represents a known function but the values of the function parameters are uncertain. Unstructured uncertainty is entirely unknown but is limited in magnitude, that is to say, it is bounded.²⁰ In some ways this difference between structured and unstructured uncertainty is analogous to that of aleatory and epistemic uncertainty. Control and dynamical systems focuses primarily on unstructured uncertainty. Unstructured uncertainty, also referred to as model uncertainty, is a generic error associated with all design models.

Systems Engineering

System engineering provides two distinct definitions/classifications for uncertainty: one that is rigorous but somewhat theoretical, the other which is more relaxed but practical. The rigorous definition classifies uncertainty as either vagueness or ambiguity. Vagueness is associated with the difficulty of making sharp or precise distinctions in the world; that is, some domain of interest is vague if it cannot be delimited by sharp boundaries. Ambiguity is associated with one-to-many relations, that is, situations in which the choice between two or more alternatives is left unspecified. Ambiguity is further separated into nonspecificity of evidence, dissonance in evidence, and confusion in evidence.²¹

The practical definition characterizes uncertainty by a distribution of outcomes with various likelihoods of both occurrence and severity. It intertwines the definition with that of risk. Risk is defined as a measure of the uncertainty of attaining a goal, objective, or requirement pertaining to technical performance, cost, and schedule. Risk level is categorized by the probability of occurrence and the consequences of occurrence. Risk is classified into technical (e.g., feasibility, operability, producibility, testability, and systems effectiveness), cost (e.g., estimates, goals), schedule (e.g., technology/material availability, technical achievements, milestones), and programmatic (e.g., resources, contractual).²² This classification is similar to the management classification of Ref. 15. The two distinct classifications are provided in Fig. 3.

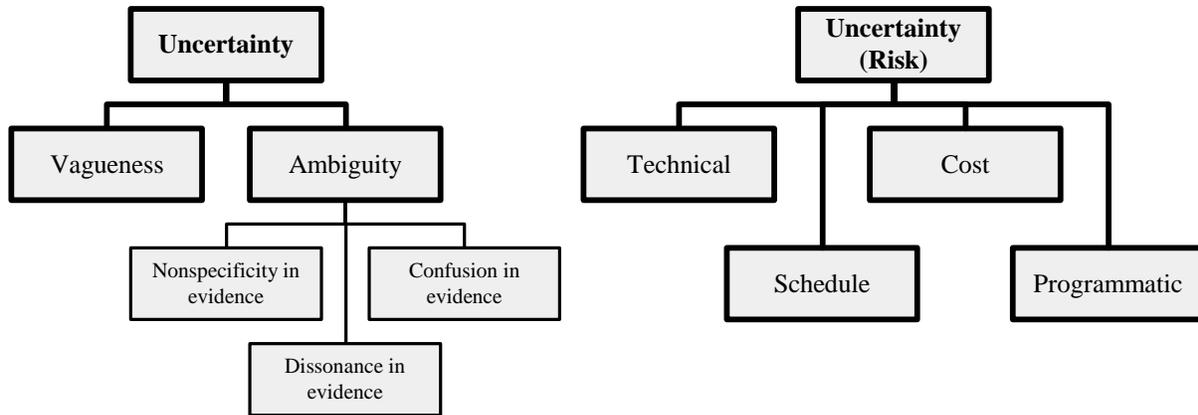


Fig. 3. Uncertainty Classification in Systems Engineering.

Civil, Structural, and Environmental

Although the fields of civil, structural, and environmental engineering are often grouped together, the classifications for uncertainty that each assume is different. The leading classification of uncertainty for civil engineering is provided in Fig. 4.²³

