Stand Density Measures: An Interpretation

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Abstract. Many stand density measures can be regarded as expressions of average area available to trees of a stand, relative to that occupied by trees growing under a standard density condition and comparable in dbh or otherwise. Relationships among a number of common relative stand density measures are discussed. Either the open-grown condition or the normal stand can be used as the standard and lead to similar results. Differences are introduced by use of stand diameter, height, volume, or site and age as alternative bases for referring observed stands to the standard condition. A ratio of observed basal area to that of a normal stand of the same age and site, frequently used as an expression of relative density, is not directly interpretable as a comparison of areas. Otherwise, most common measures appear to be practically equivalent. ***Forest Sci. 16:403-414.***

Additional key words. Competition, stocking.

ALTHOUGH stocking and stand density are terms long used in forestry, different people use these terms with varying shades of meaning and degrees of interchangeability.

Forestry Terminology (Soc. Amer. Forest. 1950) defines stocking as "an indication of the number of trees in a stand as compared to desirable number for best growth and management . . ."; and stand density as "density of stocking in terms of number of trees, basal area, volume, or other criteria, on a per acre basis." By these definitions, stocking is a comparison with the current management objective, and stand density is almost any numerical quantity obtainable by measurement of the stand and expressible on an area basis.

Yet it is evident from usage of these terms that they have connotations and are related to concepts fundamental to silviculture and not included in the above definitions: in particular, competition, area occupancy, and stand closure. This paper presents a point of view on the meaning of relative measures of stand density and points out analogies and equivalencies which exist among a number of common measures.

Growth rate of a forest stand of given genetic constitution is the resultant of (1) age, (2) site, and (3) area occupancy and intensity of competition, which are associated with number, dimensions, and distribution of trees in the existing stand. Much observation and experimentation are concerned with relationships between growth and measures which are believed to express occupancy and competition in terms of the measurable characteristics of tree number, size, and (sometimes) distribution. Such measures should be adequate descriptors of important stand characteristics, not dependent on age or site, and should be capable of visualization and interpretation in biologically meaningful terms.

In plant and animal ecology, density is commonly defined as number of individuals per unit area. Competition exists if the site resources available to the individual are reduced and development of the individual is modified by the presence of other individuals of the population. Hence, for organisms of fixed size and requirements, intensity of competition varies with density expressed as number of individuals.

In forest stands, definition of density as number of individuals per unit area is of limited usefulness, since trees increase in size more or less indefinitely and change in dimensions and ability to utilize available site resources in response to the influence of adjacent trees.

Competition at a given point in time is not directly measurable. Its visible and

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measurable manifestations are reduction in growth rate of the individual tree and change in distribution of growth among its parts, relative to potential growth in the absence of competition. Cumulative effects of past competition on development of the trees composing a forest stand can be expressed in terms of a comparison of absolute or relative dimensions of some average tree with dimensions of a tree growing under conditions believed to represent a minimum or maximum limit of possible levels of competition, but otherwise comparable in one or more respects.

Since potential growth rate is usually unknown, comparisons in terms of total tree size are not feasible. But comparisons of crown dimensions or areas occupied by trees of comparable diameter or height *are* feasible expressions of cumulative effects of competition. Smith and Bailey (1964) define stand density as "the degree of crowding of individual trees within the portion of the area actually stocked with trees." If we accept this, most of the relative measures of stand density commonly used in forestry are interpretable as ratios of some average crown area or land area occupied by or available to trees of a given stand to that occupied by trees of the same diameters, heights, or other observable characteristics under some "standard" condition of competition or "crowding." Such measures are ordinarily derived from and are only applicable to reasonably uniform, homogeneous, even-aged stands and implicitly assume similarity of diameter distributions of stands being compared.

The two standard conditions explicitly or implicitly used as the reference base for most such measures are (1) no competition, as approximated by open-grown trees, and (2) a loosely defined "average-maximum" competition represented by the "normal" stand. These conditions approximate the biological limits of stand density, without reference to optimum stand growth or management objectives.[1]

Area occupied may be assumed equivalent (plausible for species which normally form one-storied closed stands) or proportional to crown projectional area, and average crown projectional area of open-grown trees of given diameter can be estimated from measurements of crown width (Krajicek *et al.* 1961). Similarly, tree areas of average trees in normal stands may be estimated by the tree-area technique of Chisman and Schumacher (1940) or by direct measurement of tree crowns (Curtin 1964). Gingrich (1967) has shown that the curves of average maximum and average minimum tree area in relation to diameter are proportional for upland oaks in the Central States, and the same is indicated by Curtin's crown-width curves. If this is generally true, either curve is equally suitable as the reference base for a relative stand density scale, differing only by a constant of proportionality.

Some Alternative Measures of Density

Basal Area. Basal area is frequently referred to as a measure of stand density. Its widespread use as a stand statistic probably originated from its use as one of the three factors in computation of volume (basal area, height, form) rather than from any biological interpretation.

