Uncertainty Modeling and Management in Multidisciplinary Analysis and Synthesis

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Abstract

The complex, multidisciplinary nature of aerospace design problems, as well as the requirement to examine life-cycle characteristics, have exposed a need to model and manage uncertainty. In this paper, a formal approach for modeling uncertainty in such design problems is presented. The approach includes uncertainties associated with mathematical models, operation environment, response measurement, and input requirements. In addition, a new method for propagating this uncertainty (in an efficient manner) to find robust design solutions is developed and described. The uncertainty model combined with the probabilistic robust design technique is a critical advancement in multidisciplinary system design, in that it identifies solutions that have a maximum probability of success. Continued research in both uncertainty modeling and efficient robust design methods appears essential. Both the uncertainty model and robust design technique are demonstrated on an example problem involving the design of a supersonic transport aircraft using the relaxed static stability technology. At each step, validation studies are performed and initial results indicate that the robust design method represents an accurate depiction of the problem. This depiction provides critical insight into where and why uncertainty affects the family of design solutions.

Introduction

Key objectives such as system affordability (the balance of system effectiveness and the associated cost Ref. [1]), while always prominent in terminal design stages and manufacturing, have only recently been seriously examined in the setting of conceptual and preliminary design. Affordability trades need to be examined early, however, since the eventual cost of a product is determined to a large extent by decisions made in conceptual design. Yet, the difficult task of evaluating such objectives motivates the use of multidisciplinary and life-cycle analysis methods. Looking more deeply, important observations related to the fundamental formulation of complex engineering design problems in these new settings include:

- the need to directly model *uncertainty*, in its variety of forms, such as: low fidelity contributing analyses, unknown operational environment, ambiguous requirements, and human preferences
- the inappropriateness of optimizing to deterministic objectives, in light of uncertainty

Addressing affordability as an objective and uncertainty as a reality in multidisciplinary vehicle design shifts the fundamental question from "can it be built" to "should it be built" to “with what confidence might it succeed”. As pointed out in Ref. [2], from an industrial perspective, the goal of multidisciplinary design should primarily be to design a vehicle (or set of candidate vehicles) that satisfies the requirements, and then to determine the robustness of the design to changes in assumptions made along the way. Many of these assumptions involve the life-cycle disciplines required for affordability analysis, such as manufacturing and operational economics. In addition, uncertainty associated with the engineering analyses conducted is greatest in the conceptual and preliminary design phases, yet this is also the time when large numbers of possible alternatives are being excluded. Thus, the importance of proper uncertainty modeling management is heightened. Deterministic design, on the other hand, simply neglects these uncertainties by assuming all inputs and outputs to be precise. This practice is increasingly inappropriate for the development of affordable systems where the importance of cost prediction and risk mitigation is equal to that of vehicle performance. Thus, advances in decomposition, approximation, and optimization schemes for multidisciplinary systems (the current focus of multidisciplinary analysis (MDA) community according to Ref. [3]) must be equipped with the capability to handle imprecise, ambiguous, or uncertain information in the contributing analyses.

In order to address this identified need in MDA problems, a new approach has been developed and evolved over the past years. Progress in development of this probabilistic design approach has been presented by the authors at recent Aerospace Sciences Meetings, including an overview paper (Ref. [4]) and a more specific reporting of some mathematical formulations (Ref. [5]). This new method for robust,

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multidisciplinary design is based on the premise that the design activity is a process of making decisions, and that deterministic analysis, synthesis, and optimization can lead to poor decision making. *Design algorithms are sought that are based on the maximum knowledge possible, address robustness to uncertainty, and produce affordable solutions that match the customer requirements.* A consistent approach to modeling the different uncertainty types is key to enabling such algorithms, but has yet to emerge.

The paper will be particularly focused on describing the formal design uncertainty model that has been developed for aerospace systems, as well as techniques for uncertainty propagation. Such a formal model is a critical prerequisite for any successful robust design activities.

Definitions and Motivation

Uncertainty

There is a significant amount of literature concerning the definition and modeling of uncertainty in a wide range of fields. An investigation of several definitions in the literature indicates that context and intent are important factors in determining the viewpoint taken. This is not surprising since uncertainty is present in all engineering models, regardless of the type of phenomena under study. Control system design, structural design, and financial forecasting are examples (both within and outside the bounds of engineering) of the wide range of activities where uncertainty modeling and management plays a central role. In this paper, a simple, consistent general definition of uncertainty is desired that is useful for multidisciplinary aerospace design settings. Portions of an extensive literature review are summarized in Table 1 and serve as a starting point for this task.

One first observes that those in the fields of system and structural design characterize uncertainty in terms of probability and statistics. This is likely due to the predominant use of experimentation and sampling for the purpose of uncertainty quantification. In terms of establishing a formal definition, Zhao (Ref. [6]) and Tung/Yen (Ref. [7]) emphasize the concept of error between model and reality while Hazelrigg (Ref. [8]) emphasizes the consequence of such error. Likewise, the range of uncertainty source identification and classifications cover several different perspectives. The source identification provided by Oberkampf, et.al. in Ref. [10] appears to be especially complete, though they use the term “uncertainty” as one of three categories of “total uncertainty”. Definitions and classifications are important, since there is a delicate balance between accuracy and conservatism when constructing effective uncertainty models. Conservative uncertainty bounds are seen as inefficient, akin to the “safety factors” approach typical to structural design or the unstructured uncertainty model (e.g. a unit disk) typical of early formulations in modern control theory (Ref. [6]). However, in attempting to maximize the “preciseness of an uncertainty model”, one must guard against missing the full range of likely behavior. Achieving this balance, whether it be in the design of a control system, a structural component, or an entire aircraft, is uniquely connected to a comprehensive understanding of the analysis limitations, baseline models employed, and sensitivity of system outputs to the uncertainty itself.