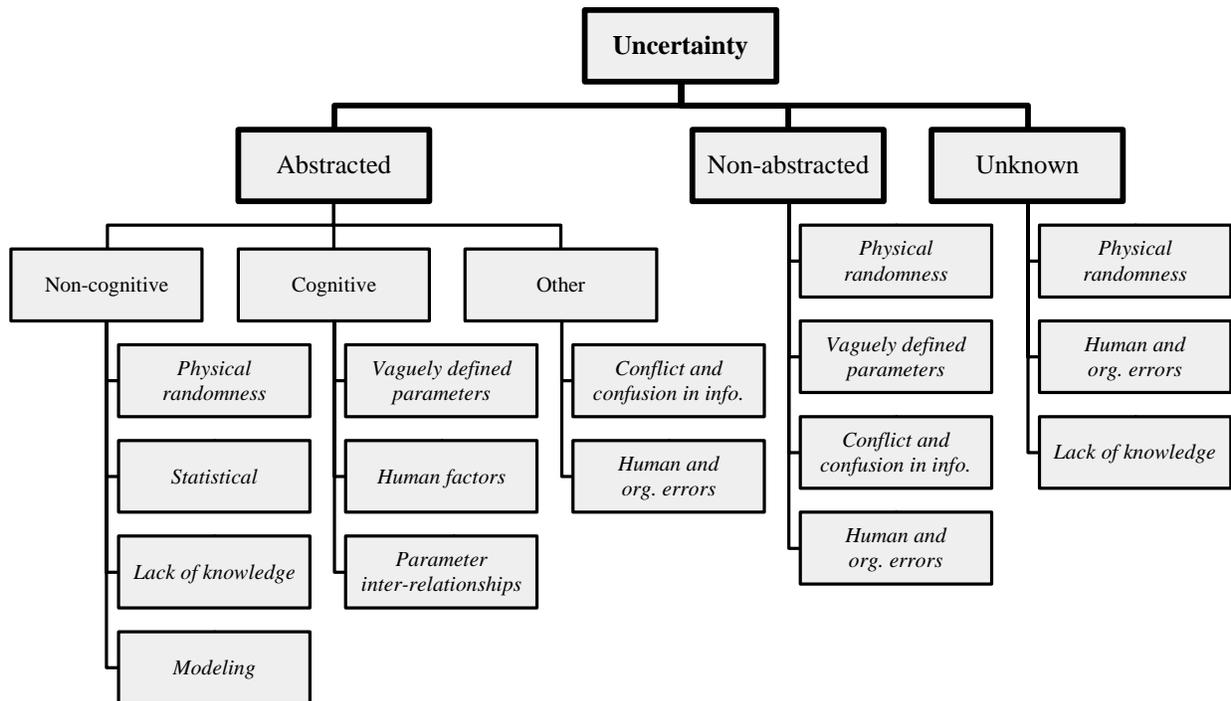


Fig. 4. Uncertainty Classification for Civil Engineering.²³

Ref. 23 specializes the rigorous uncertainty classification provided by systems engineering (Ref. 21) to civil engineering. Abstracted uncertainties arise from elements of a real system that are represented by a model. Unknown uncertainties are due to the nature, sources, contents, and impact on the system that are not known. Cognitive uncertainties arise from mind-based (subjective) abstractions of reality. Uncertainties that are neither non-cognitive nor cognitive are called ‘other uncertainties’ and include conflict in information as well as human and organizational errors. Ref. 23 states that the division between abstracted and non-abstracted aspects may not be rigid but in fact a convenience that is driven by objectives of the system modeling.

Structural engineering follows a somewhat analogous classification.²⁴ The classification and definitions of for structural engineering are provided in Fig. 5 and Table 3, respectively.

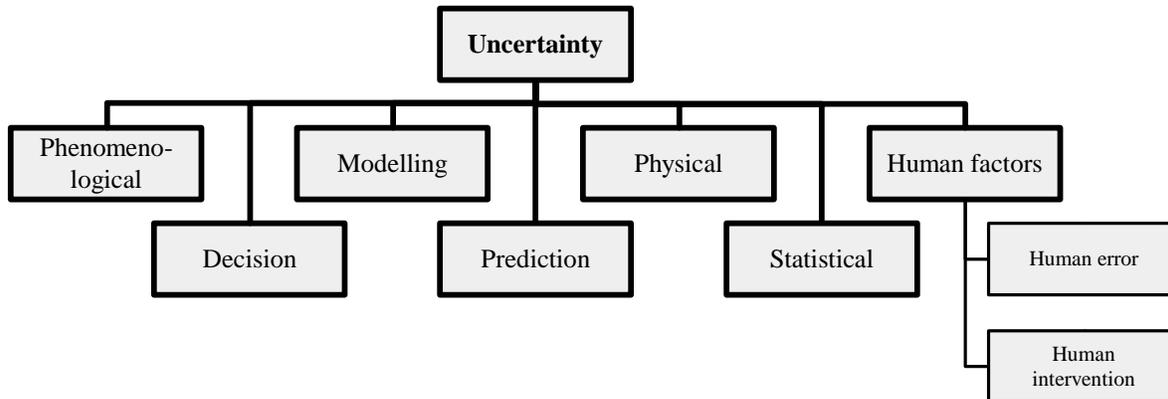


Fig. 5. Uncertainty Classification for Structural Engineering.²⁴

Table 3 Uncertainty Definitions for Structural Engineering²⁴

Uncertainty	Definition/Explanation
Phenomenological	arises whenever the form of construction or the design technique generates uncertainty about any aspect of the possible behavior of the structure under construction, service, and extreme conditions
Decision	arises in connection with the decision as to whether a particular phenomena has occurred
Modelling	associated with the use of one (or more) simplified relationships between the basic variables to represent the ‘real’ relationship or phenomenon of interest
Prediction	associated with the prediction of some future state of affairs
Physical	inherent random nature of a basic variable
Statistical	arises in the associated parameters when a simplified probability density function is implemented
Human factors	
Human error	due to natural variation in task performance and gross errors
Human intervention	associated with the intervention in the process of design, documentation, and construction and, to some extent, also in the use of a structure

Ref. 24 stresses the importance of uncertainty in human factors: the uncertainties resulting from human involvement in the design, construction, use, etc., of structures. Environmental engineering follows closely the policy and risk analysis classification and definitions provided by Ref. 17 that were discussed earlier.²⁵

Management Science

The field of management science, in particular the probabilistic risk analysis community, defines uncertainty as “that which disappears when we become certain”.²⁶ Uncertainty is classified into aleatory, epistemic, parameter, model, and volitional as illustrated in Fig. 6. The definitions for each type of uncertainty are provided in Table 4.

