Numerical Expression of Stocking in Terms of Height

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Having previously advocated the use of height to define maturity as a basis for legal control of cutting in young stands, the author now develops the use of height as a factor in the measurement and regulation of growing stock. An equation is offered which may prove to have value in defining upper and lower limits to normal stocking. The author will be glad to provide interested foresters with logarithmic-scale, letter-size sheets covering the range of stocking.

THERE are two possible approaches to the evaluation of stand density: comparison with normal-yield-table values, and numerical expression. But in some cases suitable yield tables are not available, and in any event they are a cumbersome tool, especially for the practitioner concerned with intermediate cuttings. Stands do not always fall near the median of a site-index band. Also, since present yield tables are based on natural stands, they are not applicable as to number of trees when thinning plantations or stands previously thinned. This departure from "normal" increases with each successive thinning, both as to number of trees and average stand diameter.

To find something more convenient, foresters have been turning to expression of stocking in terms of spacing, with diameter as the key factor. Though we realize that trees which have been planted with a spacing of 6 by 6 feet cannot actually be thinned to a spacing of 8 by 8 feet, there is much merit in this spacing concept. It can be helpful as a guide to marking, especially for the first thinning in dense young stands.

Since forestry literature is replete with references to the close correlation between average stand diameter and total number of trees or basal area, and since diameter is easily measured, it is natural that some foresters have expressed spacing in terms of diameter. But this correlation between average diameter and other values is based on natural stands with a normal distribution of diameter classes. As this distribution is disturbed and average diameter is increased by periodic thinnings, a spacing guide based on diameter may prove unsatisfactory. Diameter is the result rather than the cause of spacing.

But height, within reasonable limits of stocking, is negligibly affected by spacing. It is as suitable for the purpose of simple numerical expression of stocking as it is for the determination of site quality. Height has the virtue of combining the components of age and site in one measurement. It requires but little work to approximate the height of average dominants. For experimental plots this value is required for site classification. Recent advances in measurement of height from aerial photographs may compel greater use and bring fuller appreciation of the factor of height.

Yet, strangely, it has been largely ignored in expression of stocking. Only one use of it has come to the writer's attention. This is a rule which Tkatchenko[2] credits, without further reference, to Keller: that spruce stands should be thinned to the number of trees per unit of area based on a spacing of one-sixth of the height of the stand.

A Theory Of Normal Stocking

The principle underlying Keller's rule appears basically sound. For uniformly stocked, even-aged stands of any species we may write:

$$n = \frac{43{,}560}{(h\ f)^2}$$

where n is the number of trees per acre, h is the height of the stand, and f is a certain fraction of height appropriate for the species. In logarithmic form it is the equation of a straight line, provided f is constant. The lines for different values of f are parallel and their slope is defined by the equation $2 \log h = - \log n$. Expressed more simply, if f is constant, the equation means that the ratio of number of trees to the square of the average dominant height is the same for all ages and on all sites. Whether this equation reflects a law of normal stocking for closed stands can be determined only by experimentation. For present support of the equation we must turn to yield tables. In Tables 1 and 2 values for f are expressed in percent of height.

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[2]Tkatchenko, M. E. General forestry. Leningrad. 745 pp. (Russian). 1939.

The apparent conclusions are that:

1. The factor *f* decreases with tolerance. The discrepancy between red and jack pine is explained by the fact that jack pine, while definitely more intolerant, will endure extreme crowding from the side.

2. Site has no effect on *f*.

3. The factor *f* definitely increases where suppressed trees are excluded from the stand.

4. The factor *f* varies with the natural ability of species to "thin out with age."

Aspen and red oak are definitely thinning out with age, their average spacing increases as they grow older. Black spruce and red pine show the opposite tendency of becoming relatively more crowded with age. These diverse trends in spacing may be a reflection of characteristic differences in the form of crowns. Even among conifers, the red pine has begun to reverse its trend at 160 years, while the spire-crowned black spruce continues to decrease its spacing. It would appear that species tending to become more crowded, especially when originating as dense stands, should benefit most from thinnings.

TABLE 1.—SPACING OF ALL TREES IN PERCENT OF AVERAGE HEIGHT OF DOMINANT AND CODOMINANT TREES

Age	Site quality: Good	Medium	Poor
Years		Percent	
		Jack pine[1]	
30	18	20	22
40	18	19	21
50	18	19	21
60	17	19	21
70	18	19	21
80	18	20	22
		Aspen[2]	
20	12	12	12
30	14	14	14
40	15	15	15
50	16	16	16
60	17	17	17
70	17	18	18
80	18	19	20
		Red pine[3]	
40	20	21	20
60	18	18	19
80	17	17	19
100	17	17	19
160	18	18	20

[1]Gevorkiantz, S. R. and W. A. Duerr. Methods of predicting growth of forest stands. Lake States Forest Expt. Sta. Econ. Note No. 9, tables 15 and 27. 1938.
[2]Ibid., tables 20 and 27.
[3]Gevorkiantz, S. R. Unpublished material.

