Measuring and Evaluating Stocking and Stand Density in Upland Hardwood Forests In the Central States

BY
SAMUEL F. GINRICH

Abstract. Stocking and density standards are presented for upland hardwood stands (chiefly oak-hickory and mixed species) in the central states. Utilizing the open grown and forest grown tree-area requirements, limits of stocking and density for full site utilization are established. The standards presented are not influenced by site quality, stand age, or stand structure. The wide distribution of diameters in even-aged upland hardwood stands is due in part to differential species growth. Coefficients of variation, skewness, and kurtosis are given for a range of stand conditions. Under most stand conditions of even-aged upland hardwoods, stand structure can be ignored in the appraisal of stocking. Evidence is also presented to show that stand structure has very little effect on volume growth.

ON A GIVEN SITE, growing space is the dominant factor that controls the growth rate of an individual tree. Space occupied by individual trees in a stand is described in terms of stand density. The regulation of stand density by thinnings is the key ot good silviculture where the production of high-quality wood is the primary objective.

Based on analysis of data from central states' forests, this paper describes the development of new stocking and density criteria and their application in upland hardwood forests which occupy nearly one-fourth of the commercial forest land in the United States (U.S. Forest Service 1963). As the first step in determining what stand density is desirable in upland hardwood stands of oak-hickory and mixed species, we have sought to accurately define the range of densities in which stand growth is at a maximum. The choice of a stocking norm within this range can be made on the basis of management objectives.

Standard definitions are available for both density and stocking (Bickford *et al.* 1957). Because these terms are often used synonymously or at best not clearly distinguished one from the other, a brief definition of each is presented. *Stand density* is a quantitative measurement of a stand in terms of square feet of basal area, number of trees, or volume per acre. It reflects the degree of crowding of stems within the area. *Stocking*, on the other hand, is a relative term used to describe the *adequacy* of a given stand density in meeting the management objective. Thus, a stand with a density of 70 square feet of basal area per acre may be classified as overstocked or understocked, depending upon what density is considered desirable.

The tree-area ratio developed by Chisman and Schumacher (1940) is used here as the basic measure of stand density and the corresponding stocking categories.

The author is Research Forester, Northeastern Forest Expt. Sta., Forest Service, U.S. Dept. Agric., Columbus, Ohio. Manuscript received Nov. 30, 1965.

This is a simple and objective measure of stand density that is independent of stand age and site quality. The tree-area ratio is based on the premise that the growing space used by a tree depends on the size of the tree and is related to stem diameter by a second-degree parabola. This ratio is simply a means of allocating tree-area requirements, using stand data, by solving the equation where

$$\text{Tree area} = aN + b\Sigma D + c\Sigma D^2$$

a, b, and c are the regression constants; N is the number of trees per acre; ΣD is the sum of the individual diameters; and ΣD^2 is the sum of the squares of the diameters. Solving this equation for $N = 1$ will result in a measure of the tree area allocated to a single tree of diameter D under the stocking conditions represented by the stand data.

Several sources (Beers 1960, Lexen 1939, Lynch 1958) have challenged the utility of the tree-area equation in the form used here. They maintain that either the N or ΣD could be removed because of their small contribution to tree area. It is true that the basal-area component (ΣD^2 = basal area/.005454) contributes the largest amount to tree area. However, ΣD is also significant, and as will be shown later, is useful in adjusting stocking percent to account for differences in stand structure. And N is important in young stands where basal area is low. Actually, in stands less than 15 years old, the number of trees by itself is an acceptable measure of stand density. Even though N contributes least to stocking percent, dropping it from the equation results in lost precision. The standard error of estimate of the three-variable equation is ± 29.6 milacres of tree area or ± 3 percent stocking. Dropping the N component increases the standard error of estimate to ± 70 milacres or ± 7 percent stocking. Thus, careful scrutiny of each component reveals the need to retain them all in the equation.

Source of Data

Stand data used to develop the stocking-density criteria for upland hardwoods came from two sources, both selected to represent the maximum stocking of natural, undisturbed stands where the average tree has the minimum space necessary to survive. These sources are Table 37 from Schnur's (1937) yield tables for upland oak forests and 87 half-acre permanent plots currently used by the U.S. Forest Service in a regionwide (Ohio, Kentucky, Missouri, and Iowa) stand-density study.[1] The plots are in stands ranging from 25 to 75 years in age and 45 to 75 in oak site index (Schnur 1937). As will be shown later, these plots (after treatment) furnished the basis for the study of growth trends in understocked stands. Oaks predominated on the plots along with the usual mixture of other hardwoods that characterize upland hardwood forests of the eastern United States.

The tree-area equation derived from these stands was:

$$\text{Tree area (milacres)} = -.0507N + .1698\Sigma D + .0317\Sigma D^2$$

This was the equation used to compute the stocking-density criteria. Solving this equation for $N = 1$ and D = any given diameter defines the minimum tree-area requirements for that size tree. A stand having 1,000 milacres of tree area per acre is 100 percent stocked. As will be shown later, stand structure (the distribution of trees by diameter classes) is taken into account when stocking standards are converted to an acre or stand basis.

Another source of data was used to obtain tree-area requirements under a competition-free environment—where each tree has all the space it can use.

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[1] Pooling of the smoothed data from Table 37 with the raw data from the 87 plots will yield a conservative value for the standard error of estimate.

TABLE 1. Relation between basal area and average tree area.

Dbh (inches)	Basal area	Tree-area requirements Maximum	Minimum	Min./max.
	Square feet	*Milacres*	*Milacres*	*Percent*
2	0.022	0.82	0.42	51
4	.087	1.96	1.14	58
6	.196	3.56	2.11	59
8	.349	5.66	3.34	59
10	.545	8.22	4.82	59
12	.785	11.3	6.55	58
14	1.069	14.8	8.54	58
16	1.396	18.8	10.8	57
18	1.767	23.3	13.3	57
20	2.182	28.3	16.0	57

Krajicek, Brinkman, and Gingrich (1961) found that open-grown oaks and hickories of any given size have a well-defined limit of growing space they can use when grown in the absence of competition from surrounding trees. We concluded that competition for growing space begins at the point where the available growing space in a stand is just equal to the total, open-grown, tree-area requirements of all the trees in the stand. Maximum tree-area requirements were found to be independent of site quality and stand age.