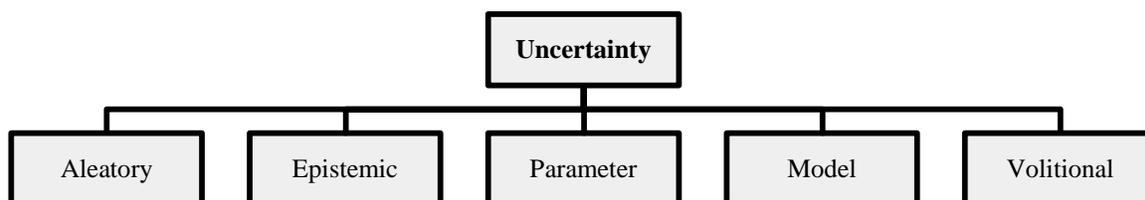


Fig. 6. Uncertainty Classification for Management Science.²⁶

Table 4 Uncertainty Definitions for Management Science²⁶

Uncertainty	Definition
Aleatory	arises through natural variability in a system
Epistemic	arises through lack of knowledge of a system
Parameter	uncertainty about the 'true' value of a parameter in a mathematical model
Model	uncertainty about the truth of the model
Volitional	uncertainty that an individual has in whether or not he will do what he agreed to do

Ref. 26 is careful to distinguish uncertainty from ambiguity. Uncertainty is removed by observation while ambiguity is removed on the level of words by linguistic conventions. Ref. 26 assumes that it is always possible to reduce any given ambiguity to a desired level but impossible to remove all ambiguity. The work of disambiguation goes on until the residual ambiguities are not worth the effort required to remove them.

Computational Modeling & Simulation

One of the more extensive efforts to classify and define uncertainty has been done by the computational modeling and simulation community.²⁷ Ref 27 is clear to distinguish variability, uncertainty, and error as shown in Fig. 7.

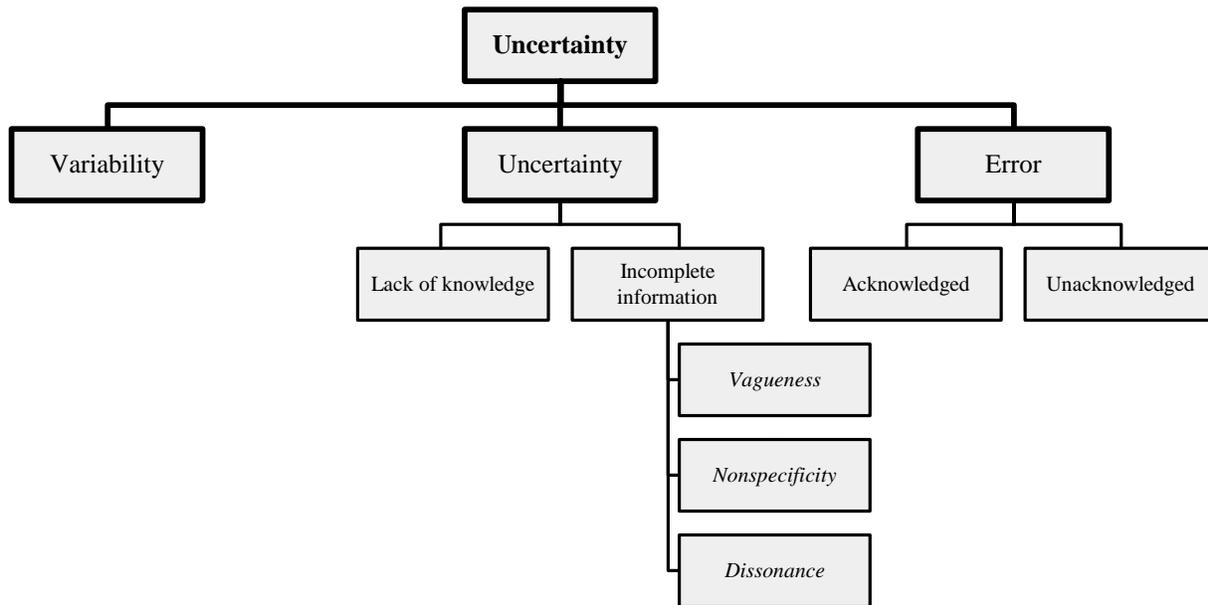


Fig. 7. Uncertainty Classification for Computational Modeling & Simulation.²⁷

Variability is defined as the inherent variation associated with the physical system or the environment under consideration. Uncertainty is defined as a potential deficiency in any phase or activity of the modeling process due to a lack of knowledge or incomplete information. Sources of incomplete information are summarized in Table 5 and follow closely the rigorous systems engineering definitions provided by Ref. 21.