In older stands, average crown width is more or less proportional to average diameter bh ($\bar{D}$), and average crown projectional area (and area occupied) to $\bar{D}^2$ and hence to basal area. However, since trees less than 4.5 ft in height occupy area even though their basal area is zero, and since total basal areas increase in a

[1] Hence, measures expressing stand conditions relative to these limits do not fit the *Forestry Terminology* (1950) definition of "stocking," since these limits do not represent the desirable number for best growth and management. Such measures are not explicitly recognized as a class by *Forestry Terminology*, and the distinction between density and stocking is neither clearcut nor consistently made in forestry literature, despite attempts at definition (e.g., Bickford *et al.* 1957). In this paper, all measures—however expressed—which are regarded as expressions of degree of crowding of stems without reference to any stated management objective are collectively referred to as measures of stand density. However, relative stand density measures expressed in the form of index numbers or ratios rather than as per-acre values are also frequently referred to in the literature as stocking measures or stocking ratios.

consistent pattern as stands develop, the proportionality constant must change as a function of characteristics such as average diameter, height, or age and site.

For a normal stand of basal area G_e and unit area,

$$\Sigma(\text{tree areas}) = 1.0 = k(G_e) = \text{unit area.}$$

And, for any observed stand of basal area G_o and the same average diameter (and similar percentage distribution of number of stems by diameter classes), $k(G_o)$ is the total area available to trees of comparable diameters in a normal stand.

Hence, G_o/G_e equals the total area available in a normal stand to trees of diameters equal to those in the observed stand divided by the unit area.

If measures of stand density are regarded as expressions of the ratio to actual area occupied by a stand, of the crown area or surface area corresponding to trees of the same diameters when grown under "standard" conditions, then the meaningful expression of stand density is not basal area itself, but relative basal area—G_o/G_e.[2]

Number of Trees in Relation to Average Diameter. Reineke's (1933) stand density index and related measures are based on the normal stand relationship,

$$N_e = a(\bar{D})^b,$$

in which—for Reineke's data—$b \sim -1.6$. Relative stand density can be expressed by the ratio N_o/N_e,[3] where N_o is the observed number of trees in a given stand and N_e is the expected number in normal stands of the same quadratic mean diameter ($\bar{D}$).

Since average area per tree is $1/N$,

$$\begin{aligned} N_o/N_e &= (1/N_e)/(1/N_o) \\ &= \text{(average area per tree in normal stands)/(average area per tree in observed stands)} \\ &= [(1/N_e)/(1/N_o)]\,[N_o/N_o] \\ &= [(N_o)\text{ (average area per tree in normal stands)]/unit area,} \end{aligned}$$

which is the ratio of total area occupied by the observed number of trees in a normal stand of the same quadratic mean diameter to the unit area available to trees in the observed stand.

And since,

$$\begin{aligned} N_o/N_e &= [N_o(k\bar{D}^2)]/[N_e(k\bar{D}^2)] \\ &= G_o/G_e \sim [G_o/(a\bar{D}^{-1.6})(k\bar{D}^2)] \\ &= G_o/[a_1(\bar{D}^{+0.4})], \end{aligned}$$

the ratio N_o/N_e is equivalent to a relative basal area—G_o/G_e—in which G_e is expected basal area in normal stands estimated as a power function of stand quadratic mean diameter.

Number or Spacing in Relation to Height. If stand height ($\bar{H}$) is proportional to $\bar{D}^{+0.8}$—which is approximately the case for a number of species[4]—then:

$$N_e \sim a(D)^{-1.6} = k(\bar{H})^{-2}.$$

Various authors (Wilson 1946, Czarnowski 1961, Braathe 1957) have used number of trees or average spacing in relation to some stand average height as expressions of stand density. These measures can be written as,

$$\begin{aligned} N_o/N_e &= N_o/[k/(\bar{H})^2] \\ &= N_o(\bar{H})^2/k, \end{aligned}$$

[2] Note that a density expression of form G_o/G_e, where $G_e = f(\bar{D})$ or f(age, site), may be implicit in a stand growth function which includes $\bar{D}$ or age and site as variables in addition to basal area, even though not explicitly stated as G_o/G_e. The common use of basal area as a variable in such functions is not necessarily inconsistent with the above reasoning and leads to useful prediction equations, although relative stand density effects are confounded with those of tree size or of age and site.

[3] Reineke used a reference curve proportional to the average curve for normal stands, representing the upper limit of stand density. He referred to N_o/N_e as percentage stocking, and defined stand density index (SDI) as number of trees in a stand of $\bar{D}$ = 10.0 inches and N_o/N_e equal to that of the observed stand. For a given species, SDI = (a constant) (N_o/N_e), whether N_e is based on maximum or normal density, and the same interpretation applies to both forms.

[4] Bruce, David. Unpublished paper presented at Lodgepole Pine Conference, Bend, Oregon, 1965

in which $\bar{H}$ may be either dominant or top height, or stand average height; and the exponent of $\bar{H}$ is commonly—though not necessarily—assumed to be 2.0 (i.e., spacing proportional to height). Obviously, this is similar in form and interpretation to Reineke's stand density index, except that stand height rather than stand diameter is the basis for comparing an observed stand with the standard density condition.