CONTROLLED EXPERIMENTAL THINNING

While thinning will always remain an art, we must observe some rules of perspective. In the case of experimental plots, rigid controls to stocking are imperative, if the results are to have meaning. An untreated plot is not always a proper control.

Data provided by the Lake States Forest Experiment Station are shown in Figures 1 and 2. It should be understood that these three jack pine plots are part of a series established for the purpose of obtaining contrast from varying degrees of thinning, ranging from light to very heavy. It is now clear that the thinning of Plot 26, though substantial, was not sufficient to secure significant response in diameter growth. The two commercial thinnings did, however, yield 9.0 cords of pulpwood, thus illustrating the values which can be secured by what amounted to salvage cuttings in anticipation of mortality.

In the case of Plot 27, the first very severe, noncommercial thinning had the effect of removing a considerable number of trees which normally would have been left to grow to pulpwood size. This accounts for the lower yield, cut plus residual, of 26.9 cords at 35 years, in comparison with 29.7 cords for Plot 26, though practically equal to the 27.1 cords on unthinned Plot 29.

TABLE 2.—SPACING OF ALL TREES AND OF DOMINANT TREES IN PERCENT OF AVERAGE HEIGHT OF DOMINANT TREES

Age	Site quality: Good	Medium	Poor	Good	Medium	Poor
	All trees			Dominants		
Years			Percent			
			Black spruce[1]			
40	13	12	13	18	18	20
60	12	12	12	17	17	18
80	12	11	12	16	16	17
100	12	11	12	15	16	17
160	11	11	11	15	15	16
			Red oak[2]			
20	16	16	16	22	22	23
40	18	17	17	22	22	23
60	19	19	19	23	23	23
80	20	20	20	23	24	24
100	21	21	21	24	24	25
160	22	23	22	25	26	26

[1]Fox, G. D. and G. W. Kruse. A yield table for well-stocked stands of black spruce in northeastern Minnesota. Jour. Forestry 37: 565-67. 1939.
[2]Gevorkiantz, S. R., Harold F. Scholz, and R. A. Farrington. Timber yields and economic returns from the mixed-oak farmwoods of southwestern Wisconsin. Manuscript.

The greatest value derived from this experiment lies in the fact that it discloses the approximate pay-dirt level of stocking for jack pine to lie between spacings of one-fourth and one-fifth of height.

The next step should be to establish many series of adjoining plots in young stands, to cover all site qualities over the commercial range of jack pine in the Lake States. Unless any series of plots is to be used as a demonstration that thinning is good practice, there is no need for including an untreated plot. But each series should include three treated plots; the median plot might be held to a stocking between 20 and 23 percent of height. The timing of subsequent thinnings would not be hampered by thinking in terms of a fixed number of years but would be determined by height growth, with longer periods between thinnings on poor sites than on good sites. When the median plot is thinned to 23 percent, the two adjoining plots would be thinned to 21 and 25 percent of height.

Similarly, for other species, the stocking range could be chosen and bracketed. Such work would reveal to what extent the equation may be sound,