Table 5 Incomplete Information Definitions²⁷

Type	Definition
Vagueness	Characterizes information that is imprecisely defined, unclear, or indistinct (characteristic of communication by language)
Nonspecificity	Refers to the variety of alternatives in a given situation that are all possible, i.e., not specified
Dissonance	Refers to the existence of totally or partially conflicting evidence

Error is defined as a recognizable deficiency in any phase or activity of the modeling and simulation that is not due to a lack of knowledge. Error is further subclassified into acknowledged error (such as finite precision arithmetic on a computer or approximations made to simplify the modeling of a physical process) and unacknowledged error (such as blunders and mistakes). The classification of uncertainty in Ref. 27 is based on the mathematical type and information content of the uncertain quantity. A different perspective of uncertainty by the same group of researchers has also been formulated. It is based on how uncertainty appears in the mathematical model, that is to say, it is a parametric or model-form uncertainty.²⁸ This classification and definitions for uncertainty is provided in Fig. 8 and Table 6, respectively.

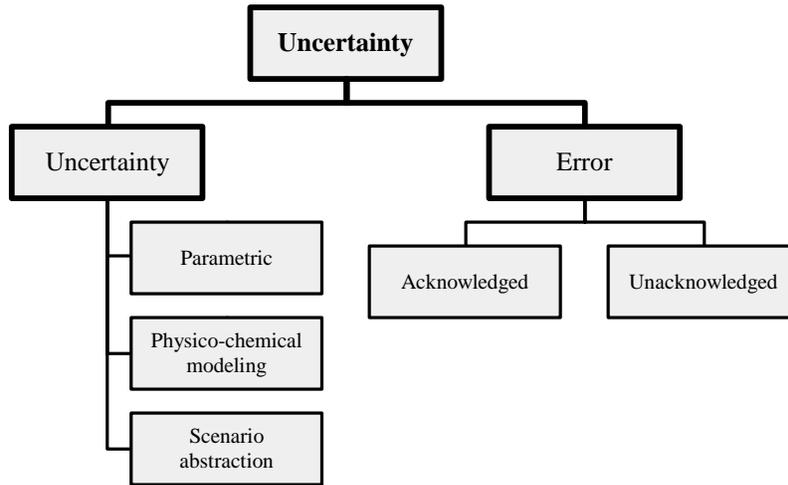


Fig. 8. Uncertainty Classification for Computational Modeling & Simulation (Mathematical Model).²⁸

Table 6 Uncertainty Definitions for Computational Modeling & Simulation (Mathematical Model)²⁸

Uncertainty	Definition
Parametric	Uncertainty in the occurrence in parameters contained in the mathematical models of a system and its environment
Physico-chemical modeling	Limited knowledge or understanding of a physical process or interactions of processes in a system
Scenario abstraction	Limited knowledge for the estimation of likelihood of event scenarios of a system

Error definitions in Fig. 8 remain unchanged from that of Ref. 27. Refs. 27 and 28 provide two different perspectives of uncertainty. A difference reference in the same field classifies uncertainty in an analogous manner as is presented in Fig. 9.²⁹

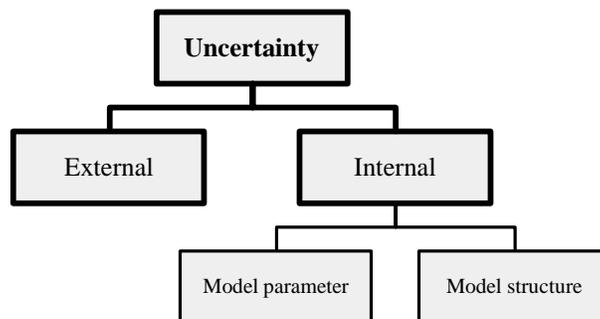


Fig. 9. Alternate Uncertainty Classification for Computational Modeling & Simulation.²⁹

External uncertainty is variability in model prediction arising from plausible alternatives for input values (also known as input parameter uncertainty). Internal uncertainty arises from two sources. One is due to both limited information in estimating the characteristics of model parameters for a given fixed model structure (model parameter uncertainty). The other is the model structure itself, including uncertainty in the validity of the assumptions underlying the model. Research into uncertainty in this field is ongoing.

Mechanical

Over a decade of research into uncertainty occurred in the field of mechanical engineering beginning in the late 1980s.^{30,31,32} Refs. 30, 31, and 32 combine to define uncertainty as imprecision (design imprecision), probabilistic uncertainty (noise, stochastic uncertainty), and possibility. Imprecision is the representation of an incomplete design description. That is to say, ranges of possibilities resulting from choices not yet made (uncertainty in choice). Probabilistic uncertainty is a random (stochastic) uncertainty. Possibility is the uncertainty in the limits in capacity within a formal model (uncertainty due to freedom). Fig. 10 summarizes this classification for mechanical engineering.