Number in Relation to Volume. Average volume per tree can also be used as a basis for comparing an observed stand with a standard condition. Thus, Tadaki (1968) expressed relative density by the ratio of average tree volume (v) to that given by the "full-density curve" $v = kN^a$, $a \sim -1.5$, representing the upper limit of the family of curves describing the "competition-density effect" (Kira *et al.* 1953, Shinozaki and Kira 1956)—the relation of average size (volume) to number of stems—as a function of stand height and number. If this equation is rearranged as,

$$N_e = k_1 v^{-a} \text{ and } 1/N_e = \text{average area per tree} = (1/k_1)v^a,$$

a ratio N_o/N_e is interpretable as a comparison of average area per tree in an observed stand with that in a full-density stand of the same average volume per tree.

Tree-Area Ratio. Chisman and Schumacher's (1940) tree-area ratio (TAR) expressed density as:

$$[aN + b\Sigma(D_i) + c\Sigma(D_i^2)]/\text{area}.$$

They reasoned that if the relation of the unknown area (TA_i) occupied by an individual tree to the tree's diameter (D_i) could be represented by an equation of form,

$$TA_i = a + bD_i + cD_i^2,$$

then for normal stands of unit area,

$$\Sigma TA_i = 1.0 = aN + b\Sigma(D_i) + c\Sigma(D_i^2).$$

And, for a series of normal stands, the least squares estimates of the equation constants are those which minimize:

$$[1.0 - aN - b\Sigma(D_i) - c\Sigma(D_i^2)]^2.$$

Once these constants are determined for normal stands, the equation gives—when applied to nonnormal stands—an estimate of the proportion of the unit area which would be occupied by the trees of the stand if each were assigned the average area occupied by a tree of the same diameter in normal stands.

Crown Competition Factor. Krajicek *et al.*'s (1961) crown competition factor (*CCF*) assumes that area occupied by an open-grown tree is proportional to its crown projectional area and that the relationship of crown width of the open-grown tree to diameter is of the form: crown width $= a + bD_i$. Their measure of stand density is,

$$\begin{aligned} CCF &= [k_1\Sigma(\text{maximum crown area})]/\text{area} \\ &= [k_2\Sigma(\text{maximum crown width})^2]/\text{area} \\ &= [k_2\Sigma(a + bD_i)^2]/\text{area} \\ &= k_2[a^2N + 2ab\Sigma(D_i) \\ &\quad + b^2\Sigma(D_i^2)]/\text{area} \end{aligned}$$

which, for unit area, is proportional to:

$$\Sigma(a + bD_i)^2 = a^2N + 2ab\Sigma(D_i) + b^2\Sigma(D_i^2).$$

This differs from Chisman and Schumacher's tree-area ratio only in the method of estimation of the two equation constants versus the three of tree-area ratio, and in the use of the open-grown tree rather than an average of trees in normal stands as the reference base.

The crown width-diameter line is generally fitted as a straight line (Fig. 1), with positive y-intercept and negative x-intercept; since a tree of diameter zero at bh still has measurable crown width.

For unit area, *CCF* is proportional to the expression,

$$\begin{aligned} \Sigma(a + bD_i)^2 &= [(b^2)\Sigma(a/b + D_i)^2] \\ &= b^2\Sigma(D_i + c)^2 \end{aligned}$$

where $(-c) = (-a/b) =$ the x-intercept.

Except for a proportionality constant, *CCF* can thus be regarded as a modified basal area obtained by a translation of axes, $D_i \rightarrow (D_i + c)$, providing a common origin for the crown width and diameter scales. For unit area, Krajicek *et al.*'s original equation can be written as:

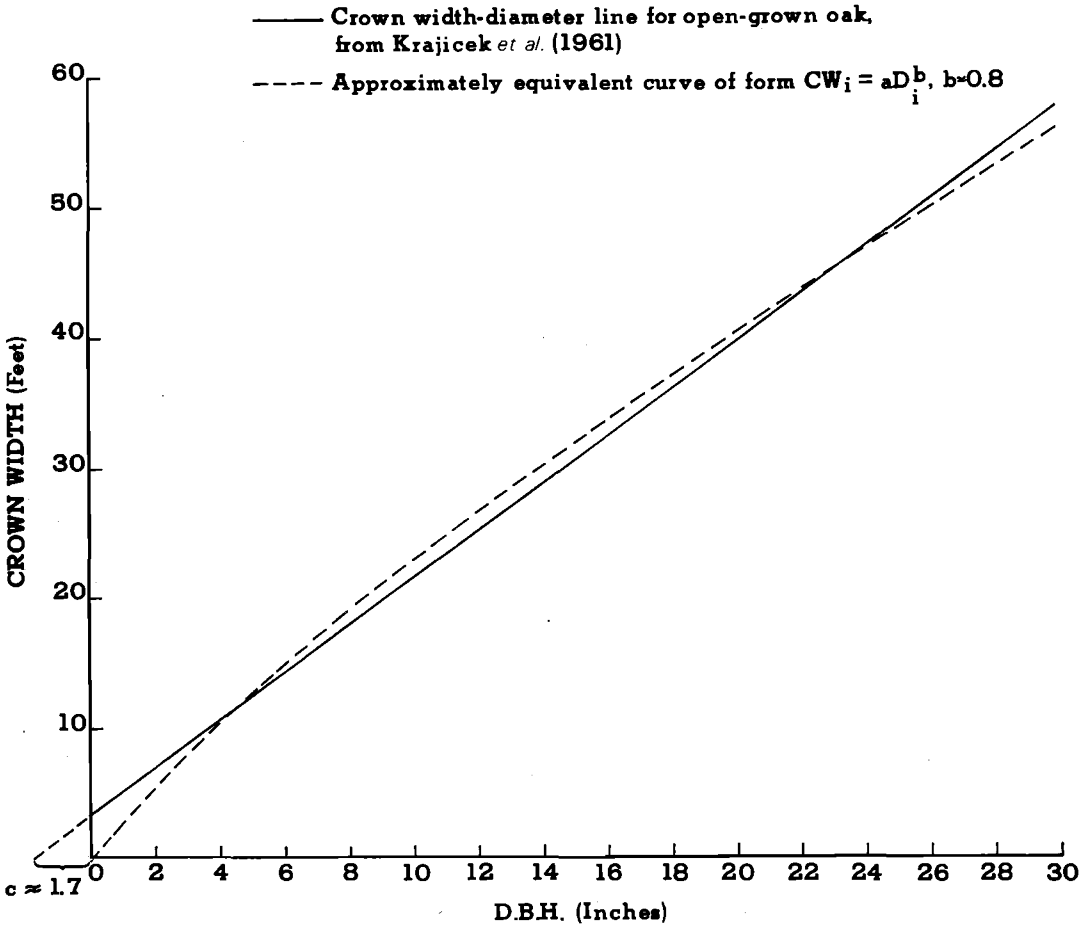

FIGURE 1. *Typical crown width-diameter curves for open-grown trees.*

$$CCF = .0060\,[\Sigma(D_i + 1.7)^2] \propto \Sigma(D_i + 1.7)^2.$$

Compared to conventional basal area, this assigns relatively higher values to stands of small diameter and tends to compensate for the trend of basal area with average diameter. It is evident that a constant CCF corresponds to a "$(D + c)$ times a constant" spacing rule, rather than the "D plus" and "D times" rules with which Krajicek *et al.* (1961) compared it.

Interpretation of Stand Density Measures

Although the preceding discussion by no means exhausts the list of measures of density, it has pointed out the essential similarity of many such measures when regarded as expressions of average area occupied or available per tree, relative to some standard condition.

Minimum, Maximum, and Constant Stand Densities. If one assumes linear relationships among linear measures, as done by Krajicek, Curtin, and many others, relationships in a system based on diameter can be represented as shown in Figure 2. AO represents maximum crown width or $\sqrt{\text{tree area}}$ in relation to diameter, as indicated by development of open-grown trees. AN is the minimum development of average trees, as indicated by stands of normal or near-maximum density. For any given closed stand, density may be

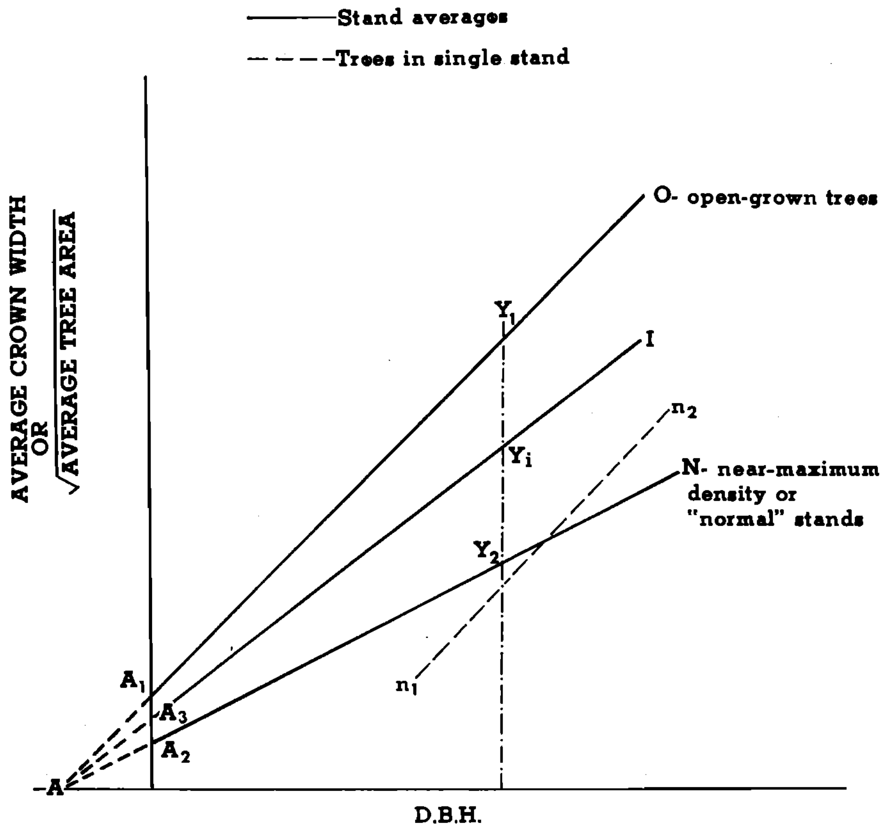

FIGURE 2. *Average crown width-diameter and $\sqrt{\text{tree area}}$-diameter reference curves.*

specified by position relative to either AO or AN; i.e., the density of stand (Y_i) may be measured by either Y_i/Y_1 or Y_i/Y_2, which are proportional for all stands at any specified level of density, represented by the line A_3I.