Table 1: A Sampling of Uncertainty Definitions and Classifications

Source/Perspective	Definition	Classification/Sources
Zhao [Ref. 6] *Controls Perspective*	Uncertainty refers to the differences or errors between models and reality.	• Unstructured, representing that which is un-modeled or not possible to model (e.g. high frequency dynamics) • Structured, representing that for which information on the likely behavior is available (e.g. model parameters)
Hazelrigg [Ref. 8] *Systems Design Perspective*	In an experiment, when the sample space contains more than one element with non-zero probability, there is uncertainty	• Insufficient knowledge of the laws of nature • Inability to assess or measure a phenomenon • Inability to measure initial or boundary conditions • Inherent randomness of a physical process
Wershing [Ref. 9] *Structural Design Perspective*	(No formal definition)	• Inherent (variability in nature of phenomena) • Statistical (results from incompleteness of statistical data, e.g. too small sample size) • Modeling (use of simplified analysis models) • Human error (in calculation, fabrication, judgment, etc.)
Oberkampf, et.al. [Ref. 10] *Computational Modeling & Simulation*	Uncertainty is a potential deficiency in any phase or activity of the modeling process that is due to a lack of knowledge.	• Uncertainty- due to incomplete information • Error- recognizable deficiency in modeling/simulation that is *not* due to lack of knowledge • Variability- Inherent variation associated with physical system or its environment
Tung/Yen [Ref. [7]] *Complex Systems Design Perspective*	Uncertainties attributed mainly to the lack of perfect understanding with regard to phenomena or processes	• Natural (inherent to physical process) • Model (inability to perfectly model nature via mathematics) • Input (stochastic inputs) • Measurement/data transfer and manipulation • Operational/environmental

With these insights, the following definitions are adopted for this research. First, *knowledge* is defined as a piece of information in context (Ref. [11]). For example, the statement "Aircraft gross weight = 100,000lbs" is information, while "The gross weight of a 3000nm range commercial transport with aspect ratio 2.5 is 100,000lbs" is knowledge. Uncertainty is then defined as follows.

Definition 1: Uncertainty
Uncertainty is the incompleteness in knowledge (either in information or context), that causes model-based predictions to differ from reality in a manner described by some distribution function.

It is clear from this definition that uncertainty implies the possibility of multiple outcomes and an ability to mathematically model these range of possibilities.

Robust Design

The purpose of robust design is to find good design solutions in the presence of uncertainty. This can be a difficult task, especially when the system is complex with multiple interacting disciplines and when the definition of 'good' is not obvious. Thus, MDA methods and robust design methods need to work hand-in-hand. The authors in Ref. [12] correctly point out that, strictly speaking, MDO has two distinct aspects: formulation (how a problem is decomposed and recomposed) and optimization (mathematical technique to solve the formulated problem). To exploit multidisciplinary characteristics, one must first understand the formulation aspect: defining the links that characterize the interaction and understanding what is unknown or uncertain about these links. Recent research reported by the authors of this paper (see Ref. [13]) as well as the authors in Ref. [14] has resulted in approaches to representing uncertainty, generally at the system level, and propagating that uncertainty into some response which measures the goodness of design alternatives. This measure is usually associated with the concept of robustness, and the area of research is generally referred to as *robust design methodologies.*

Refs. [15, 16, 17] give a detailed description of some robust design concepts proposed in recent years. Within a robust design framework, the objective function is characterized by (at least) two different components: a most likely value and a variance around that value. Robust design, therefore, should be guided by a multi-attribute objective function. The relevant input variables fall into two general classifications. Control variables are those that a designer is able to select precisely. Noise, or uncontrollable variables, are those inputs whose values cannot be chosen precisely for some reason. The variance in the objective is due to either uncontrollable factors ("noise" can be a misleading term, since it implies random variability, which is not always the case) or possible eventual deviations in design factors, both of which represent incomplete knowledge as mentioned in Definition 1. In Ref. [18], Chen delineates robust design problem into those concerned with downstream activities (i.e. noise) as Type I and those associated with changes in design variables as Type 2. Our approach is to define a formal uncertainty model that can address all uncertainty types and allow flexibility and traceability in doing so.

Early approaches for robust design separated control and noise variables, and then sought to determine the control variable settings that produce desirable values of the objective function mean, and which also minimize the variability of the objective function caused by the noise variables. More recent approaches for robust design have focused on the use of "combined arrays." This method groups the control and noise variables together into one response model which can subsequently be used in a variety of ways for robust design. The common thread of the combined array approaches is the formation of metamodels (a model of the actual analysis model, Ref. [19]), to facilitate the numerous function calls usually required in a robust design optimization exercise. The usefulness of the combined-array approach for robust design in aerospace problems was demonstrated by researchers at Georgia Tech as reported in Refs. [20], [21]. A common form of metamodel is the Response Surface Equation (RSE), which has been used by the authors and others in the MDA community in recent years.

The various "combined array" approaches are most easily delineated from each other in how they determine robust solutions. The work found in the literature falls under two general categories: those that base robustness on a weighting of mean and variance and those that base robustness on a "probability of success" found from a cumulative distribution function (CDF). After extensive research into the subject, the following serves as the authors' definition for characterizing a robust design solution. The likelihood of success is, of course, dependent on the specification of a set of criteria (from management, customers, etc.).

Definition 2: Robust Design Solution
A design solution that maximizes the likelihood of success while satisfying constraints in the presence of a specific uncertainty model.

With a general definition for uncertainty and robustness in hand, a novel approach for uncertainty modeling and management is described next.

New Approach for Design Uncertainty Modeling

The design process is defined, fundamentally, as the process of making decisions along a timeline. As such, there is a sequence of modeling (which produces analysis tools), prediction (which employs tools to produce data), uncertainty characterization (which estimates error in the data and underlying models), decision making (which operates on the data), and feedback (which hopefully revisits the models based on comparisons of prediction and reality). The construct represented in Figure 1 is a generic model for uncertainty in a design analysis/synthesis activity. The importance of first establishing a generic model cannot be overstated, since the goal is to develop an underlying theory of addressing uncertainty in complex systems design. Classifying types is an attempt to determine the mechanism by which sources of uncertainty enter the analysis and synthesis process. To facilitate this task, a generic picture of the relation between types of uncertainty is shown in Figure 1, adapted from the well-known control system model. In the field of control, addressing uncertainty has been achieved due to formal mathematical constructs of uncertainty. In aerospace vehicle synthesis and design, no such established framework exists. Therefore, the following design uncertainty types analogous to the control model are proposed: input, operational/environmental, model parameter, and measurement. These types are organized in the figure, with parenthetical references to their aircraft control system "analogies".

