Crown Competition—A Measure of Density

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THE MEASUREMENT of stand density has been one of the most trying problems in forestry. Many measures have been proposed but a completely satisfactory one has not yet been found.[1] In an effort to help solve the problem, a new measure of stand density, developed at the Central States Forest Experiment Station, is presented here.

Density, as foresters commonly use the term, is the relation between the number of trees or some volumetric or areal unit to a specific area, usually 1 acre. In effect, then, a density measure is intended to help determine the relation of the average tree in the stand to the maximum growing space it could utilize on the one hand, and the minimum growing space necessary to live on the other. It is within this range that the forester wishes to maintain density to fully utilize the site for maximum production of desired usable volume.

The maximum growing space that can be used by a tree of small diameter is, of course, less than that for a tree of larger diameter. The problem of determining the growing-space/tree-size relations is complicated by the effects of present and past density on both stem diameter and crown size. So for a given species of specified d.b.h., the size of the crown may vary greatly. (Although trees compete for moisture and nutrients as well as for light, it would be impractical to measure and relate size of root systems to degree of competition between trees. In fact, it is probable that crown area may be an indicator of root area and thus a measure of such subterranean competition.[2]) The only place where the crown/d.b.h. relations are not confounded by competition from other trees is in the *open-grown* tree. So the new method described here is based on the relation of crown area to d.b.h. of open-grown trees.[3]

[1]The authors recognize the vast amount of literature on the subject of stocking and density, but believe a review of this work would add very little to the presentation of the new measure given in this report. Good discussions of stocking and density measures are given by:

Spurr, Stephen H. Forest inventory. The Ronald Press Co. New York. 476 pp. 1952 and

Bickford, C. Allen, et al. Stocking, normality, and measurements of stand density (Report of S.A.F. committee on stocking). J For. 55:99-104. 1957.

[2]Büsgen, M. and Münch, E. The structure and life of forest trees, page 273. New York, 1931.

[3]A brief presentation of this subject is given in Station Note 108 "Crown development: an index of stand density" by Krajicek and Brinkman. Central States Forest Experiment Station, U. S. Department of Agriculture, 1957.

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Development of Crown-Width/D.B.H. Relations

To establish the relation between width of crown and d.b.h. for open-grown trees, 340 open-grown trees were measured in eastern Iowa: 88 white oaks (*Quercus alba* L.), 60 black and red oaks (*Q. velutina* Lam. and *Q. rubra* L.),[4] 35 hickories (*Carya ovata* (Mill.) K. Koch), and 157 Norway spruce (*Picea abies* (L.) Karst.). As nearly as could be determined, each tree had developed without competition throughout its entire life. To maintain uniformity in selecting sample trees, only those that met the following specifications were included in the sample:

1. Crown free of competition on all sides.
2. Limbs extending to the ground on small trees and nearly so on larger trees.
3. Lowest branches the longest, or at least as long as those above. (This eliminated trees that had been released from competition in the past and on which the lower portion of the present crown originated from epicormic branches.)
4. For small trees (less than 16 feet tall), no forking in the entire length; for larger trees, no forking of the bole below 16 feet. (This limited the sample to trees that were of the type that would be favored in the forest stand, except for limbiness and excessive taper.)

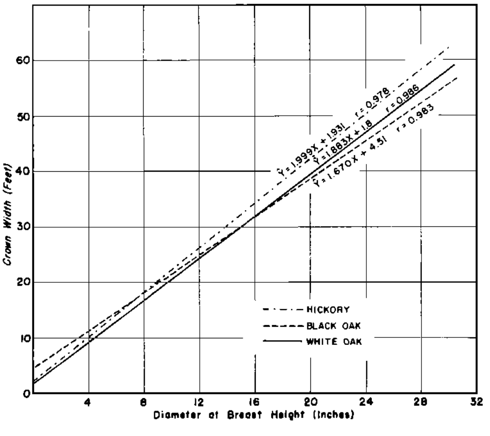

FIGURE 1. *Relation of crown width to d.b.h. for open-grown hickory, white oak, and black oaks.*

5. No evidence of pruning, shearing, browsing, decadence, storm damage, or serious insect damage.
6. Tree apparently not of sprout origin.

The smaller hardwood sample trees were found in old fields, on road rights-of-way, and on recently cutover, ungrazed areas where the sample trees originated after cutting. The larger hardwood sample trees were found in timbered pastures, cemeteries, and, to a limited extent, in residential areas. Norway spruce sample trees were found principally in cemeteries and parks located in or near small towns, in recently established windbreaks, and occasionally in residential areas.