The relationships of crown width-diameter and $\sqrt{\text{tree area}}$ diameter among trees composing a single stand (e.g., n_1n_2) are not necessarily proportional to these average curves nor are they necessarily linear on the same axes. At the open-grown limit, the individual within-stand curve coincides with the average among-stands relationship, but its slope will increase relative to the corresponding among-stands curve with increasing density and probably also with increasing stand average diameter.

The region between A_1O and A_2N represents the range of stand average crown width-diameter or $\sqrt{\text{tree area}}$-diameter values, and corresponding densities, within which closed stands are possible, although all stands within this range are not necessarily closed.

Stand Density and Stand Closure. In closed stands—which approximate complete area occupancy—relative stand density measures express cumulative crowding or competition effects in terms of an average crown (or tree) area-diameter relationship and associated average crown development, relative to some standard condition. This interpretation is directly applicable to stands with no recent cutting and with nearly complete crown cover, or to stands treated with frequent light thinnings, in which crown closure is nearly complete and fluctuates only within a narrow range. Conventional measures of stand density by themselves do not provide a description of average tree characteristics comparable among stands with different degrees of closure, as may be illustrated by comparison of three hypothetical stands of

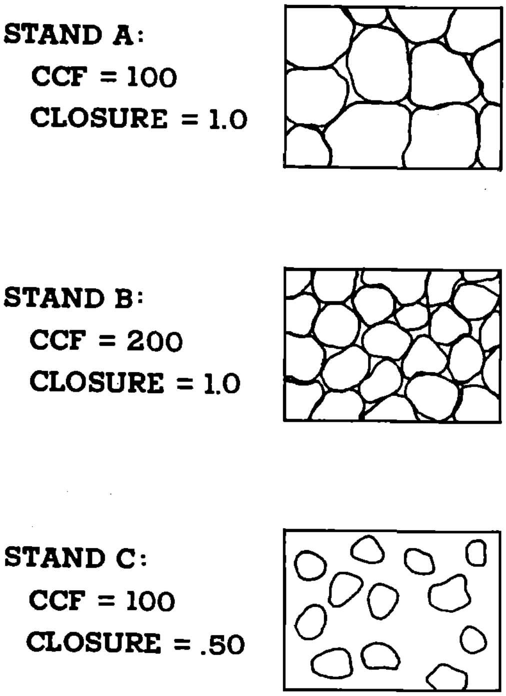

FIGURE 3. *Comparison of three hypothetical stands of identical age and site but differing crown competition factor (CCF) and stand closure.*

identical age and site—shown in Figure 3—in terms of crown competition factor (*CCF*).

Stand *A*: Uniformly distributed trees, in a stand which has developed without appreciable competition in the past but has now reached the point of full crown closure. *CCF* = 100.

Stand *B*: Uniformly distributed trees, in an uncut normal stand. Crown development of individual trees much restricted by competition. *CCF* = 200.

Stand *C*: Past history identical to stand *B*, but recently cut, reducing *CCF* from 200 to present value of 100.

Trees in stand *C* must differ markedly from those in stand *A*, both in relative crown development and in current level of competition. The stands probably differ considerably in present and potential growth, although identical in age, site, and *CCF*. A value interpretable as an expression of average crown area relative to standard conditions—similar to the interpretation of *CCF* for closed stands—is obtained by dividing *CCF* by crown closure (fraction of the area occupied by tree crowns).[5] For stand *C*, this is 100/0.5 = 200, equivalent to that in stand *B*.

In practice, measurement of stand closure is difficult and inaccurate. Since considerable crown overlap may exist with tolerant species on good sites, whereas other species never reach complete crown closure on certain sites, crown closure corresponds only approximately to area occupancy. The point is that conventional measures of stand density are directly interpretable as expressions of cumulative crowding or competition effects on development of average trees only in reasonably uniform, homogeneous stands of approximate full closure or comparable degrees of closure. Full description of stand conditions would require both density and stand closure, and inferences based on growth-stand density relations in stands of a given degree of closure may not apply to stands differing markedly from this condition.

Briegleb (1952) pointed out that ability of a tree to utilize growing space depends not only on stem diameter but also on crown development, which is determined by previous history of the stand. He showed that crown dimensions of Douglas-fir were related to both diameter and height and developed a stocking standard in terms of numbers of trees or corresponding basal areas for stands of given average diameters and heights. The method expresses cumulative competition effects by the combination of stand average diameter and average height and gives for each combination the numbers and basal areas corresponding to a subjectively chosen standard stocking.