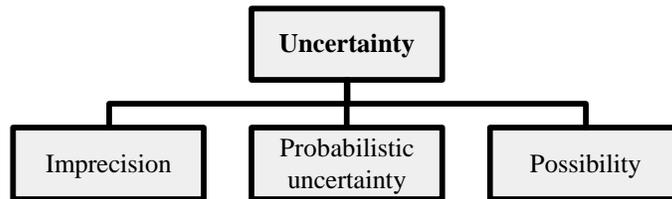


Fig. 10. Uncertainty Classification for Mechanical Engineering.³²

Aerospace

Only recently has an effort been made in aerospace engineering.^{33,34} Ref. 33 defines uncertainty as “the incompleteness in knowledge (either in information or context), that causes model-based predictions to differ from reality in a manner described by some distribution function”. Using an analogy to a control system problem, uncertainty for aerospace vehicle synthesis and design is classified into input, model parameter, measurement, and operational/environmental. This classification is illustrated in Fig. 11.

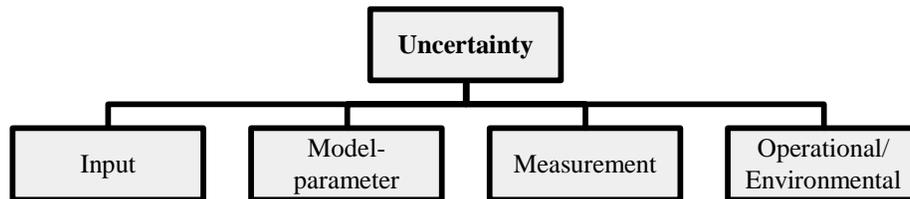


Fig. 11. Uncertainty Classification for Aerospace Vehicle Synthesis and Design.³³

Input uncertainty arises when the requirements that define a design problem are imprecise, ambiguous, or not defined. Model parameter uncertainty refers to error present in all mathematical models that attempt to represent a physical system. Measurement uncertainty is present when the response of interest is not directly computable from the mathematical model. Finally, operational/environmental uncertainty is due to unknown/uncontrollable external disturbances. This classification is redefined somewhat for the specific field of aircraft system design where uncertainty is now delineated into operational/environmental, system-level, and discipline-level uncertainty.³⁵ This classification is presented in Fig. 12.

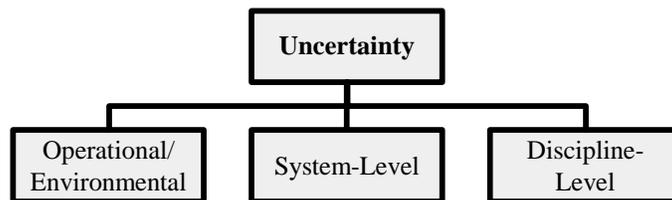


Fig. 12. Uncertainty Classification for Aircraft Systems Design.³⁵

Operational/ environmental uncertainty is concerned with modeling how a vehicle or fleet of vehicles will be utilized over its useful life. System-level uncertainty is concerned with the requirements, synthesis, and predicted performance of a vehicle. Finally, discipline-level uncertainty is concerned with the various contributing analyses that are required to synthesize vehicle alternatives.

Uncertainty research in space system design is very recent. Ref. 34 defines uncertainty as “inability to quantify precisely; a distribution that reflects potential outcome”. Uncertainty is classified into development, operational, and model.³⁴ Fig. 13 illustrates this classification and Table 7 defines these uncertainties.