Input uncertainty arises when the requirements that define a design problem are imprecise, ambiguous, or not defined. Model parameter uncertainty refers to error present in all mathematical models that attempt to represent a physical system. Measurement uncertainty is present when the response of interest is not directly computatble from the math model (i.e. it must be inferred). Finally, operational/environmental uncertainty is due to unknown/uncontrollable external disturbances. Each of these types can cause the "model-based predictions to differ from reality" as described in the definition above.

With an uncertainty taxonomy in place, efficient ways must be found to utilize this within a multidisciplinary environment to produce required responses. Each particular uncertainty is represented by a random variable, and together they are stored in a random vector **Y**. A vector defined as a vector of design objectives that are a function of a vector of deterministic design variables (**X**) and the vector of random variables (**Y**). Similarly, **W** is a vector of design constraints that are also a function of **X** and **Y**.

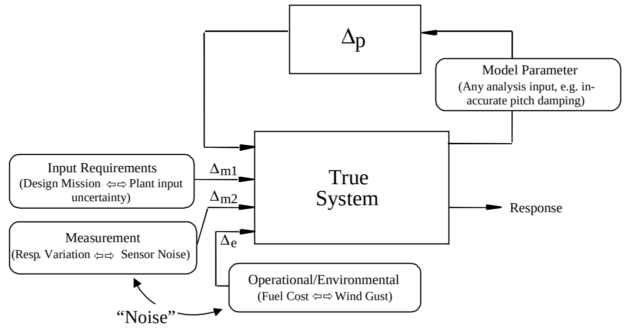

Figure 1: Uncertainty Types, with Flight Control System Analogies

In aircraft design, the luxury of having a synthesis code which coordinates and combines the disciplines in a non-arbitrary, physics based manner can be exploited in computing **Z** and **W**. The information flow proceeds as in Figure 2. The subscripts on $\mathbf{X}_i$ and $\mathbf{Y}_i$, respectively, indicate the subset of the respective vectors that are passed to that particular discipline. Similarly, vector $\mathbf{R}_i$ represents quantities computed by the disciplines and transferred to the synthesis/sizing module. Vector **Z** contains the system objectives and constraints computed from sizing, plus possible components of $\mathbf{R}_i$ computed from the disciplines. Within each discipline resides a set of metamodels that are a function of the elements of **X** and **Y** (as well as possible local variables). These local disciplinary variables can be fixed or can vary with the disciplinary analysis performed to generate the approximations. The main advantages of this method are expected to be that it provides an efficient way to bring sophisticated analyses to synthesis, replacing the obsolete databases; it allows the possibility for concurrent discipline analyses; and, it facilitates the newly developed probabilistic robust design procedure. The potential drawbacks include the accuracy of the metamodels (especially in terms of derivative information, as pointed out in Ref. [16]) and potential limitation in the number of variables modeled. The former issue is addressed in the current method. For example, uncertainty due to metamodel approximation is captured as a model parameter uncertainty. The overall merits of this approach are discussed in Ref. [15].

The overall approach depicted in Figure 2 represents an all-at-once procedure, as there is no subspace optimization or bartering between disciplines. *A fundamental difference is that disciplines in many current MDO methods are asked to compute and return a locally feasible (or optimal)* ***point****, whereas the disciplines in the approximation approach are asked to construct a model of the feasible* ***space*** *once, then simply query these efficient models thereafter.*

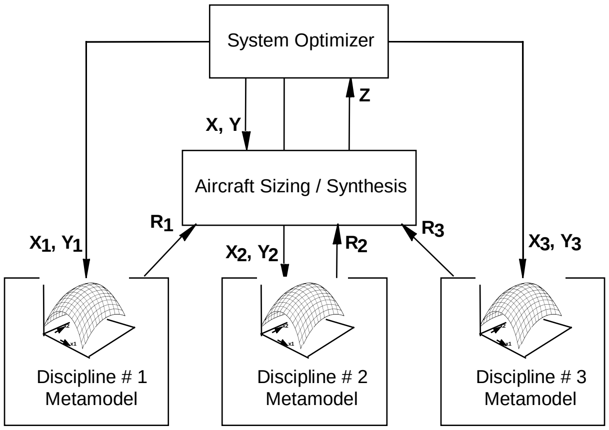

Figure 2: Framework for Multidisciplinary (All-at-Once), Robust Design

Uncertainty Management- Generating Probabilistic Measures of Robustness

With a formal structure for uncertainty modeling and a framework for searching for robust designs in a multidisciplinary setting, attention is directed toward two key issues. First, a proper robust design objective must be formulated. Second, a means for efficiently computing this objective through the space of design options is needed.

The definition for robustness (Definition 2) offered earlier captures the belief that an overall probability measure of success is preferable for measuring robustness and managing uncertainty. It allows for the modeling of decision-maker preferences and their impact what is a measure of goodness. It is proposed here that the cumulative distribution function (CDF) of the identified objectives of a problem is the key to a flexible robust design process. Formulations that weight mean and variance in a composite objective are seen to be less realistic, especially for modeling preferences of a human decision-maker. The probabilistic design model presented in this paper is focused on computing these CDFs.

Computational efficiency in robust, multidisciplinary design is critical, since robust design problems require substantially more function evaluations than traditional designs. The authors in Ref. [16] estimate that typical robust design problems may take at least 2 orders of magnitude more evaluations. For the most general class of robust design problems, this estimate might be conservative. The probabilistic design model, based on formation of distribution functions for objectives and constraints, requires several orders of magnitude increase in the analysis in comparison to the deterministic model. If a deterministic search takes 'n' simulations, a brute-force Monte Carlo approach might require '1000*n' or more simulations to achieve a sufficiently rich distribution for the objective function. A promising solution is offered in Ref. [16] through the use of automatic differentiation to obtain efficient sensitivities for guiding robust design optimization. However, only robustness to design variable uncertainty (Chen's Type II problem) was tackled there, and the use of this approach for all types of uncertainty mentioned in Figure 1 needs further examination.