The equation derived to express the maximum amount of area that trees can use was:

$$\text{Tree area (milacres)} = .175N + .205\Sigma D + .060\Sigma D^2$$

Thus, two tree-area equations were derived that determine the range of the stocking-density criteria for upland hardwoods when the growing space is fully utilized.

The relation between these two equations is significant. Minimum tree-area requirements range from 57 to 59 percent of maximum usable area for trees from 4 to 20 inches in diameter (Table 1). This means that the minimum stocking for full utilization of the growing space is from 57 to 59 percent of the density of fully stocked stands.

A comparison of tree-area requirements with basal area reveals the basic weakness of basal area alone as a measure of stand

TABLE 2. The effect of changing dbh on the relation between basal area and tree area.

Actual change in dbh (inches)	Expansion factor[1] Basal area	Average tree area[2]
2 → 4	4.0	2.55
4 → 8	4.0	2.91
6 → 12	4.0	3.14
8 → 16	4.0	3.28
10 → 20	4.0	3.38

[1] The increase (in multiples) in basal area and tree area corresponding to the changes in dbh.

[2] Determined from the average change in minimum and maximum tree area (Table 1).

TABLE 3. A comparison of average diameter-growth rates[1] of selected upland hardwood species and projected average tree diameters at 30, 60, and 90 years.

Species	Trees bored	10-year diameter growth: Seedlings[2] and saplings	Poles[2]	Sawtimber[2]	Projected average dbh: 30 years	60 years	90 years
	No.	*Inches*					
Yellow-poplar	569	2.26	2.40	2.55	6.9	14.3	21.9
Black walnut	409	1.98	2.24	2.06	6.1	12.6	18.8
Scarlet oak	677	1.67	2.00	2.30	5.0	11.0	17.9
Red oak	400	1.54	1.83	2.32	4.6	10.1	16.8
White ash	929	1.57	1.77	2.09	4.7	10.0	16.1
Black oak	1,850	1.60	1.78	1.96	4.8	10.1	15.9
Sugar maple	939	1.29	1.60	1.82	3.9	8.4	13.5
Beech	609	1.12	1.55	1.63	3.4	7.4	12.1
White oak	2,519	1.20	1.37	1.84	3.6	7.5	11.8
Hickory	2,123	1.16	1.27	1.42	3.5	7.1	11.0
Chestnut oak	568	0.92	1.32	1.69	2.8	5.8	9.7

[1] Regional average of Forest Survey Resource Reports from Ohio, Indiana, Illinois, Kentucky, Missouri, and Iowa. Central States Forest Experiment Station. (Basis—11,592 trees bored.)

[2] Seedlings and saplings—trees less than 5 inches; poles—trees 5 to 11 inches; sawtimber—trees over 11 inches.

BA Not a good measure of density in managed stands

density. Over a period of time trees increase in diameter, but if stands are continually cut to maintain a constant basal area, the stocking condition, or degree of competition, actually decreases. Doubling the diameter of a tree increases its basal area four times but increases the tree-area requirements only about three times (Table 2). Thus, as tree diameter increases the basal area of a stand must also increase if the percent stocking is to remain the same.

Relation of Site Quality, Stand Age, and Species to Tree-Area Requirements

The basic data for fully-stocked stands came from stands ranging from 25 to 75 years in age and 45 to 75 in oak site index. Within this range, tree area was independent of site quality and tree age. Krajicek, Brinkman, and Gingrich (1961) reported similar results in studying open-grown trees. In fact, this feature of the tree-area ratio has been consistent wherever it has been used. This means that the average tree of a given diameter will utilize the same area at a given level of density regardless of the site or the age of the tree. A 40-year-old, 10-inch black oak (*Quercus velutina* Lam.), for example, that is growing on a good site requires the same area as a 65-year-old, 10-inch black oak on a poor site. The tree on the good site will grow faster, however, because the same amount of area on a good site contains more of the factors necessary for growth.

The species composition of the fully stocked study plots varied from pure white oak (*Quercus alba* L.) to a mixture of northern red oak (*Q. rubra* L.),

TABLE 4. The effects of differential species growth on the structure of upland hardwood stands.

Age (years)	Average tree diameter	Coefficient of variation: Due to composition[1]	Coefficient of variation: Total[2]	Amount due to species composition alone
	Inches	*Percent*	*Percent*	*Percent*
30	4.3	24	42	57
60	9.0	24	37	65
90	14.2	23	30	77

[1] Based on Table 3.

[2] As determined from the 87 stocking plots.

scarlet oak (*Q. coccinea* Muenchh.), black oak, chestnut oak (*Q. prinus* L.), hickories (*Carya* Nutt. spp.), red maple (*Acer rubrum* L.), yellow-poplar (*Liriodendron tulipifera* L.), and others. The oaks and hickories had similar tree-area requirements under fully stocked and open-grown conditions. Some of the above species were not sufficiently represented to compute reliable tree-area requirements but we did learn that a yellow-poplar of the same diameter always occupied less area than oak and hickory; and beech (*Fagus grandifolia* L.), more. Variations in stocking among the plots was not related to differences in species composition within the range examined. Since the oaks and hickories predominate in the upland hardwood type their requirements largely determine the average requirements for the type.

Although no adjustment in tree-area requirements on account of species composition was necessary, composition does influence stocking through its effect on stand structure. Different growth rates among species widen the range of diameters within any stand. This makes it difficult, sometimes impossible, to distinguish even-aged and uneven-aged[2] stands by their appearance.