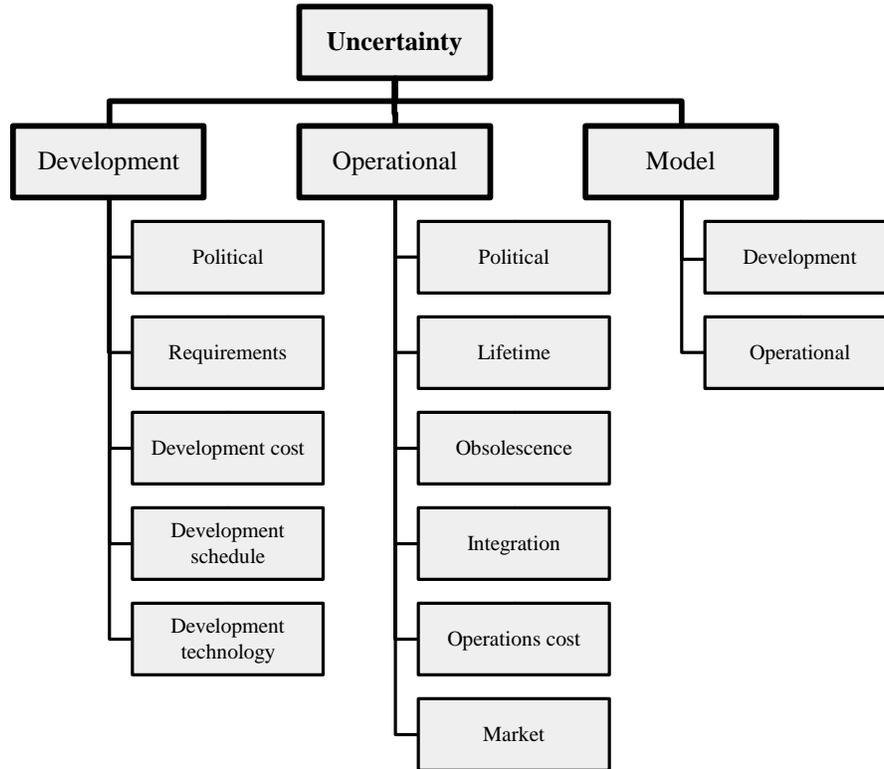


Fig. 13. Uncertainty Classification for Space Architectures.³⁴

Table 7 Uncertainty Definitions for Space Architectures³⁴

Uncertainty	Subclassification	Uncertainty of ...
Development	Political	development funding instability
	Requirements	requirements stability
	Cost	developing within a given budget
	Schedule	developing within a given schedule profile
	Technology	technology to provide performance benefits
Operational	Political	operational funding instability
	Lifetime	performing to requirements in a given lifetime
	Obsolescence	performing to evolving expectation in a given lifetime
	Integration	operating within other necessary systems
	Cost	meeting operations cost targets
	Market	meeting demands of an unknown market
Model	n/a	<i>no formal definition</i>

This classification and associated definitions appears to build on the management classification provided by Ref. 15. Ref. 34 does not provide significant details on uncertainty types beyond these definitions and primarily concerns itself with mitigating uncertainty by portfolio management (drawing on an analogy to economics).

Uncertainty Classification for the Design and Development of Complex Systems

The various classifications described provide both common and distinct classifications and definitions for uncertainty. Unfortunately, none of the previous classifications seem exactly applicable to the design and development of complex systems. Although the classifications provided in the computational modeling and aerospace engineering fields are thorough (Refs. 27, 28, 33, and 34), they still lack important uncertainty types. Furthermore, neither provides a comprehensive method to handle each type of uncertainty. The definition and classifications of uncertainty from the various fields provided earlier motivate a new classification for the design and development of complex systems: ambiguity, epistemic, aleatory, and interaction. This new classification is provided in Fig. 14.

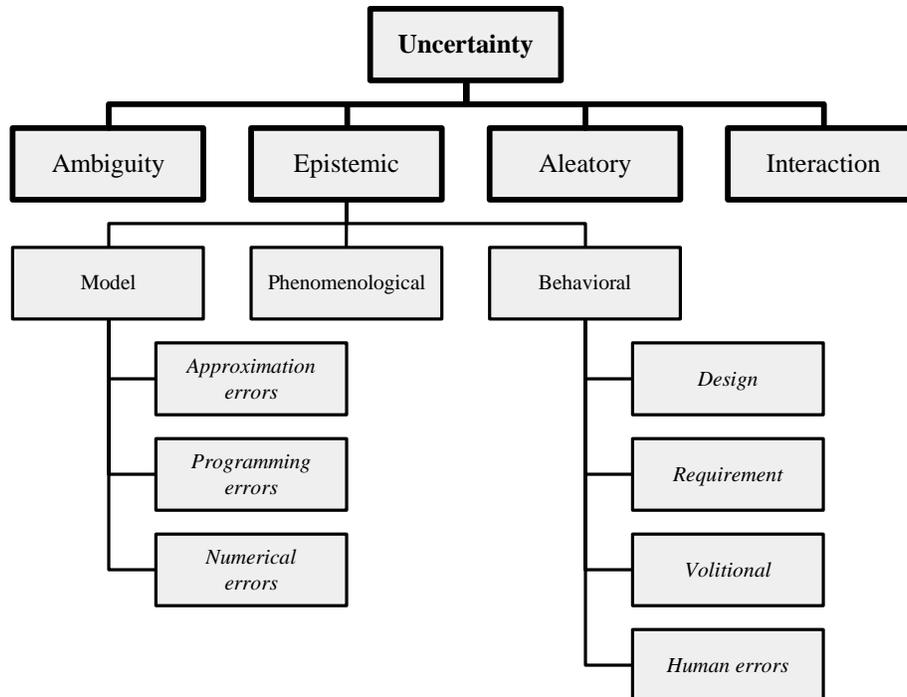


Fig. 14. Uncertainty Classification for the Design and Development of Complex Systems.

A definition for each type of uncertainty follows. Although some of the definitions were provided earlier for a given field, these definitions are repeated in full in this section for clarity. Addressing each type of uncertainty defined is the subject of ongoing research.