The range of options to deal with this dilemma is summarized in Figure 3. Given an analysis routine that computes the objective function, the most exact method to obtain the CDF is through a Monte Carlo simulation around this analysis. This is typically too expensive for complex systems analysis. Thus, one can either approximate the analysis itself (through metamodels), Option II in Figure 3, or approximate the probabilistic simulation (through techniques such as Fast Probability Integration, Ref. [22]), which is the Option III. Insights into the settings in which each option may be appropriate are discussed in detail in Ref. [23].

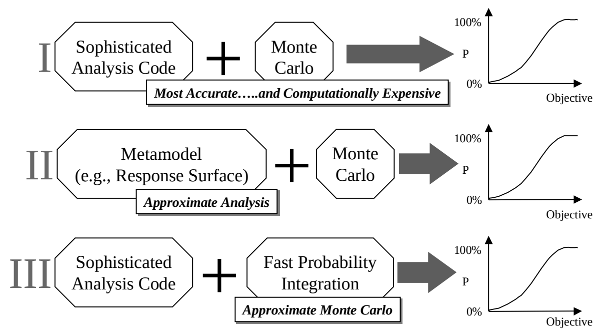

Figure 3: Options for Probabilistic Design

Any of the three options, however, have the disadvantage that the analysis must be "re-run" for each set of design options. *In essence, the mapping of design variables to objective function CDF must be performed over and over.* Such a task can quickly become either inefficient or overly dependent on approximation of the analyses in an effort to reduce the computational time. Though there are numerous alternative ways to construct the CDF (or PDF) of a response, the issue of excessive computational effort arises. *A functional relationship between response CDFs (F(**Z**)) and primitive design variables (**X**) is needed. A method for forming such a CDF metamodel is not found in the literature.*

A New Procedure

The following five-step procedure has been developed to construct these important relationships. The procedure utilizes the combined Design of Experiments (DOE)/Response Surface Methodology (RSM) to relate the CDF of a response random to the design variables under a selected uncertainty model. The procedure is also explained pictorially in Figure 4. Background on the RSM can be found in Ref. [24].

1. Select the desired set of design variables and create a corresponding experimental design table. Step 1 begins with selection of the design variables to be considered ($\mathbf{X}$), ranges for those variables that define the extent of the design space, and the regression model equation. Next, a design of experiments (DOE) table is selected with the resolution required by the model equation. Multiple center points are included in the DOE to quantify experimental error since the presence of uncertainty leads to non-repeatable results for simulation runs using the same design variable settings. The error term, ε, in the regression now includes experimental error. This is certainly so for key statistics such as the sample mean, defined in Eq. (1).

$$\overline{Z} = sample\ mean = \sum_{i=1}^{n_s}\left(\frac{z_i}{n_s}\right) \qquad \textbf{(Eq. 1)}$$

2. Construct an uncertainty model for the problem. Using the generic model shown in Figure 1, Step 2 calls for the establishment of a specific uncertainty model, accomplished by assigning PDFs to the uncertain parameters represented by vector $\mathbf{Y}$. This assignment is based on such things as historical data, experimental results, expert opinion, and analysis.

3. For each row in the experimental design table, generate a CDF for the objective and constraints. Step 3 consists of the execution of an analysis code that produces response Z given values for $\mathbf{X}$ and $\mathbf{Y}$. For each row in the experiment table, the values of $\mathbf{X}$ remain fixed as the PDFs for each random variable in $\mathbf{Y}$ are sampled, culminating in a CDF for Z.

4. Discretize each CDF into n_p values corresponding to n_p probability levels for each objective and constraint; these n_p values for each objective and constraint are the responses in the experiment. In Step 4, the constructed CDFs are discretized into n_p-1 intervals, resulting in n_p particular values of Z (termed z_o) and W (w_o), as shown in Eq. (2) and Eq. (3). These n_p values become the responses for which response surface equations are to be formed. An appropriate value for n_p depends on the accuracy desired in representing the CDF, though no less than $n_p=5$ is advised.

$$(z_o)_i \ \ for\ P(Z \le z_o) = \frac{i}{n_p},\ i = 1, n_p \qquad \textbf{(Eq. 2)}$$

$$(w_o)_i \ \ for\ P(W \le w_o) = \frac{i}{n_p},\ i = 1, n_p \qquad \textbf{(Eq. 3)}$$