Although Briegleb referred to this as a standard of density, in terms of the concepts discussed in this paper it is better considered an expression of stand closure and is of considerable interest as an attempt to express a stand characteristic not

[5] This corresponds to Smith and Bailey's (1964) and Smith's (1965) use of the term "stocking," avoided here because of confusion with other usages.

measured by the common stand density expressions and basically different from these in concept.

Possible Modifications of Common Measures

Tree-Area Ratio and CCF. It has been shown that

$$CCF = k_1\Sigma(D_i + c)^2.$$

The analogy between *CCF* and tree-area ratio, and the apparent proportionality of tree area-diameter curves for open-grown trees and for average trees in normal stands suggest that an expression of form

$$\text{tree-area ratio} = k_2\Sigma(D_i + c)^2,$$

where c, k_1, and k_2 are constants estimated from normal stand data, would be proportional to *CCF*. Figure 4 shows that although tree areas estimated by the equations of Chisman and Schumacher (1940) for loblolly pine and those of Gingrich (1967) for oaks have a slightly curvilinear relationship to diameter on logarithmic axes, in each case a suitable translation of axes, dbh $\rightarrow$ (dbh + c), yields a linear relationship with a slope of approximately 2.0.

Power Functions of Diameter Breast Height. Basal area is proportional to ΣD_i^2. However, comparisons of basal area with tree area (Gingrich 1967) show that 2.0 is not the power of diameter most closely related to average tree area.

From Reineke's equation, average area per tree = $1/N \sim a_1\bar{D}^{1.6}$. That is, over a series of normal stands, average area per tree is approximately proportional to the 1.6 power of average diameter. And, if the smallest trees are neglected, the tree area curves of Figure 4 are approximated on logarithmic axes by straight lines with slopes considerably less than 2.0. This suggests that a sum of some power of diameters—less than 2.0 and probably near 1.6—would provide a tree-area ratio of form $k_i\Sigma(D_i^c)$. This would be analogous to basal area but without the latter's inherent dependence on stand diameter and hence on site and age. Current work with Douglas-fir in the Pacific Northwest indicates that this is indeed the case.[6]

And, if tree area is proportional to the 1.6 power of diameter, its square root (and, presumably, crown width) is proportional to the 0.8 power of diameter. For much published data for the crown width-diameter relationship in open-grown trees, a curve of this form is scarcely distinguishable from the straight line commonly used (Fig. 1), suggesting that crown competition factor could be expressed as:

$$CCF = k_2\Sigma(D_i^c),$$

proportional to a tree-area ratio of the same form.

Since trees under 4.5 ft in height occupy growing space, power functions—which assign zero area to trees of zero diameter—should logically be based on diameters measured at a fixed relative height rather than at a fixed absolute height. Measurement of diameters at a fixed height of 4.5 ft (breast height), though necessary in practice, must introduce errors for very small trees. Likewise, the assumption of a linear relationship between crown width (or $\sqrt{\text{tree area}}$) and diameter bh is somewhat arbitrary. Differences introduced by assuming one or the other form are often negligible within the range of diameters of practical interest (Fig. 1).

Tree Areas Within Stands. The measures which have been discussed express position of a *stand* relative to a limiting density condition (A_2N or A_1N in Fig. 2), in terms of some form of average area per tree. They give no information about the allocation of growing space among tree size classes *within* a stand (line n_1n_2 in Fig. 2). In principal it should also be possible to express the contribution of particular components of a stand to the overall stand value of the density measure, and one procedure for this has been given by Stage (1968).

Diameter vs. Height vs. Site and Age As Bases for Density Measures

The preceding discussion has been primarily concerned with those measures which

[6] Manuscript in preparation.