Crown width of each sample tree was measured twice, the second measurement at right angles to the first, and the results averaged. Regression analyses of the data showed definitely that the crown width of an open-grown tree is closely related to its d.b.h.

Site quality was not determined directly for the locations where the sample trees were found. For the species examined, open-grown trees were somewhat shorter than their forest-grown counterparts of the same diameter on similar soils and aspects. Most of the hardwood sample trees were found on forest soils where site indices ranged from 45 to 75. The rest were found on prairie or transitional soils in which the productivity of the similar forest soils was known to range from poor to good.

Differences in the relation between crown and stem diameter among the hardwood species sampled were small (Fig. 1) The differences between the red oaks and white oak and between white oak and hickory were not statistically significant at the 5-percent level. The difference between hickory and red oaks was significant, but just barely so. For simplicity in application all hardwood data were combined (Fig. 2).

Crown width and d.b.h. for Norway

[4]Hereafter "red oaks" refers to both *Q. velutina* and *Q. rubra*.

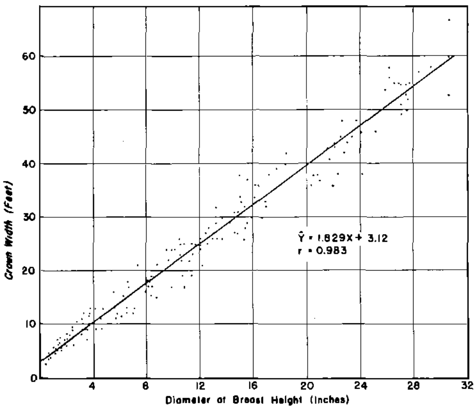

FIGURE 2. *Open-grown crown width–d.b.h. relation for combined hickory, white oak, and black oaks.*

spruce also were highly correlated (Fig. 3.). The differences between the hardwoods and Norway spruce were highly significant at the 1-percent level.

Since such a close relationship exists between the crown diameter and d.b.h. of open-grown trees, and this relationship is nearly constant within a species, it may be inferred that the crown of a tree of given d.b.h. cannot occupy more than a certain area even with unlimited growing space.

Development of an Index of Crown Competition

Knowing the average crown width and assuming that crowns of open-grown trees tend to be circular, the approximate area covered by the crown can be readily computed. The percentage of an acre occupied by a vertical projection of the crown is obtained by dividing the area in square feet by 435.6. For example, an open-grown Norway spruce with d.b.h. of 16 inches has a crown width of approximately 26 feet (Fig. 3), or crown area of 531 square feet (1.22 percent of an acre). This value has been termed "maximum crown area" (MCA), because it expresses, *in terms of percent of an acre,* the maximum area that could be occupied by the crown of a tree of specified d.b.h.

To determine MCA values for each tree in a stand would be time consuming. However, from the least squares solution of crown width and d.b.h., a formula can be derived that simplifies the procedure:

The MCA value of a tree with a crown diameter of CW is

$$MCA = \frac{\dfrac{\pi\,(CW)^2}{4}}{435.6}$$

$$= 0.0018\,(CW)^2$$

From the regression analysis of the combined oak-hickory data (Fig. 2),

$$CW = (1.829D + 3.12)$$

where D = d.b.h.

and

$$(CW)^2 = 3.345D^2 + 11.413D + 9.734$$

Therefore,

$$MCA = 0.0060D^2 + 0.0205D + 0.0175$$

Similarly, the formula of MCA values for Norway spruce is

$$MCA = 0.0031D^2 + 0.0239D + 0.0460$$

With the MCA values determined, it is not difficult to compute the *minimum* number of open-grown trees per acre that are theoretically required to produce a complete canopy (assuming that crowns are distorted sufficiently to fill all voids in the canopy, but without reducing the crown area of the individual trees). If a tree has a d.b.h. of 16 inches and an MCA of 1.22 (as in the previous example), then 82 open-

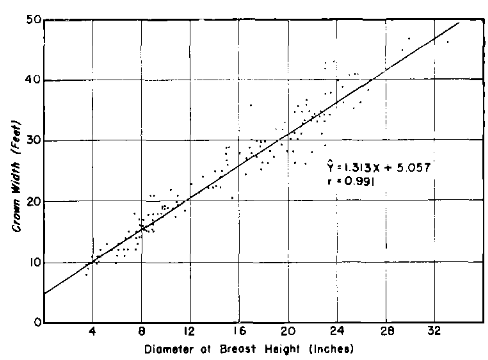

FIGURE 3. *Relation of crown width to d.b.h. for open-grown Norway spruce.*

TABLE 1. Determination of the CCF for an even-aged upland oak stand, site index 60, age 50 (Schnur 1937).