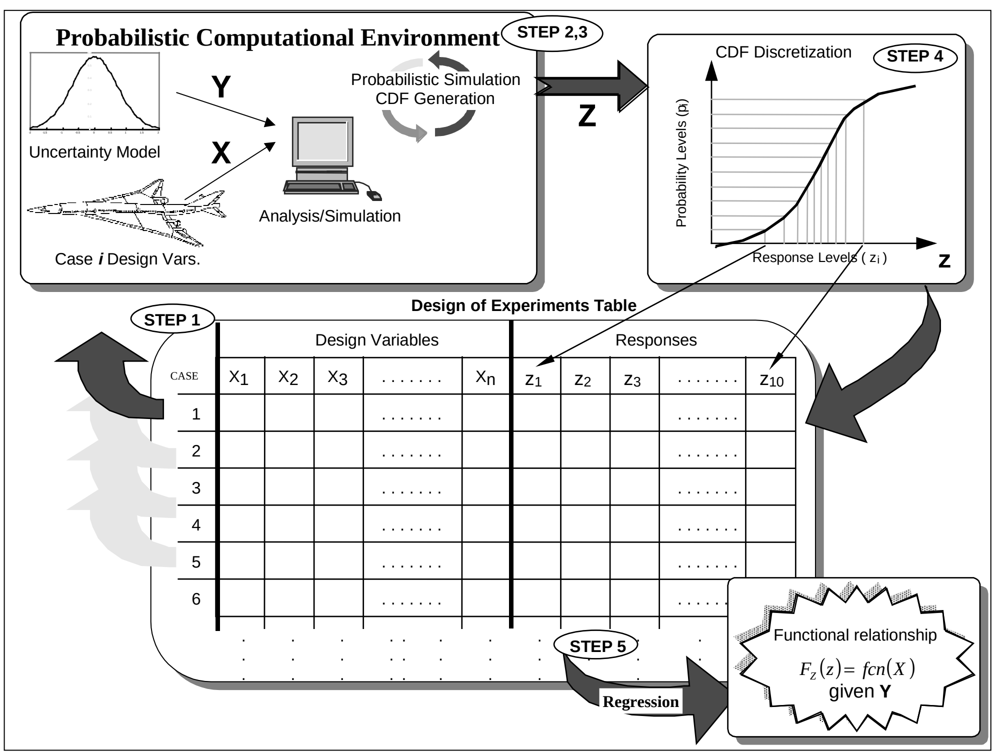

Figure 4: Summary of Steps for Generating Parametric CDFs

5. Form metamodel (response surface equation) for each response by multiple nonlinear regression. The procedure culminates in Step 5 with the formation of the desired functional relationships between design variables and the CDFs via regression on the DOE data. For any set of design variable values, the RSEs can be used to reconstruct the CDFs and perform *any variety of robust design optimizations*. The analysis does not have to be re-executed each time a new problem is to be solved, as long as problem falls within the original design space selected.

Application: Robust Design with Aerodynamic and Stability and Control Disciplines

Problem

The specific proof-of-concept application for this research concerns the investigation of the benefits of Relaxed Static Stability (RSS) during conceptual design of a notional supersonic transport. RSS is defined as the reduction or elimination of inherent static and dynamic vehicle stability requirements. RSS may allow a reduction in trim drag via placement of the supersonic CG location in relation to the aerodynamic center such that the static margin is zero, indicating no need for a trim deflection at the supersonic cruise condition. However, it is likely that longitudinal instability will ensue when the aerodynamic center shifts forward in low speed subsonic flight as a result of this rearward CG placement. In addition, examination of RSS must be done in the presence of multiple disciplines and a variety of uncertainties. This challenge coalesces to the following question: *What is the optimum level of stability relaxation corresponding to optimum wing and horizontal tail geometry, size, and position which maximizes robustness in system affordability while meeting stability, handling quality, and control authority constraints at critical points in the flight envelope*?

The design space under study in this paper consists solely of parameters related to the geometry and relative position of the wing and horizontal tail, respectively, as summarized in Figure 5 and Table 2. Later, two variables associated with a simple control system will also be included. The unique shape of the envisioned supersonic transport wing includes a leading edge (LE) “kink”, at which point a sweep angle change occurs (refer to point (X1,Y1) in Figure 5). A sweep change is also present on the trailing edge (TE). A sampling of possible wing planform shapes modeled within the design space appears in the upper right of Figure 5 to exemplify the extent to which quite different configurations are to be studied. The ranges for the wing design variables are normalized by the wing semi-span for convenience. This normalization is especially useful in employing the RSM as outlined in Ref. [25]. The baseline aircraft is defined by setting the design variables in Table 2 at their midpoint values.

Affordability is the ultimate payoff, and thus the objective function chosen here is the required average yield per revenue passenger mile ($/RPM). This metric in essence represents the ticket price an airline must charge in order to achieve specified return on investments to the airline and manufacturer. Therefore, $/RPM is to be minimized.

Table 2 : Design (Control) Variable Definitions, X

Description	Name	Lower Bound	Upper Bound
Kink, LE X-location	X1	1.54	1.69
Tip, LE X-location	X2	2.10	2.36
Tip, TE X-location	X3	2.40	2.58
Kink, TE X-location	X4	2.19	2.36
Root Chord	X5	2.19	2.40
Kink, LE Y-location	Y1	0.44	0.58
Wing Area (sq. ft.)	SW	8500	9500
Wing Apex Position (% fus)	XWING	0.26	0.29
Horiz. Tail Area (sq. ft.)	STAIL	400	750
Horiz. Tail Apex Position (%fus)	XTAIL	0.87	0.92
Center of Gravity (%fus)	CG	0.575	0.61

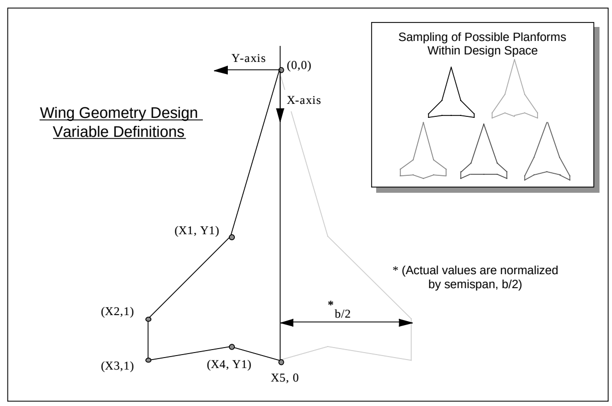

Figure 5: Parameterization of Wing

Approach

The problem essentially is solved by a system optimizer over the set **X** of design variables which seeks to maximize the probability of achieving a target for the objective, while satisfying the handling quality constraints. The probabilistic robust design decision making methods developed (Figure 4) are combined with the disciplinary metamodels and new multidisciplinary strategy introduced in Figure 2 to form the finalized experimental testbed to be used to test the research hypotheses. A simplified view of this finalized concept is illustrated in Figure 6.

The method begins with the building of metamodels for disciplinary metrics as a function of elementary design variables (e.g. configuration geometry), based on engineering analysis. For the specific problem in this paper, in order to evaluate the benefits of RSS in a robust design setting, a rapid method for computing the stability and control characteristics of candidate configurations is needed. After a review of available options in the public domain, the High Angle of Attack Stability and Control (HASC95) program is selected to estimate the longitudinal forces, moments, and associated S&C derivatives in this research. HASC95 is documented in Ref. [26], and the reasons for its selection are described in Ref. [15]. This program is the "simulation engine" used to generate the regression data for the metamodel building exercise to be described. For the mission aerodynamics, RSEs developed in Ref [25] over the same design space are employed. These RSE's, which capture the individual discipline physics for a class of aircraft, are then integrated into a sizing/synthesis code (here the Flight Optimization System (FLOPS), Ref. [27]), which sizes the vehicle for a given mission. After uncertainty models are established, probabilistic analysis techniques are used to determine the system feasibility and viability via the construction of cumulative probability distributions for key system constraints and objectives. The ultimate objective of these probabilistic feasibility and viability investigations is to find *robust solutions*.