Average diameter-growth rates of the 11 species that comprise 90 percent of the commercial cubic-foot volume of upland hardwoods in the central states vary widely (Table 3). These data come from a different source than the plots used in this study, but the ranking of the species by growth rate is strongly confirmed by the study plots and even shows good similarity to that found by Trimble (1960) working in a different area.

Diameter-growth rates were projected to 30, 60, and 90 years (Table 3) to determine their effect on diameter dispersion about the mean. Stands of typical species composition were then synthesized and species were weighted according to their occurrence in the "average" upland hardwood stand. The coefficient of variation of mean diameter for these "stands" was compared with that of stands of average uniformity as determined from the stocking plots (Table 4). This comparison shows that species composition alone may account for half to three-fourths of the diameter dispersion in upland hardwood stands.

So, species composition greatly influences the growth and productivity of a stand, although as yet there is no way to predict the amount of influence. Since differences in stand structure in the absence of cutting result from differences

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[2] Smith (1962) says, "... there is not, in the strictest sense of the term, any such thing as an uneven-aged stand ... The uneven-aged stand is an artificial entity required for the comprehension of what might otherwise be a chaos of little stands."

in composition, however, a method that adjusts stocking to account for differences in structure will largely account for differences in composition also. Thus, if a stand has a preponderance of slow-growing species it is not yet possible to predict how much less its growth will be than a stand of fast-growing species. But maintaining the stand at the lowest stocking that fully utilizes the growing space will lead to the fastest diameter growth possible for that particular stand.

Stand Structure and Stocking

It has been more or less implicit in the literature of both silviculture and management that there is some optimum stand structure for "best" growth and yield. Therefore, two questions arise: should stocking and density criteria be adjusted for differences in stand structure, and does the stocking equation describe differences in stand structure?

Stand structure was first used in America when stand tables showing the distribution of trees by diameter classes were presented with normal yield tables. Because diameters were rarely normally distributed around the mean, smooth frequency distributions were often developed from such distribution functions as the Charlier and Pearson curves, the Pearl-Reed growth curves, and the Fourier series. Coefficients of skewness, kurtosis, and variation adequately described the structure of normal, even-aged stands. A significant and consistent conclusion reached by Meyer (1930), Schumacher (1928), and the U.S. Forest Service (Unpublished) was that the computed parameters (coefficients of skewness, kurtosis, and variation) were more closely related to average tree diameter than to site and age. This fact is incorporated into more recent yield tables that present stand tables in terms of average tree diameter, rather than in terms of site and age. All this work emphasized the importance of average tree diameter in describing stand structure.

The tree-area equations were used to determine the growing space requirements of *single trees* under fully stocked and open-grown conditions. Converting this to *stand* measures of stocking thus requires a knowledge of the structure of the specific stand. In view of the close relation between average tree diameter and structure, and because the variables in the equations are those necessary to obtain average tree diameter, the equations were examined further to see if any additional adjustment for stand structure was needed.

Two obviously different stand structures can have exactly the same stocking variables and coefficient of variation and therefore be equally stocked (Fig. 1). Type I represents a bimodal structure, not commonly found in upland hardwoods, while Type II, is a near-normal structure with low positive skewness and kurtosis. Under similar conditions of site and species composition the stands should produce the same volume of wood because a tree of a given diameter in either stand theoretically has the same amount of growing space. Nevertheless, their structures differ and skewness and kurtosis coefficients are needed to describe these differences.

However, if two stands have the same number of trees and basal area per acre but different coefficients of variation, then differences in stand structure create differences in the sum of diameters and the equation gives different stocking percents (Fig. 2). Consequently, for greatest accuracy the sum of diameters must be included in the calculation of stocking percents to account for variations in stand structure.

Of the three variables in the equation, the sum of diameters squared (basal area/.005454) and the total number of trees (N) are most readily obtained in the field. Unfortunately, the sum of the diameters is much more difficult to get. It requires a diameter measurement or careful estimate of each tree—a procedure now less common than before the advent