D	N	DN	D^2N	Check using MCA values	
(d.b.h.)	(Number of trees per acre)			MCA per tree	Total MCA
				Percent	*Percent*
1	10	10	10	0.044	0.440
2	29	58	116	.082	2.378
3	50	150	450	.133	6.650
4	63	252	1,008	.196	12.348
5	77	385	1,925	.270	20.790
6	78	468	2,808	.356	27.768
7	66	462	3,234	.455	30.030
8	49	392	3,136	.566	27.734
9	31	279	2,511	.688	21.328
10	17	170	1,700	.822	13.974
11	7	77	847	.969	6.783
12	3	36	432	1.128	3.384
13	1	13	169	1.298	1.298
Total	481	2,752	18,346		174.905 (CCF)

$CCF = 0.0060(18{,}346) + 0.0205(2752) + 0.0175(481) = 174.91$

grown trees of that size could cover an acre completely ($82 \times 1.22 = 100$ percent, or sum of MCA values equals 100). Using MCA values for other d.b.h. classes, any combination of open-grown trees of various diameters whose MCA values total 100 for an acre theoretically could have a closed canopy.

To avoid confusion in using one term—MCA—to express the area requirement of both a single tree and a sum of trees, another expression, "Crown Competition Factor" (CCF), has been coined to represent the sums of the MCA values for an acre. Crown Competition Factor is defined as the sum of MCA values for all trees in the stand, divided by the area in acres. It is used as an expression of stand density. These summations can be expressed by using the previous formulas in the following form:

For oak-hickory stands,

$$CCF = \frac{1}{A}\left[0.0060(\Sigma D_i^2 N_i) + 0.0205(\Sigma D_i N_i) + 0.0175(\Sigma N_i)\right]$$

and for Norway spruce,

$$CCF = \frac{1}{A}\left[0.0031(\Sigma D_i^2 N_i) + 0.0239(\Sigma D_i N_i) + 0.0460(\Sigma N_i)\right]$$

where

D_i = individual d.b.h. or d.b.h. class
N_i = number of trees in d.b.h. class
A = area in acres

An example of the application of the CCF to a specific stand is given in Table 1. All the trees in the stand are used to determine the CCF. In the stand of trees shown in Table 1, nearly 60 percent of the Crown Competition Factor is contained in trees less than 8 inches d.b.h. To consider only those trees larger than a specified lower diameter limit will result in a low CCF and will not represent the actual competition in the stand. It is known, of course, that the larger trees have an adverse effect on the growth rate of the smaller. It is more difficult to assign an effect of the small trees on the growth of larger ones. Where moisture is a limiting factor at any time during the growing season, the smaller trees probably have an adverse effect on the growth of the larger trees.

One other precaution is necessary. When computing the CCF from stand tables, the actual number of trees in each diameter class should be used rather than the diameter of the tree of average basal area and total number of trees. If the latter procedure is followed, excessive errors may result when the diameter distribution deviates greatly from the normal curve.

Results and Discussion

It is obvious that maintaining stand density below CCF 100 from time of establishment to maturity would normally produce low-quality trees, especially if they were not pruned artificially. Greater density (with a higher CCF value) would be accompanied by a reduction of the actual or effective live crown of the average tree in the stand.

Density will not increase indefinitely, of course. A number of even-aged pure oak stands in Iowa consistently showed a ceiling density of about CCF 200, when all trees in the stand were included. Similarly, the average CCF for 52 even-aged oak plots, each 0.2 acre in size, located throughout southeastern Ohio and selected for full stocking was exactly 200.[5] A consistent ceiling probably exists for all other species also, but it is probably different from that found for oaks.

[5]These unpublished data on which the computations are based were collected and furnished by Willard Carmean and Stephen G. Boyce, Soil Scientist and Research Forester, respectively, Central States Forest Experiment Station.

At least three things account for differences among species in the maximum CCF they can obtain. One is their crown area development when there is no competition. Another is the basic shape of the crown, such as conical, parabolic, etc. Also, tolerance may be a factor. For example, measurements over an 8-year period in heavily stocked oak-hickory plots show a slightly higher CCF than those found in pure oak stands, probably because of the greater tolerance of hickory. Some of the Ohio plots cited above also had CCF values in excess of 200, but these too contained some of the more tolerant species.