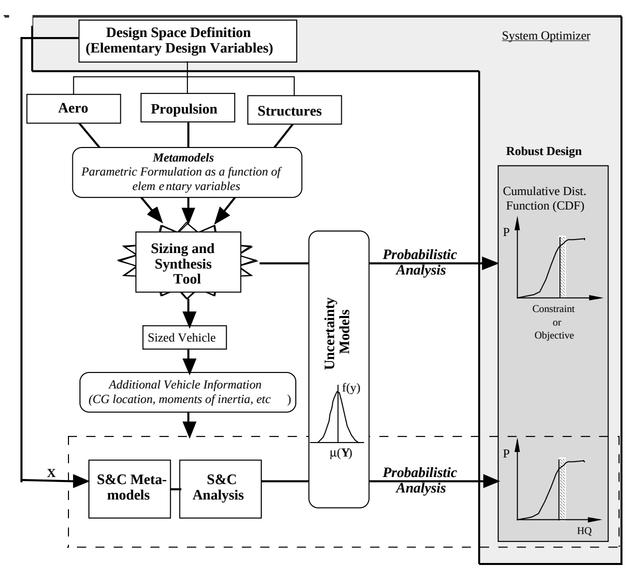

Figure 6: Probabilistic, Multidisciplinary Design Method Employed for Aircraft Design

S&C Derivative Uncertainty Structure

Earlier, the formal uncertainty model established four types of uncertainty for aerospace systems design: model parameter, input, measurement, and environmental. The full study in Ref. [15] on which this paper is based tackled uncertainties in model parameter, input, and environmental. For the purposes of brevity, *only model parameter uncertainty will be addressed in this paper*. The formulation is similar for the other types. In this paper, model parameter uncertainty takes the form of uncertainty in each of the required longitudinal stability derivatives. This uncertainty emanates from three sources.

First, error due to code fidelity is the error emanating from a computer code's inability to completely capture physical phenomena. Analysis results from the HASC95 code are only approximations of the actual forces and moments, since a linear, vortex-lattice method can only approximate the true aerodynamic behavior. This *fidelity uncertainty* is demonstrated in the comparison of HASC95 results for the pitching moment versus angle-of-attack derivative, $C_{M\alpha}$, to wind tunnel data discussed in Figure 7. The structure and magnitude of this uncertainty term is based on these validation plots. Since this type of error has skewsness, a series of beta distributions shall be used in each case. The fidelity uncertainty models are summarized in Table 5 and Table 6.

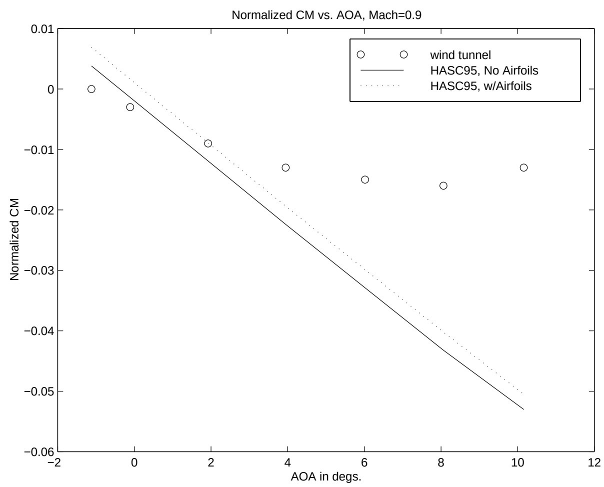

Figure 7: Analysis Fidelity Uncertainty Computation for $C_{M\alpha}$

Second, the use of metamodels to capture the behavior of HASC95 in estimating the derivatives over the whole design space results in *approximation uncertainty*. The magnitude of this uncertainty term is based on the predictive validation results, an example of which is shown in Figure 8. The validation of the RSEs for $C_{M\alpha}$ constructed from HASC95 is displayed in the figure, with 15% error bands shown. These plots are used to detect any tendency of the RSE to under or over-predict the response; however, neither tendency is predominant in any of the results for the stability derivatives shown in the figures. Thus, Gaussian distributed random variables are used to model the error due to approximation. The random variable PDFs represent the error as a percentage of predicted value. Thus, each PDF has a mean of zero and a variance selected such that the error band values from the validation occur at probabilities of approximately 0.01. The approximation uncertainty models are summarized in Table 5 and Table 6. The pitch-damping derivative is not required for trim drag analysis to be conducted at the supersonic condition.

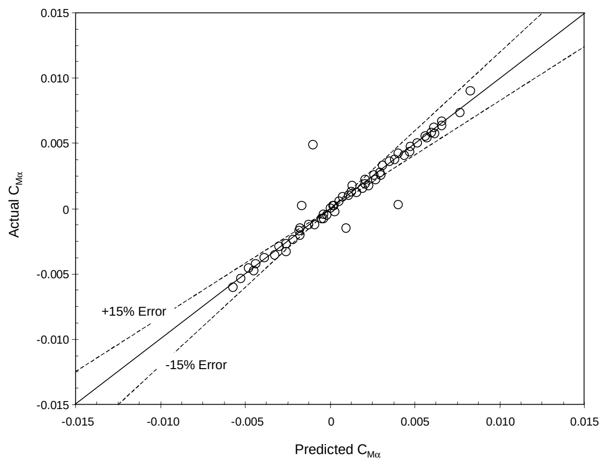

Figure 8: Approximation Uncertainty for $C_{M\alpha}$ Response Surface Equation

Third, in a manner similar to the fidelity uncertainty associated with the aerodynamics, the effects of flexibility are not addressed in the HASC95 analysis (or the wind tunnel experiments), resulting in imprecision due to *neglecting flexibility*. Error due to flexibility arises when the influence of static aeroelastic deformations on the airframe (which subsequently influence both stability and control derivatives) are neglected. Careful study of verification/validation results, interpretation of the statistical quality of the regression, and historical data regarding rigid/flex ratios are all used to add structure (or information) to the uncertainty in derivative estimates. This is important in assessing robustness as well as tracking the impact of analysis improvements.