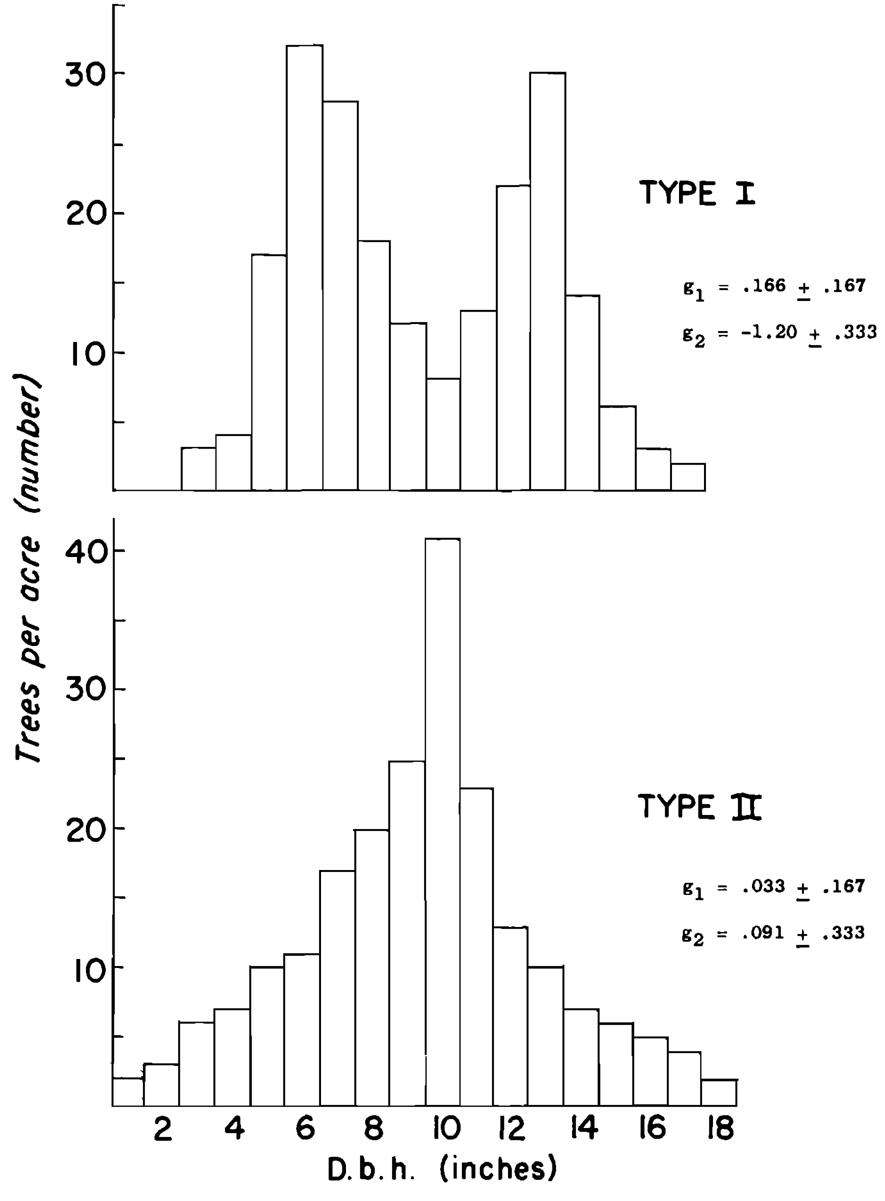


FIGURE 1. *A comparison of two stands of different structure but equal in all the components of stocking. Type I structure is indicative of a two-storied, several-aged stand. Type II structure is typical of an even-aged stand composed of several different upland hardwood species. g_1 and g_2 are the coefficients of skewness and kurtosis respectively.*

$N = 212$

$\Sigma D_2 = 1{,}995$

$\Sigma D = 21{,}215$

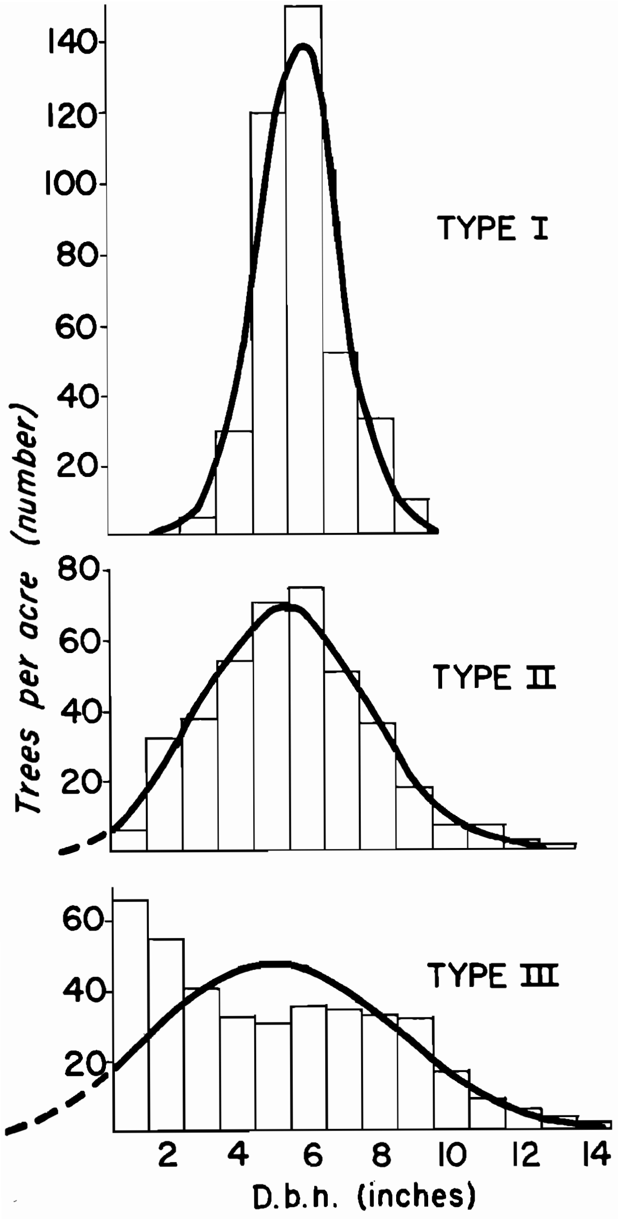
$BA = 115.7$

$\bar{D} = 9.41$ *inches*

$\bar{D}_B = 10.00$ *inches*

$CV = 36$ *percent*

$S\% = 100$



Sum of diameters = 2353
Average diameter = 5.88
Standard deviation = ± 1.19
Coefficient of variation = 20%
Stocking percent = 83.5
g_1 = .297 ± .122
g_2 = .560 ± .243

Sum of diameters = 2221
Average diameter = 5.55
Standard deviation = ± 2.28
Coefficient of variation = 41%
Stocking percent = 81.3
g_1 = .419 ± .122
g_2 = .206 ± .243

Sum of diameters = 2020
Average diameter = 5.05
Standard deviation = ± 3.24
Coefficient of variation = 64%
Stocking percent = 77.9
g_1 = .458 ± .122
g_2 =−.756 ± .243

FIGURE 2. *A comparison of stocking in three stands of different structure showing the effect of structure on the sum of the diameters, and hence stocking. Each stand has a fixed number of trees (400) and basal area (78.5 square feet). The diameter of the tree of average basal area is 6 inches. The smooth frequency curves represent the normal distributions with Sheppard's adjustment for grouping continuous variables.* g_1 *and* g_2 *are the coefficients of skewness and kurtosis respectively for the actual frequency distributions.*