Ambiguity

Because little precision is required for general communication, individuals often fall into the habit of using imprecise terms and expressions. When used with others who are not familiar with the intended meanings or in a setting where exactitude is important, this imprecision may result in ambiguity. Ambiguity has also been called imprecision, design imprecision, linguistic imprecision, and vagueness.^{30,17,21} Although it can be reduced by linguistic conventions and careful definitions, ambiguity remains an unavoidable aspect of human discourse. A clarity test has been proposed as a conceptual way to sharpen up the notion of well-specifiedness.³⁶ Imagine a clairvoyant who could know all facts about the universe, past, present, and future. Given the description of the event or quantity, could the clairvoyant say unambiguously whether the event will occur (or had occurred)? Could the clairvoyant give the exact numerical value of the quantity? If so, the description of the event or quantity is well-specified. A statement such as “Jack is tall” would not pass the clarity test. However, “Jack McCullough (social security number 123-45-6789) is six feet three inches tall at this instant” would pass the clarity test. There is some debate as to whether ambiguity is a form of uncertainty.²⁶ Although in theory it is possible to reduce any given

ambiguity to any desired level, this is often not done because of the effort required. Fuzzy logic has been used as a formal method to represent ambiguity.³⁷

Epistemic

Epistemic uncertainty is any lack of knowledge or information in any phase or activity of the modeling process. The key feature that this definition stresses is that the fundamental cause is incomplete information or incomplete knowledge of some characteristic of the system or the environment. Epistemic uncertainty also goes by the names: reducible uncertainty, subjective uncertainty, model form uncertainty, state of knowledge, type B uncertainty, and *de dicto*.^{28,26,2} Epistemic uncertainty can be further classified into model, phenomenological, and behavioral uncertainty.

Model

Model uncertainty is the accuracy of a mathematical model to describe an actual physical system of interest. Model uncertainty, also known as model-form, structural, or prediction-error uncertainty, is a form of epistemic uncertainty. That is to say, model uncertainty is often due to a lack of knowledge. Model uncertainty is associated with the use of one or more simplified relationships between the basic variables used in representing the ‘real’ relationship or phenomenon of interest.²⁴ All models are unavoidably simplifications of the reality which leads to a disturbing conclusion: every model is definitely false. However, some models are better than others. A model that represents the phenomena of interest over a range of interest is termed a requisite model. Model uncertainty arises from approximation, numerical, and programming errors.

Approximation Errors

For physical processes that are relatively well understood, deficiencies in certain models are often called *approximation errors* rather than model uncertainty. For example, in the modeling of the specific volume of a gas, the models can be ordered in terms of increasing accuracy (decreasing model uncertainty) as follows: ideal-gas law, van der Waals equation, Beattie-Bridgeman equation, and Benedict-Webb-Rubin (BWR) equation. The ideal gas law neglects intermolecular forces between molecules and uses only one constant. The van der Waals equation uses two constants to allow for interaction and volume effects. The Beattie-Bridgeman equation uses five constants and is accurate over a much larger range. The BWR equation uses eight constants and is even more versatile. In general, this ordering is appropriate, but for individual gases there is no guarantee that any one model will be more accurate than any other because even the ideal gas law can be accurate for specific conditions such as low pressures and high temperatures.

Numerical and Programming Errors

Model uncertainty also arises from numerical and programming error. *Numerical error* can arise due to finite precision arithmetic and can be reduced by using higher precision computers and software. *Programming error* occurs during development of the model due to mistakes or blunders by the programmer. Although there is no straightforward method for estimating programming errors, they can be detected by the person who committed it, resolved by better communication, or discovered by redundant organizational and operational procedures and protocols.²⁷

Phenomenological

Phenomenological uncertainty arises whenever the design technique or form of development generates uncertainty about any aspect of the possible behavior of the system under development, operation, and extreme conditions. Some relevant information cannot be known, not even principle, at the time of making decisions during design and development. Phenomenological uncertainty is particularly important for novel projects or those which attempt to extend the ‘state of the art’. Often these projects fail due to an apparently ‘unimaginable’ phenomenon (so called “unknown unknowns”). Evidently, only subjective estimates of the effect of this type of uncertainty can be given.²⁴

Behavioral

Behavioral uncertainty is uncertainty in how individuals or organizations act. Behavioral uncertainty arises from four sources: design uncertainty, requirement uncertainty, volitional uncertainty, and human errors. *Design uncertainty* includes variables over which the engineer or designer has direct control but has not yet decided upon. An example is the choice an engineer has in selecting a given component among a set of possible components. Design uncertainty is eliminated when a system is complete as all choices have been implemented. *Requirement*

uncertainty includes variables that some organization or individual initially determines independently of the engineer or designer. An example may be the orbit of a satellite that is explicitly specified by the customer. The question of whether an uncertain variable is a design or requirement depends on the context and intent of the model it is being used in and who the decision maker is. For example, a new standard for cleanliness in a restaurant is a design variable from the perspective of the county and/or city health department but it is a requirement variable from the perspective of the restaurant owner and management.