Since both the wind tunnel and HASC95 analyses described do not account for the effects of structural flexibility on the S&C derivatives, uncertainty models for flexibility are applied in the robust design setting. Since conducting the actual aeroelastic analysis for the supersonic transport design space is beyond the scope of this research, the structure of the uncertainty model is based on previous studies of supersonic transport flexibility. According to Ref. [28], static flexibility effects on stability derivatives of supersonic transports are significant and can vary with Mach number and altitude. As a result, the uncertainty due to un-modeled flexibility is a function of flight condition.

Flexible-to-rigid ratios for the pitch stability and lift-curve slope (C_{L_α}) derivatives reported in Refs. [28] and [29] are shown in Table 3 and Table 4. The ratios in both references are static only (no dynamic flexibility effects). Thus, they are “in phase” with the rigid body modes and can be applied to that model directly. Unfortunately, since no other references were found with such information, the extent to which flexibility effects are addressed here is very limited. Uncertainty due to flexibility is limited to a modeling of the error in C_{M_α} at the Mach 2.4 flight condition. A conservative error model is chosen, in the form of a Gaussian PDF with mean of 0.75 and standard deviation of 0.05.

Table 3: Supersonic Transport Flex/Rigid Ratios ([28])

Derivative	M=2.4	M=0.8
C_{L_α} (flex/rigid ratio)	0.77	0.8
(dC_M/dC_L) increment	0.118	0.065

Table 4: Supersonic Transport Flex/Rigid Ratios at M=2.7 (Ref. [29])

Derivative	rigid	flex	rigid/flex
C_{L_α}	0.029008	0.026645	0.9185
C_{M_α}	-0.002851	-0.000486	0.1705

To summarize, uncertainty models for the pitch stability, lift-curve slope, pitch damping (C_{M_q}), and elevator control power ($C_{M_{\delta e}}$) are developed for use in the design of a supersonic transport with RSS. The uncertainty models for each derivative and for each condition are summarized in Table 5 and Table 6. By explicitly representing this uncertainty during analysis, it becomes possible to *investigate the sensitivity of the design to variations in the uncertainty model* (i.e. answer the question “How important is uncertainty in the longitudinal static stability parameter in arriving at the robust design solution, as propagated through aircraft synthesis?”).

Table 5: Summary: Subsonic S&C Derivative Uncertainty Models

	Low Subsonic (Mach 0.3)		**High Subsonic (Mach 0.9)**	
Deriv-ative	**Approx. Uncert.**	**Fidelity Uncert.**	**Approx. Uncert.**	**Fidelity Uncert.**
C_{L_α}	N(0,0.02)	Beta(4,2,0.1)	N(0,0.02)	Beta(3.6,2,0.06)
C_{M_α}	N(0,0.05)	Beta(4,2,0.84)	N(0,0.05)	Beta(4,2,0.7)
C_{M_q}	N(0,0.025)	N/A	N(0,0.025)	N/A
$C_{M_{\delta e}}$	N(0,0.025)	N/A	N(0,0.025)	N/A

Table 6: Summary: Supersonic S&C Derivative Uncertainty Models for Trim Drag Model

Derivative	Approx. Uncert.	Fidelity Uncert.	Flexibility Uncert.
C_{L_α}	N(0,0.09)	Beta(4,2.5,0.02)	N/A
C_{M_α}	N(0,0.32)	Beta(4,2,0.34)	N(0.75,0.05)
C_{M_o}	N(0,0.08)	Beta(4,2,2.2)	N/A
$C_{M_{\delta e}}$	N(0,0.025)	Beta(4,2,0.15)	N/A

Total uncertainty in each derivative is reconstructed from the components through the additive uncertainty model shown in Eq. (4). An additive model implies that the error estimate from each of the three uncertainty sources (represented through three random variables) are combined and removed from the nominal estimate to obtain the true derivative estimate for that condition. In Eq. (4), the two $E(C_{xy})$ terms are random variables which each represent percentage error (expressed as a decimal) due to approximation and fidelity error, respectively. Error due to approximation is the error introduced by the response surface representation, $E(C_{X_y})_{RSE}$. Error due to analysis fidelity is represented by in Eq. (4) by $E(C_{X_y})_{fidelity}$. The term $(r)_{flex/rigid}$ is a random variable that represents the range and associated likelihood of values of the flex-to-rigid ratio. In summary, when the S&C derivative estimates are required during analysis, the appropriate RSE is evaluated for the current design variable values and the three random variable distributions present in Eq. (4) are sampled from the distributions summarized in Table 5 and Table 6. The two percentage error random variables are multiplied by the RSE evaluation to get actual error. This actual error is subtracted from the RSE evaluation and this result is multiplied by the flex-to-rigid value (when available) to obtain the final S&C derivative estimate. This final estimate is returned to the analysis program.

$$(C_{X_y})_{actual} = \left\{(C_{X_y})_{RSE} - (C_{X_y})_{RSE}\left[\pm E(C_{X_y})_{RSE} \pm E(C_{X_y})_{fidelity}\right]\right\} * (r)_{flex/rigid} \quad \text{(Eq. (4))}$$

Synthesis and Robust Optimization Results

The design space and uncertainty models are framed in the robust design setting to form a mathematical problem statement defining the task of examining RSS for a supersonic transport with robustness. The procedure will be performed twice, once with RSS active and then with RSS inactive, in order to see the impact of the RSS technology on robustness. When RSS is active, the number of design variables increases from 7 to 9, to include the longitudinal closed loop control system parameters (ω_{sp}, ζ_{sp}).