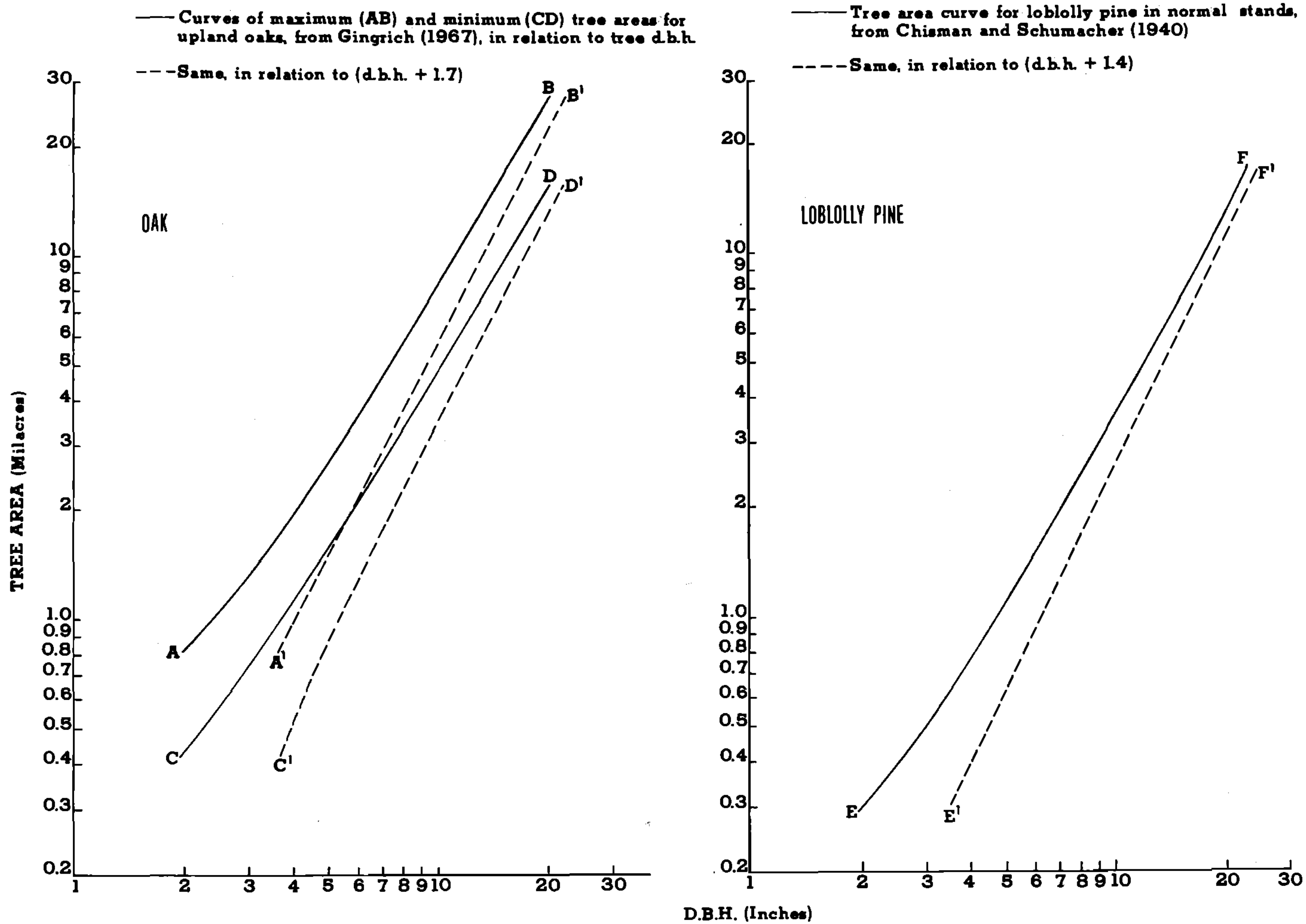

FIGURE 4. *Linearization of tree-area curves for oaks by translation dbh → (dbh + 1.7), and of loblolly pine tree-area curve by translation dbh → (dbh + 1 4), on logarithmic axes*

express relative density as a ratio of observed total tree area, number, or basal area to some function of average diameter or sums of powers of diameters which provides an estimate of the corresponding value for "standard" conditions. Such measures have a common interpretation as comparisons of areas occupied by trees of similar diameters under standard conditions with area available to trees in the observed stand.

Density may also be expressed as a ratio of observed number of trees to number in stands of standard density and equal height. This form, interpretable as a comparison of observed area per tree with that in a standard stand of the same height, has had considerable and apparently satisfactory use in Europe and some in this country, particularly in plantations. In plantations, thinned stands, and stands of low initial density, average diameters are ordinarily considerably greater than in normal stands of the same height. Therefore, numerical values of height-based measures will not in general be the same as those of diameter-based measures, although values can be expected to coincide at the standard condition and to change in the same direction as stand density changes. Since height represents a joint effect of age and site (sometimes modified by competition), expressions using stand height as a means of reference to the standard condition are related to similar expressions using age and site, but not necessarily equivalent.

Since the relationship between stand average diameter and average height is itself associated with stand density, combinations of diameter and height found in dense stands probably do not exist for open-grown trees, and combinations found in open stands may not exist in "normal" stands. Therefore, expressions (e.g., Briegleb 1952) using both diameter and height to estimate expected number of trees or tree-area do not have the same interpretation—as comparisons of an average observed area per tree with that of some average tree in a comparable normal stand or open-grown condition—as do those which use the single measurable attribute of diameter or height as the basis of comparison with the standard condition. This does not imply that they may not be useful, but simply that they represent a different class of measures than those with which this paper has been primarily concerned.

A ratio of observed basal area to basal area of normal stands of the same age and site, G_o/G_e, where G_e = a function of age and site index, has often been used as a measure of density. Since basal area is proportional to the product $N\bar{D}^2$,

$$\begin{aligned} G_o/G_e &= N_o\bar{D}_o^2/N_e\bar{D}_e^2 \\ &= [(1/N_e)/(1/N_o)][\bar{D}_o^2/\bar{D}_e^2] \\ &= [\text{average area per tree, normal/average area per tree, observed}][\bar{D}_o^2/\bar{D}_e^2]. \end{aligned}$$

And, since in general $\bar{D}_o \neq \bar{D}_e$, this—unlike a relative basal area based on stand average diameter—cannot be directly interpreted as a ratio of average tree areas. Its values will differ from those of measures based on stand diameter (Fig. 5)[6] by a factor representing the squared ratio of observed to expected stand quadratic mean diameter. Differences between the two bases of comparison may partially explain the sometimes puzzling observation that stands which have developed under regular thinning regimes or from wide initial spacings and which are obviously not over-dense may have basal areas approaching those of normal stands of the same age and site.