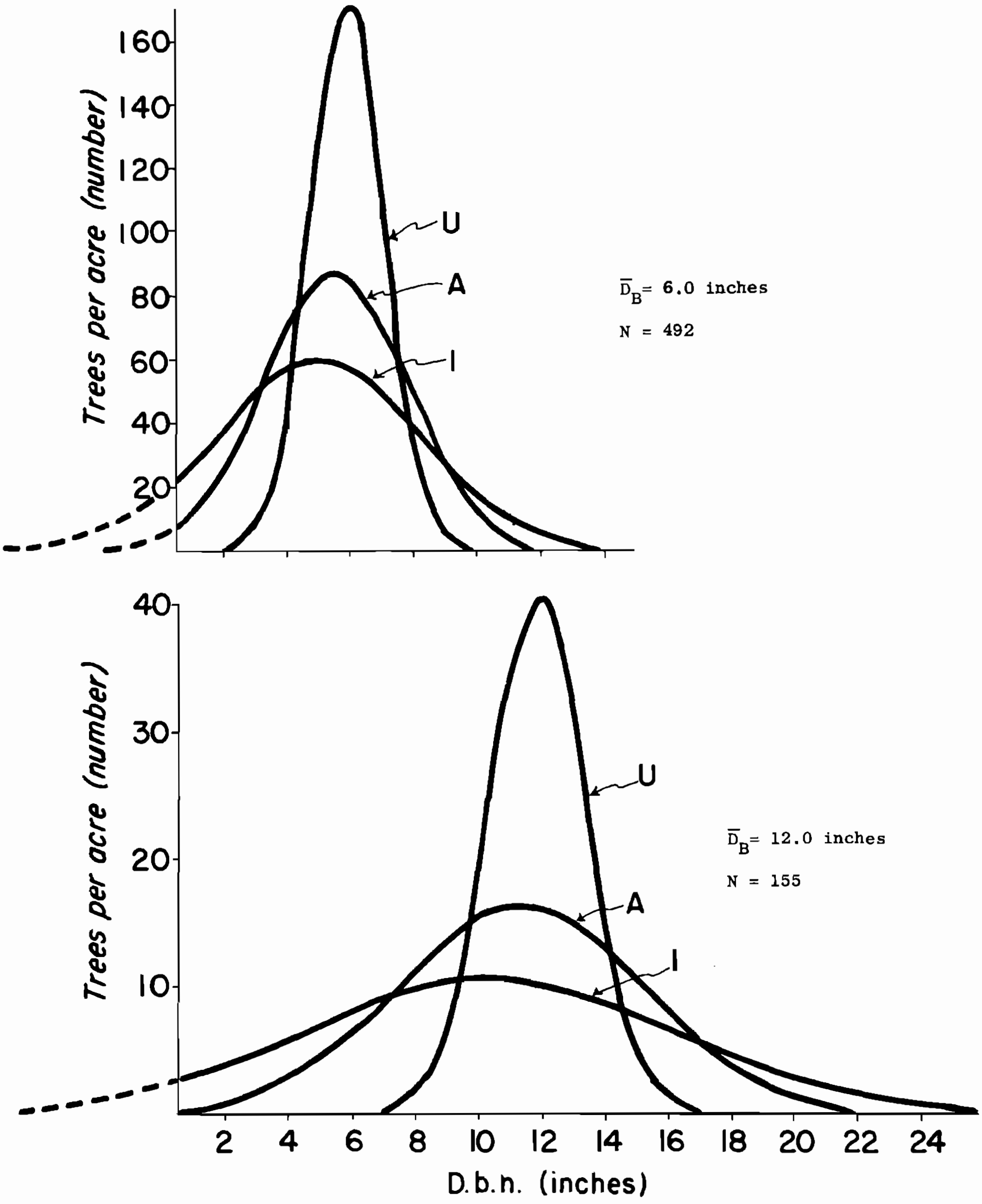


FIGURE 3. *The actual shape of normal frequency curves depends upon diameter dispersion. Assuming normality (i.e.,* g_1 *and* $g_2 = 0$*) the above sets of curves represent stands of uniform structure (U), average structure (A), and irregular structure (I). The area under each set of curves is equal. Sheppard's correction for grouping continuous variates has been applied in all cases. Corresponding stocking percents and coefficients of variation are given in Table 6.*

in locating plot or stand on chart

of the basal-area prism. It would be helpful, therefore, if using only ~~basal area~~ and number of trees would provide sufficient accuracy for field use. The structure of stands was examined to see how much accuracy might be lost, on the average, if sum of diameters were omitted. This required stand-structure parameters for "average" stands to be formulated.

Stand-structure parameters. Skewness and kurtosis coefficients were computed for the 87 stocking plots. These coefficients indicate the degree of similarity of a given frequency distribution to the normal distribution. Skewness is a measure of the lack of symmetry and is based on the sum of the third-power deviations from the mean. A distribution with a longer tail to the right of the mean (Fig. 2, Type III) has positive skewness. When the tail is to the left of the mean the skewness is negative.

Kurtosis is a measure of the extent to which the actual distribution curve exceeds the height of the normal distribution curve and is based on the fourth moment. A sharp peak in the curve represents positive kurtosis (Fig. 2, Type I) whereas flat-topped curves have negative kurtosis (Fig. 2, Type III). Normal distributions can be variously shaped, depending upon the dispersion of diameters around the mean (Fig. 3).

Skewness and kurtosis for even-aged upland hardwood stands of average uniformity[3] were calculated (Table 5). Skewness is positive in young stands with small mean diameters, but skewness decreases with age. This can be explained in part by the fact that even-aged stands show a wide range in diameters at an early stand age. It would be mathematically impossible, for example, for a stand with an average diameter of 6 inches, a coefficient of variation of 41 percent, and a total frequency of 492 trees (stocking = 100 percent) to have a normal frequency distribution, since part of the area under the curve would fall to the left of the ordinate (Fig. 3). When the average stand diameter is large enough to accommodate the frequency curve in the positive range of the ordinate a normal frequency distribution exists. By the time the average stand diameter reaches 8 inches a normal diameter distribution usually develops. This explanation is more mathematical than biological but it is confirmed by the stand data.