Probabilistic Robust Design Mathematical Problem Statement	
Maximize:	*P(* $/RPM(**X**, **Y**)<$z_o$)
Given:	*Metamodels for discretized CDF of* $/RPM *as a function of* **X** *for given* **Y**
	X = *vector of nine design variables: [*X1, X5, Y1, SW, XWING, ST, CG, ω_{sp}, ζ_{sp}*]*
	Y = *vector of random variables, defining uncertainty models, summarized in Table 5 and Table 6*
	z_o = *customer's target (a particular value of* $/RPM*)* $\rightarrow$ *optional*
Subject to:	**W**$\le$*0;* **W** = *vector of constraint random variables, including:*
	P(GW < 1,000,000 lbs.) =1
	Four HQ constraints defined in Ref. [15]
	$\mathbf{X}_{\text{lower bounds}} \le \mathbf{X} \le \mathbf{X}_{\text{upper bounds}}$ *(Table 2)*

The five-step procedure for generating the mapping of design variables to objective/constraint CDFs is performed. At issue next is determining how well these mappings represent the true nature of the underlying relationship. For this purpose, a validation is performed on the CDF RSEs for $/RPM. For example, the validation graph presented in Figure 9 is for the particular value of $/RPM response corresponding to P(0.625). The bands shown in each figure are the ±5% error bands.

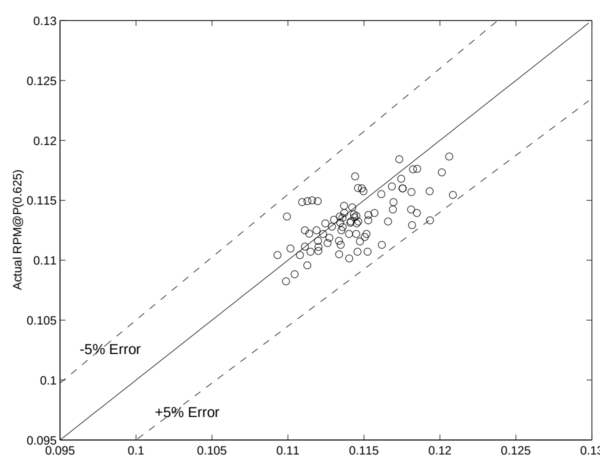

Figure 9: Predictive Validation for the P($/RPM)≤0.625) RSE

Based on these results and similar results for other probability levels (documented in Ref. [15]), the metamodels are seen to be quite reliable predictors and can be used in the robust design studies.

The validated RSEs for the $/RPM distributions are used in conjunction with a gradient-based search algorithm to find robust solutions. Cases are examined for two different targets and for the case with and without the employment of RSS. Results are reported in Table 7. Modest improvements are seen by incorporating RSS, as evidenced by the 0.5% improvement in achievable probability for a target $/RPM of $0.11. An interesting interpretation of this modest improvement is that the performance gains resulting from RSS were tempered by the uncertainty introduced by its employment. The objective function of probability of meeting a target captures these concerns in one response.

Table 7 : Robust Design Results

	With RSS	***Without RSS***
Dec. Fcn. ➔ *Des. Var.* ➘	*Max. P($/RPM<0.11)*	*Max. P($/RPM<0.11)*
X1	-1.00	-1.00
X5	-0.0821	-0.184
Y1	1.00	0.929
SW	0.577	0.514
XWING	1.00	1.00
STAIL	1.00	1.00
CG	-1.00	-1.00
$(\omega_{sp})_{cl}$	-1.00	-
$(\zeta_{sp})_{cl}$	1.00	-
$P(W_1<0)$	100%	92.8%
$P(W_2<0)$	100%	100%
Soln	0.86097	0.8565

A geometric comparison of robust solutions with and without RSS technology is shown in Figure 10. Again, for this particular objective, the geometry differences are minimal. In effect, the optimal wing shape for robustness is unaffected by the presence of RSS. For other objectives, the shape differences were significantly greater described in Ref. [15].

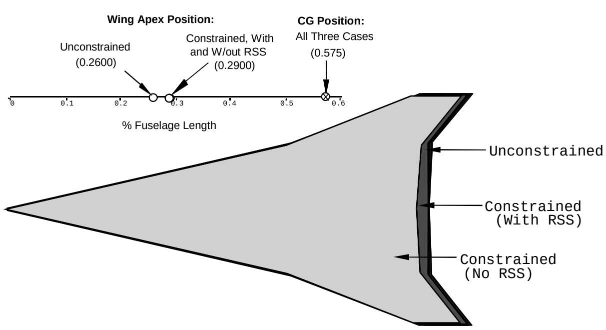

Figure 10 : Depiction of Robust Solutions Compared to Baseline

Conclusions

A taxonomy for uncertainty in aerospace systems design has been introduced in this paper. It provides a mechanism for consistent modeling of uncertainty from any conceivable source. This has been found to be an important aspect of modern (and future) aerospace problems, where emphasis on life-cycle disciplines will introduce new uncertainties and require robust solutions. *It is clear that uncertainty identification and consistent modeling are prerequisites to performing robust design studies.*

In addition, a new technique for propagating this uncertainty through system synthesis is also developed to efficiently obtain robust design solutions. Important elements of the new robust design technique include the use of a “probability of success” measure based on the cumulative distribution of the objective function. Also, the substantial cost of constructing these distribution functions at every point in the design space is mitigated by the use of the response surface method to approximate the functions at several discrete points. *Once created, these efficient metamodels can be used again and again with different robustness objectives, avoiding the need to re-execute the expensive probabilistic analysis.*

The approach developed is tested for validity on a multidisciplinary aircraft design problem involving the use of relaxed static stability technology for a supersonic commercial transport aircraft. Wind tunnel data is used to examine the fidelity of stability and control analysis of the transport. Results of this examination are then used to form uncertainty models related to that analysis. Uncertainty is also modeled for the approximations of this analysis (via metamodels) used during aircraft synthesis. The flexibility in shaping distributions based on knowledge of the uncertainty is a noted advantage of the approach.

Finally, the probabilistic robust design method is exercised resulting in a comparison of robustness results with and without the technology. It is found that modest gains in robustness are obtained with RSS, certainly less substantial than the deterministic performance gains one would expect. The efficiency of the robust design approach employed, however, makes post-optimization sensitivity studies as well as the examination of different objectives feasible.

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