Conclusion

In preceding sections, analogies and equivalencies among a number of commonly used relative measures of stand density have been pointed out. These measures have been given a common interpretation as expressions of average area available per tree, compared to either the open-grown condition or the normal stand.

[6] Figure adapted from Bruce (1969). The high values of relative basal area for age and site for ages 20 to 30 may in part be due to the known peculiarities of the normal yield table in this portion of the age range.

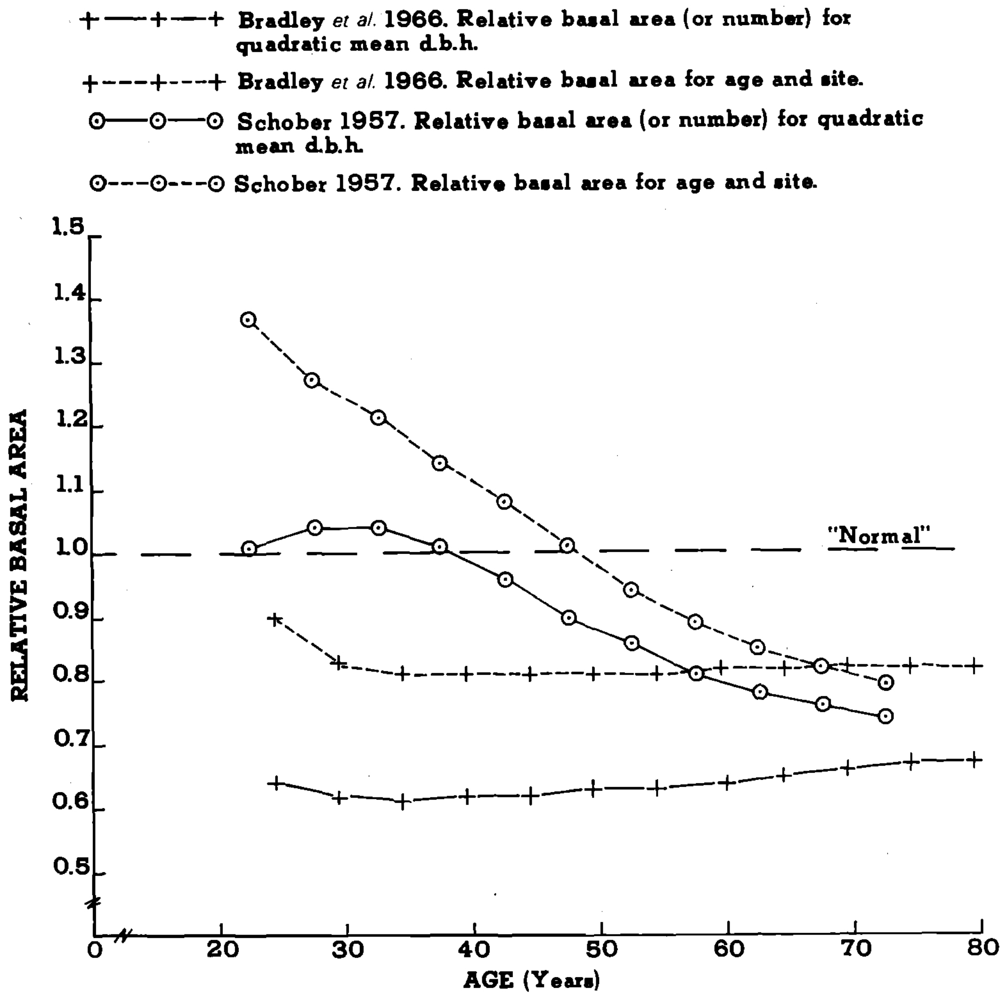

FIGURE 5. *Comparison of relative basal area based on quadratic mean dbh with relative basal area based on age and site, for normal stands in the Pacific Northwest and for intensively managed Douglas-fir stands in Great Britain and Germany, on sites equivalent to United States site III. Values for European stands are for midpoint of thinning period. Normal basal areas for quadratic mean dbh and for age and site in European tables taken from McArdle et al. (1961).*

They are applicable only to relatively uniform, homogeneous stands. In closed stands or in conjunction with some measure of stand closure, they are viewed as expressions of average crown development relative to a standard condition, and hence, expressions of cumulative competition effects on tree development.

Little of this is really new. Yet the various measures of stand density often seem to be regarded as distinct and separate entities. The interpretation given in this paper provides a unifying view of a number of common relative measures of stand density as expressions of the same basic relationship, which differ mainly in details of algebraic form and method of estimation of the constants. Minor improvements may be possible in the form of expression. Diameter, height, or site and age are not equivalent bases for referring stand totals of basal area or number to the corresponding standard condition, and density measures using these alternative bases will differ in numerical values and may not be equally suitable as expressions of relative stand density. And the estimates of site index and stand age which are required for use of measures based on site index and age introduce sources of error not present in measures based on diameter or height.

Otherwise, choice among these measures is a matter of information available, convenience in computation, and ready understanding and visualization rather than of fundamental differences in meaning or precision.

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