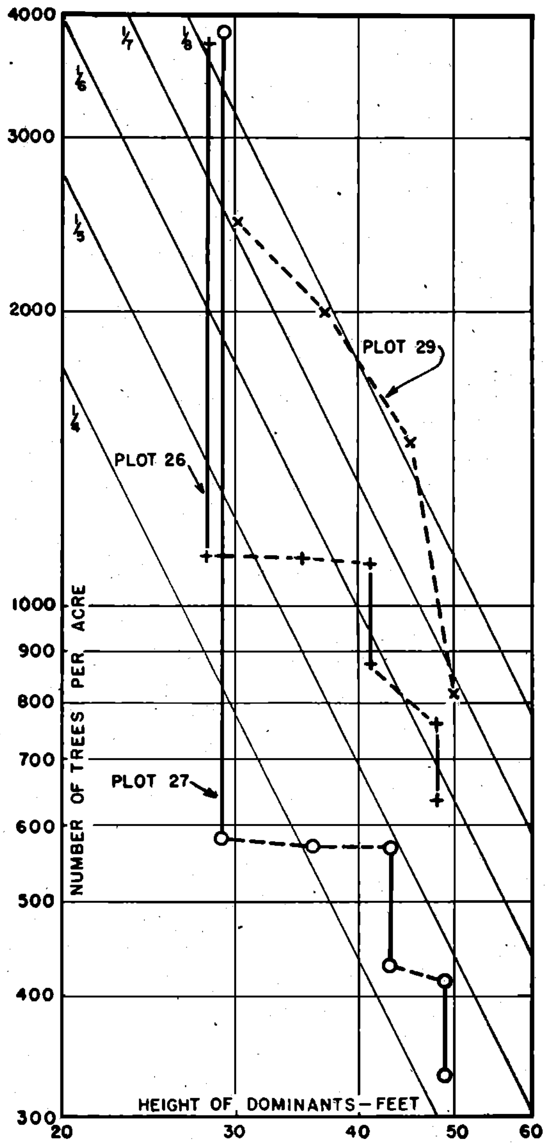

Fig. 1.—Reduction of stocking with reference to spacing in fractions of height, by 5-year periods between 20 and 35 years of age.

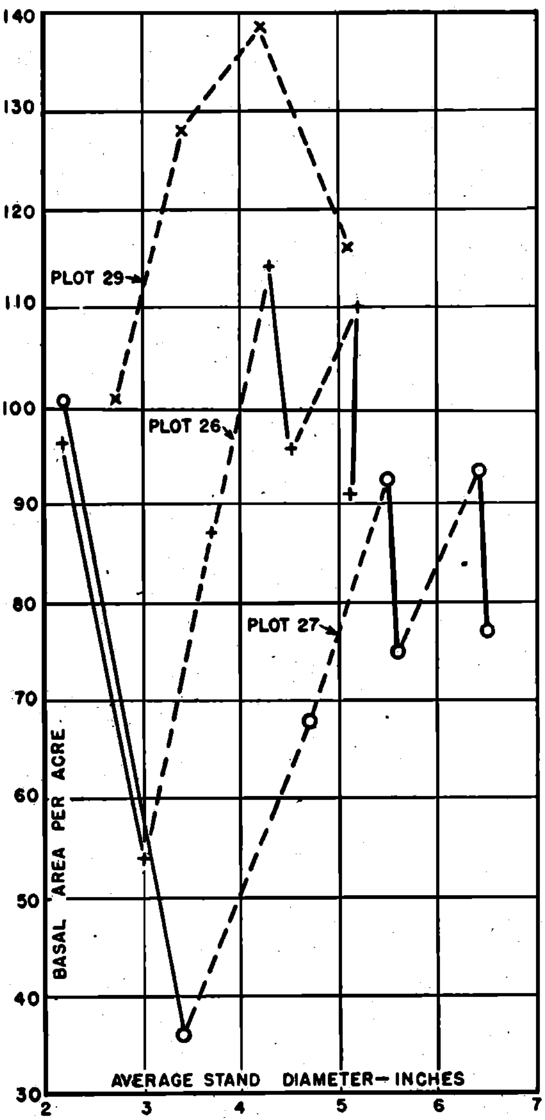

Fig. 2.—Response in basal area and diameter increment, by 5-year periods between 20 and 35 years of age.

whether single values for f_1 and f_2 to define upper and lower limits to normal stocking may hold for the rotation and over the range of sites.

In any event, we should have a controlled experiment which would provide sound data for the compilation of normal yield tables for managed stands.

Current Thinning Practice

For the present it might be well to try out rules of thumb based on height rather than on diameter. Spruce and the true firs should do well within stocking limits defined by one-sixth and one-seventh of height. White pine could be kept between one-fifth and one-sixth, and jack pine between one-fourth and one-fifth of height. In general, a correct evaluation of tolerance is the best guide. The results of thinning experiments are valuable, provided at least some of the plots were thinned heavily enough to indicate proper lower limits to stocking.

A noncommercial thinning might well aim to leave the number of trees which would represent the upper limit to stocking when the stand attains a height indicating the possibility of a commercial thinning. Where safety to the stand or silvicultural reasons would prohibit so severe a thinning, that number of trees might be treated as crop trees. The latter course would also reduce the cost of a noncommercial thinning.

Since markets govern silviculture, the spread between upper and lower limits to stocking must be greater in less accessible areas, with longer periods of return for heavier cuts. Wider spacing could be adopted on poor sites where quality would be less important than shortening the rotation. European yield tables for managed stands support such practice. A paper company, interested only in pulpwood, might adopt wider spacing than a lumber company growing the same species.

Until properly controlled thinning experiments provide valid data, we can do worse, and many of us have done worse, than to be guided by intelligently chosen limits to stocking expressed in terms of the height of the stand.