Kurtosis was not evident in the stocking plots. Only in very young stands was the coefficient of kurtosis statistically significant (Table 5). Young upland hardwood stands have low positive kurtosis, indicating a moderately peaked distribution. As stand age increases the effect of differential species-diameter growth results in broad, flat, frequency curves and negative kurtosis.

It can be concluded from the study of stand-structure parameters of even-aged upland hardwoods that stands of average structure have a near-normal frequency distribution. Although the conventional stand-structure parameters are less accurate when applied to uneven-aged stands with the inverse, J-shaped distribution, some cursory examinations of such upland hardwood stands revealed a close similarity to even-aged stands in skewness and kurtosis. Unlike northern hardwood stands, upland hardwood stands (except for short periods after heavy cutting) rarely have high frequencies in the small diameters because such stands lack the tolerant species needed to maintain the inverse, J-shaped distribution. Indeed, there is no evidence that the common species comprising upland hardwood stands can maintain the inverse, J-shaped distribution naturally, and formal research attempts to maintain it by cutting

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[3] The structure parameters presented in Table 5 represent a near-median condition between uniform, plantation-like, stands of single species on the one extreme and irregular, uneven-aged stands on the other. The parameters shown are the average of the 87 stocking plots which were initially selected to contain a mixed species composition. No plots were used that had any record of fire, grazing or past cutting.

TABLE 5. Stand-structure parameters for even-aged upland hardwood stands of average uniformity—all stands 100 percent stocked.

Ave. tree diameter[1]	Trees	Coefficient of variation	Skewness coefficient g_1	Standard deviation g_1	t	Kurtosis coefficient g_2	Standard deviation of g_2	t
Inches	*Number*	*Percent*						
3	1,430	45	0.677	.065	10.4**	0.250	0.129	1.9*
4	925	43	.588	.080	7.4**	.201	.161	1.2
5	655	42	.499	.091	5.5**	.156	.191	.8
6	492	41	.426	.110	3.9**	.102	.219	.5
7	383	39	.351	.125	2.8**	.057	.249	.2
8	308	38	.290	.140	2.1*	.002	.277	.0
9	253	37	.232	.153	1.5	−.041	.306	.1
10	212	36	.181	.167	1.1	−.092	.332	.3
11	180	34	.143	.181	.8	−.145	.359	.4
12	155	33	.102	.194	.5	−.202	.387	.5
13	135	32	.068	.209	.3	−.263	.414	.6
14	119	31	.034	.222	.2	−.324	.440	.7
15	105	29	.000	.236	.0	−.387	.467	.8

** Significantly different from the normal frequency distribution at the 1 percent level.
* Significantly different from the normal frequency distribution at the 5 percent level.
[1] Diameter of the tree of average basal area.

have failed. It is common, on first glance, to identify an upland hardwood stand as uneven-aged, then upon closer examination, find it is even-aged with the typical, near normal, but wide-spreading diameter distribution (Table 3).

Since the stocking equation was computed from stands with a moderate range of diameters, necessary adjustments of stocking for stand structure are minor, if needed at all (Table 6). The equation underestimates the stocking of uniform stands by 0.9 to 4.4 percent and overestimates the stocking of irregular stands by 2.0 to 6.3 percent. Adjustments of this magnitude may be appropriate in research but are hardly justified in day-to-day field use. Furthermore, since the basic stocking equation for stands of average uniformity had a standard error of estimate of ± 29.6 milacres or ± 3 percent stocking it becomes even more apparent that stocking adjustments will rarely be necessary. Most fully stocked stands, whether uniform or irregular, will be within this range of stocking (97 to 103 percent) (Table 6). Only very young irregular stands are likely to differ from the average more than 5 percent, and such stands are extremely rare.

That differences in structure may generally be disregarded in field use is confirmed by other research. Maintaining upland hardwood stands to meet various stand-structure objectives has failed to show that structure is related to growth. For example, a study was established in an uneven-aged upland hardwood stand in southern Illinois to compare three structural classes—large, medium, and small tree structures—and three stand density classes—40, 60, and 80 square feet of basal area—on good and poor sites. The inverted J-shaped curve was the stand

Checking whether density determined from JAR mods to be adjusted for

stand structure, kurtosis, skewness, SD
if the ED parameter left out of the equation

TABLE 6. The effect of stand structure in upland hardwood stands on stocking percent. Milacres of stocking = $-.0507N + .1698\Sigma DN + .0317\Sigma D^2N$.

			Stands of average uniformity			Uniform stands[4]			Irregular stands[5]		
$\bar{D}_B$[1]	*N*	*BA*	*CV*[2]	$\bar{D}$[3]	Stocking percent	*CV*[2]	$\bar{D}$[3]	Stocking percent	*CV*[2]	$\bar{D}$[3]	Stocking percent
3	1430	70.2	45	2.74	100	24	2.92	104.4	68	2.48	93.7
4	928	80.9	43	3.67	100	23	3.90	103.7	66	3.34	94.9
5	657	89.5	42	4.61	100	22	4.88	103.0	65	4.19	95.3
6	492	96.5	41	5.55	100	20	5.88	102.7	64	5.05	95.8
7	383	102.5	39	6.52	100	19	6.88	102.3	62	5.95	96.3
8	308	107.5	38	7.48	100	18	7.87	102.0	61	6.83	96.6
9	253	111.9	37	8.44	100	17	8.87	101.8	60	7.72	96.9
10	212	115.7	36	9.41	100	15	9.89	101.7	58	8.65	97.2
11	180	119.0	34	10.4	100	14	10.9	101.5	57	9.56	97.4
12	155	121.8	33	11.4	100	13	11.9	101.3	56	10.5	97.6
13	135	124.4	32	12.4	100	12	12.9	101.2	54	11.4	97.7
14	119	126.6	31	13.4	100	11	13.9	101.0	53	12.4	98.0
15	105	128.9	29	14.4	100	10	14.9	100.9	52	13.3	98.0

The relation between $\bar{D}_B$, $\bar{D}$, and the standard deviation is simply $\bar{D}_B = \sqrt{\bar{D}^2 - S_D^2}$

[1] The diameter of the tree of average basal area $\bar{D}_B = \sqrt{\frac{\Sigma BA}{N} 183.35}$

[2] Coefficient of variation [standard deviation (S_D) of mean diameter in percent].