To further confirm the consistency of the values found in field tests, the formula for oak-hickory was applied to stand tables for even-aged upland oaks. Except for high values in younger stands on the poorer sites, the CCF values show great uniformity (Table 2).

No Norway spruce stands were available for field testing of the formula. The similarity of crown widths among the oak species suggested that perhaps a given crown width/d.b.h. relation may be rather common within a genus. With this thought, the Norway spruce formula was applied to stand table data for Sitka spruce and west-

TABLE 2. Application of CCF to stand table data[1] for even-aged upland oaks.

Age	(CCF values by age class and site index) Site index				
(years)	40	50	60	70	80
10	301	249	219	190	166
20	218	210	188	180	181
30	184	183	178	176	167
40	171	170	168	165	160
50	169	169	175	173	173
60	171	171	176	175	169
70	165	168	172	170	165
80	168	176	173	172	164
90	175	179	177	174	164
100	178	182	177	175	166

[1]Schnur, G. Luther. Yield, stand, and volume tables for even-aged upland oak forests. U. S. Dept. Agric. Tech. Bull. No. 560, Table 37, 1937.

ern hemlock (Table 3). These data are primarily for stands in Washington and Oregon, yet the CCF values for the age-site classes enclosed by the broken line are quite consistent, except for age 140, site index 160. The latter may be an error in the original table.

A comparison of oak-hickory and Norway spruce stands of the same density as measured by crown competition reveals a striking difference in their structure. For example, a Norway spruce stand composed of 16-inch trees would have more than half again the number of trees and basal area per acre as an oak-hickory stand with trees of like size and the same CCF.

Comparison With Other Measures

A thorough comparison of CCF with other measures of stand density would require much space. A brief comparison serves to illustrate the relation between CCF, basal area per acre, and Reineke's stand-density index (Table 4). It is of interest to note that when CCF is held constant over a range of average stand diameters, basal area per acre changes at a rate that is typical of most even-aged stands. CCF does not show a consistent relationship with stand-density index. Although a CCF of 100 (which might indicate minimum density for full site utilization) is approximately associated with a stand-density index of 110 percent for stands with average diameters as shown, the relationship of number of trees to average stand diameter is curvilinear on log-log paper for any given CCF value, whereas stand-density index is linear. However, this does not represent a deterrent to the application of CCF. Approximate full stocking in terms of stand-density index (240) results in CCF values from 197 to 262 as shown in column 6 of Table 4. Again, one could conclude *in general* that a CCF of 200 (which might represent maximum density for full site utilization) is approximately equivalent to a stand density index of 240 for stands with an average diameter of from 5 to 15 inches.

In comparing "D plus" and "D times" spacing[6] with CCF spacing, it is apparent

[6]Rules of thumb used as thinning guides. Using the "D plus" rule, spacing of trees in feet is equal to the diameter of the trees in inches plus a constant; using the "D times" rule, spacing in feet is equal to tree diameter in inches times a constant.

TABLE 3. Application of CCF to stand table data[1] for even-aged Sitka spruce and western hemlock. (CCF values by age class and site index.)

Age	Site index							
(years)	60	80	100	120	140	160	180	200
40	1184	635	412	345	320	292	275	269
60	427	351	332	299	288	276	270	267
80	356	327	306	293	284	283	280	276
100	326	299	296	288	284	285	285	285
120	304	296	291	286	285	285	287	289
140	291	289	286	286	286	241	290	287
160	286	285	287	287	287	291	296	------
180	283	282	281	283	288	288	292	------
200	279	280	280	284	289	293	294	------

[1]Meyer, W. H. Yield of even-aged stands of Sitka spruce and western hemlocks. U. S. Dept. Agric. Tech. Bull. No. 544, Table 36, 1937.

Dash lines indicate limits of basic data.

TABLE 4. Comparisons of CCF with basal area and stand-density index[1] for theoretical oak-hickory stands of various average diameters.