Volitional uncertainty is uncertainty about what the subject him/herself will decide.²⁶ Other people's future actions and conduct are not entirely predictable, particularly in dealing with other organizations. Multiple organizations are often required to design and develop complex multidisciplinary systems since a single organization often lacks the knowledge base to complete the entire system on their own. The lead organization hires contractors and/or consultants to help in design and development. Contractors and consultants may provide full assemblies, components, analysis, and/or labor. Estimates for these products and resources are often underestimated to the lead organization and result in potentially significant engineering and management problems. Although an individual or organization cannot quantify their own volitional uncertainty, one individual or organization could do it for another.

Human errors occur during development of a system or project due to blunders or mistakes by an individual or individuals. Similar to the programming errors previously discussed, human errors are difficult to estimate. However, facilitative measures such as education, a good work environment, a reduction in task complexity, and improved personnel selection as well as control measures such as self-checking, external checking, inspections, and legal sanctions have proved successful in reducing human errors.²⁴

Aleatory

Aleatory uncertainty is inherent variation associated with a physical system or environment under consideration. Aleatory uncertainty goes by many names: variability, irreducible uncertainty, inherent uncertainty, stochastic uncertainty, intrinsic uncertainty, underlying uncertainty, physical uncertainty, probabilistic uncertainty, noise, risk, type A uncertainty, and *de re*.^{28,31,26,12,2} Aleatory uncertainties can typically be singled out from other uncertainties by their representation as distributed quantities that can take on values in an established or known range, but for which the exact value will vary by chance from unit to unit or time to time. The mathematical representation most commonly used for aleatory uncertainty is a probability distribution.²⁷ This distribution could be based on a frequency distribution quantified by actual measurements, statistical estimations, or by expert opinion. A decision-maker (such as an engineer or designer) has little control over aleatory uncertainty in the design and development of complex systems. Examples include the strength or exact dimension of a component where the manufacturing processes are well understood but variable and the parts have yet to be produced.

As was discussed in the first half of this paper, there is much disagreement about the distinction between aleatory and epistemic uncertainty. It has been argued that all uncertainty is epistemic: that aleatory uncertainties, represented by distributions, are used purely because of our lack of knowledge or understanding of a fundamental underlying process or because we choose not to learn about that underlying process. As an example, consider the tossing of a fair coin. This activity is represented by the discrete binomial (Bernoulli) distribution: either it lands heads (1, true, yes, etc.) or tails (0, false, no, etc.). However, flipping a coin is not truly a random activity. In theory, a sophisticated model based on which side of the coin is initially facing up, the strength and angle of the coin flip, the wind resistance, gravity, and so on could be created to accurately determine whether the coin lands heads or tails. Although this sophisticated model would likely be influenced by minute differences in initial conditions, the remaining uncertainty in the coin flip would now be epistemic. Likewise, a quantity may legitimately be random to one person, but deterministic to another who knows and understands the underlying model or process. For example, a random number generated by a computer does indeed appear random to the vast majority of people but completely predictable to those who know the algorithm being used to generate the value. Depending on the model being used and the criticality of the variable, it might not be worth developing sophisticated models such as the coin-flip model described and instead represent that variable as an aleatory uncertainty.

Interaction

Interaction uncertainty arises from unanticipated interaction of many events and/or disciplines, each of which might, in principle, be or should have been foreseeable. Potential techniques to deal with this type of uncertainty are

simulation, multidisciplinary design optimization (MDO), and complexity science. Interaction uncertainty can also arise due to disagreement between informed experts about a given uncertainty (such as a design or requirement) when only subjective estimates are possible or when new data is discovered that can update previous estimates. Weighted averages, Bayesian techniques, and Dempster-Shafer theory have been used to handle this type of uncertainty.^{17,28} Interaction uncertainty is significant in complex multidisciplinary systems such as spacecraft which may have many subsystems, variables, and experts involved in the design and development.

Conclusions and Future Works

Uncertainty remains a fertile ground for research due to its broad applicability to so many diverse fields. The first half of the paper provided a summary of classifications and definitions of uncertainty in social sciences, physical sciences, and engineering. In the case of engineering, specifically the conceptual (preliminary) design and development of complex systems, a balance must be made between a theoretically rigorous classification and definition and a classification and definition that can actually be implemented in a real-world setting. The classification provided in the second-half of the paper attempts to achieve this balance. It should be noted that in conceptual design it is arguably more important to determine the significant sources of uncertainty than identifying and quantifying all uncertainty sources. Hence, some of the uncertainties that were defined may be unimportant in certain cases. However, it is unclear on how to know this *a priori*. Research into this issue by the author has commenced in conjunction with continued investigation into mathematical techniques to propagate and mitigate the different types of uncertainty in conceptual (preliminary) design.^{38,39,40} The ultimate goal of this research will be a formal method for propagating and mitigating the effect of uncertainty that can be applied to any complex multidisciplinary engineering system.

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