[3] Mean diameter ($\Sigma D/\Sigma N$).

[4] Stands with a small range in tree diameter (most tree diameters near the mean).

[5] Stands with a large range in tree diameter (typical of uneven-aged and two-storied stands).

structure objective. No significant relation between structure and growth was found—at least up to 6 years of growth—the latest measurements available.[4] Nelson (1964), working with 10-year-growth data for loblolly pine (*Pinus taeda* L.) using the Pearl-Reed growth curve and the gamma distribution, concludes that diameter-distribution functions offer low probabilities of markedly reducing unexplained variation in cubic-foot growth of managed stands.

The Stocking Chart

Since stand structure has little effect on stocking percent, the need for individual tree-diameter measurements is eliminated for practical field use, and a close approximation of stocking percent can be read from a graph containing only basal area and number of trees per acre (Fig. 4). Thus a count of 200 trees per acre and a basal area of 90 square feet per acre represent a stocking of 80 percent in a stand where the tree of average basal area is 9 inches in diameter. On the chart the line of 100 percent stocking, hereafter referred to as A-level stocking, represents a normal condition of maximum stocking

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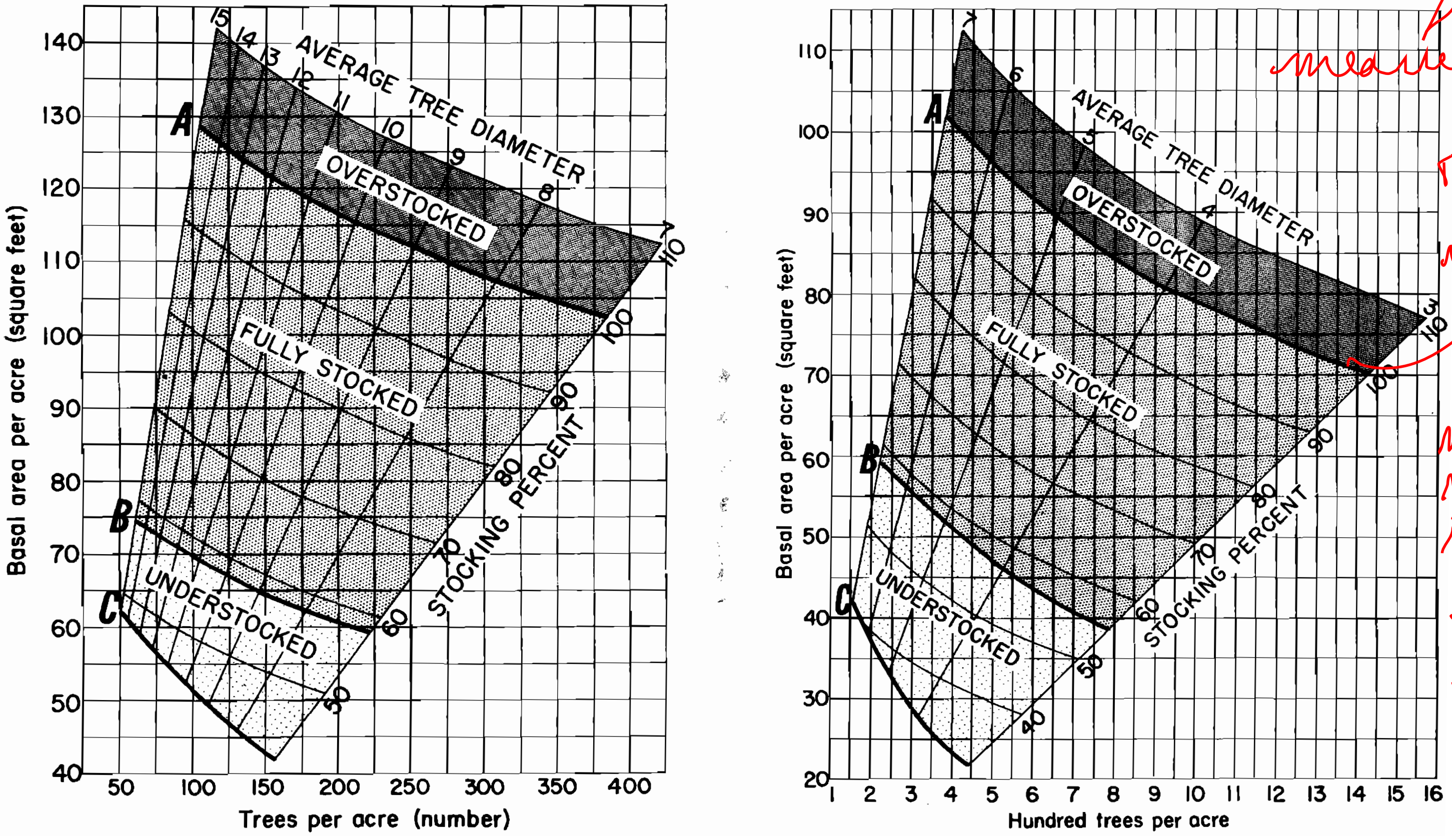
[4] Minckler, L. S. Six-year growth results of Fountain Bluff replication of upland stocking-structure study. 1961. (Report on study number FS–1–f7–4–3 on file at Northeastern Forest Expt. Sta., U.S. Forest Serv., Columbus, Ohio).

Percent stocking is relative to tree area ratio is a stand density measure



regression of normal stand,

no need to refine the stocking levels.