Average d.b.h.[2] (inches)	When CCF is 100:			When stand-density index is 240:		Basal area per acre when:	
	Trees per acre	Basal area per acre	Stand-density index	Trees per acre	Equivalent CCF	CCF = 200	SDI = 240
	Number	*Square feet*	*Percent*	*Number*	*Percent*	*Square feet*	*Square feet*
2	1,220	26.8	92	3,177	262	53.7	69
5	370	50.3	122	730	197	101	100
10	122	66.5	122	240	197	132	131
15	60	73.6	115	125	209	146	153
20	35.3	77.0	107	78.9	223	154	172
25	23.4	79.8	102	55.1	236	159	188

[1]Reineke, L. H. Perfecting a stand-density index for even-aged forests. J. Agric. Res. 46: 627-638. 1933.
[2]Calculated from average basal area per tree. (Used in the example to facilitate comparisons with stand-density index. As stated in the text, the actual number of trees in each diameter class should be used to compute CCF.)

TABLE 5. Spacing of trees for a CCF of 100 and equivalent "D plus" and "D times" constants for theoretical stands of oak-hickory and Norway spruce of various average diameters.

Average d.b.h.[1] (inches)	Oak-hickory			Norway spruce		
	CCF spacing	"D plus" constant	"D times" constant	CCF spacing	"D plus" constant	"D times" constant
	Feet	*Feet*	*Unit*	*Feet*	*Feet*	*Unit*
2	6.0	4.0	3.00	6.8	4.8	3.40
5	10.8	5.8	2.16	10.3	5.3	2.06
10	18.9	8.9	1.89	16.1	6.1	1.61
15	26.9	11.9	1.79	21.9	6.9	1.46
20	35.1	15.1	1.76	27.7	7.7	1.38
25	43.1	18.1	1.72	33.6	8.6	1.34

[1]Calculated from average basal area per tree.

that spacing based on CCF will not result in a constant factor for stands of different average diameters (Table 5). An examination of these "constants" substantiates the findings of Stahelin[7] and others that diameter plus a constant results in too much basal area with large trees and too little with small trees.

[7]Stahelin, R. Thinning even-aged loblolly and slash pine stands to specified densities. J. For. 47:538-540, illus. 1949.

It should be emphasized that CCF is not essentially a measure of crown closure. Theoretically, complete crown closure can occur from CCF 100 to the maximum for the species (in oaks, approximately CCF 200). Instead of estimating crown closure, CCF estimates the area available to the average tree in the stand in relation to the maximum area it could use, if it were open grown.

Conclusions

There is good evidence to indicate that the CCF method of measuring density can be applied to even-aged stands of hardwoods and conifers. Moreover, although field tests of the method have been confined to even-aged stands, in theory it can be used for uneven-aged stands equally well. The method adjusts accurately to variations in diameter distribution and to the potential crown space requirements for the individual trees.

Traditionally, researchers have insisted that the ideal measure of density should be correlated with growth and yield and independent of stand age. On this premise CCF has shown up well in preliminary tests. In stand density studies in even-aged oak net basal area growth (accretion less mortality) is at a maximum when CCF is 135. This holds true for stands 25, 50, and 90 years old, with corresponding average basal area per acre of 71, 86, and 96 square feet respectively. The growth data cover an 8-year period for the two younger age classes and a 4-year period for the other.

Several important points should be emphasized:

1. Relatively few measurements of *truly* open-grown trees are required to provide a basic CCF equation for a species.

2. The ceiling of about CCF 200 for oak density should not be assumed for other species, even when crown widths are found to be similar. Relative tolerance and shape of crown (conical, parabolic, etc.) determine the maximum densities attainable by a species.

3. In stands having mixtures of species with different crown characteristics and tolerance, the determination of CCF may need to be handled by proportional representation.

The CCF is as simple to apply as basal-area tables, but, unlike basal-area control, interpretation of values does not depend upon diameter distribution of a specific type or range. The Crown Competition Factor, based on a verified relationship between crown width and d.b.h., should measure competition more acurately than the methods presently used.

Summary

Measurements of several hundred oak, hickory, and Norway spruce disclosed a high degree of correlation between crown width and d.b.h. for open-grown trees of the same species. Apparently this relation is independent of age and site quality. From the open-grown-tree data, formulae were derived which express the maximum percentage of an acre the crown of an open-grown tree of a given diameter and species could occupy.

Theoretically, competition for crown space between trees in a stand begins when all crown space is occupied and each tree crown is equal in area to that of an open-grown tree of the same d.b.h. This condition is taken as a base of 100 for expressing competition among crowns for growing space. From this base, stand density may be expressed as a percentage. For convenience, this percentage is termed the Crown Competition Factor (CCF), because it is an estimate of the competition among crowns for growing space. Preliminary trials indicate that the method is applicable to even-aged stands, and theoretically can be used to advantage in uneven-aged stands. These trials show strong evidence that site and stand age do not influence CCF.