FIGURE 4. *Relation of basal area, number of trees, and average tree diameter to stocking percent for upland hardwood forests of average uniformity. Tree-diameter range 7–15 (left), 3–7 (right). The area between curves A and B indicates the range of stocking where trees can fully utilize the growing space. Curve C shows the lower limit of stocking necessary to reach the B level in 10 years on average sites. (Average tree diameter is the diameter of the tree of average basal area.)*

Basal area alone is close enough to stand density calculated w/ TAR, which is used to calculate the Relative den for undisturbed stands of upland hardwoods of average structure and was determined from the tree-area ratio for fully stocked stands as previously described. The significance of this line, in the absence of cutting or other disturbances, is that the trend of stocking, whether currently above or below the A level, is toward it. Thus any stand falling above this level is considered overstocked.

The tree-area ratio for open-grown trees was used to determine the lower imit of stocking for full-site occupancy. This level, hereafter called the B level, ranges from 55 to 58 percent of A-level stocking. Unlike normal yield table concepts, where full stocking is synonymous with average maximum stocking, the entire range in stocking between the A and B level is referred to as fully stocked because the growing space can be fully, and perhaps more efficiently, utilized within this range (assuming proper spacing among the trees).

Early growth measurements on the permanent stand-density plots confirm that the fully stocked range represents the limits where total wood production (cubic-foot or basal-area growth) is about equal. Obviously individual tree growth will differ within the fully stocked range. Dominant upland red oaks growing near B-level stocking will average 4 to 6 rings per inch whereas at A-level stocking growth is reduced to 10 to 12 rings per inch. Dominant upland white oaks average 8 rings per inch at B level and 16 to 18 at A level. Dominant hickories ranged from 10 to 25 rings per inch across the A–B range. Optimum stocking and growth for a given product or management objective lie somewhere between the broad A–B range of full stocking.

The C level of stocking (Fig. 4) represents the lower level of stocking necessary for a stand to reach B-level stocking in 10 years on average sites (site index 55–75 for oaks). Most of the study plots were within this range of site index, which includes most of the upland hardwood type. The time interval between the C- and B-level stocking may be as long as 12 to 15 years on extremely poor sites or as short as 5 to 8 years on very good sites. The trend toward full stocking is much faster in young stands where the number of trees is an important component of stocking. As stands become older, average tree size is a more important stocking component and the range between C- and B-level stocking decreases.

Conclusions and Recommendations

The tree-area ratio provides an acceptable index of stand density and stocking for upland hardwood forests where the predominant species are oaks and hickories. There are broad but definite limits of stocking and density within which the growing space is fully utilized from the standpoint of area occupancy and growth. Maintaining stands above or below these limits of stocking will result in a loss of growth.

Usable estimates of stocking and density can be obtained from a measure of basal area and a tree count. Where a high degree of accuracy is needed in measuring stocking the diameter of every tree in the plot or stand should be measured and the basic tree-area equation used to compute stocking. Otherwise stocking can be read directly from the chart.

The distribution of tree diameters in a stand has a minor effect on stocking and total growth. Only under extreme conditions of irregular species composition or age-class distribution can adjustments of the stocking estimate because of stand structure, be justified. Even-aged, upland, hardwood stands exhibit moderately skewed diameter distributions in young stands. As stand age increases the frequency distributions are normal but wide-spreading largely because of differential species growth.

Understocking in young stands is not serious provided the trees are well spaced.

Stands with an average tree diameter of 3 inches need be only 30 percent stocked (C level) in order to become fully stocked in 10 years on average sites.

About 40 percent of the basal area can be removed from stands that are 100 percent stocked without loss of total stand growth, and with a consequent increase in diameter growth of residual trees.

The predominant species of the upland, central-hardwood forests exhibit big differences in diameter growth under the same degree of stocking. Thus, there is a good possibility of increasing growth and yield by altering species composition.

Field use. The stocking chart presented has direct application for managing upland hardwood stands. In even-aged stands the chart is very accurate; in uneven-aged stands it is a very close approximation whose error will rarely be more than 5 percent and usually much less. This chart, or the two tree-area equations, provides limits of stand density and stocking within which stands should be maintained. For example, when maximum growth is the management objective, at no time would it be advisable to thin a stand below B level for to do so would result in a loss of growth. Also, stocking below B level encourages the development of bole branches on the merchantable stem resulting in lower quality trees. A diagnostic tally that separates the trees into various classes of basal area, together with a tree count, will provide the data needed to prescribe certain treatments—all of which might depend upon the management objective. An example of the application of these stocking and density standards can be found in a recently published timber management guide (Central States Forest Experiment Station 1962).

The tree count is based on the total number of living trees even though many of the trees counted contribute little to stand basal area. When a basal-area angle gauge or wedge prism is used a total tree count can be obtained by estimating the diameters of the trees tallied through the angle gauge and applying appropriate expansion factors by 1- or 2-inch-diameter classes. For example, every 4-inch tree tallied through a 10-factor angle gauge represents 115 trees per acre. Where fixed-radius plots are used the tree count is perhaps more arbitrary. Conditions will vary from stand to stand. Experience has shown that trees smaller than 3 inches in diameter, if present at all, can be ignored in sawtimber stands. In stands composed of pole-sized trees the count should include 2-inch trees. In all other stands, trees 1 inch in diameter should be counted. Shrubs such as dogwood, sumac, alder, sourwood, and so forth, should not be counted because they will seldom reach the main-canopy level.

Research use. The stocking criteria presented here will be useful in providing a more precise index of stand density for formal growth studies. Exact stocking conditions can be determined from the tree-area equation when individual tree diameters are measured—a conventional procedure in research involving permanent plots. In the past, basal area alone has often been used as a measure of stand density but, as has been shown, this is inaccurate. For example, a stand with an average tree diameter of 3 inches and 70 square feet of basal area is 100 percent stocked. Diameter growth is slow and competition for growing space is severe. But, a stand with the same basal area and an average tree diameter of 11 inches has the minimum stocking for full utilization of growing space (B level). Diameter growth is rapid and there is little competition for growing space. For research use, a standard of density is needed to describe uniform environments for tree growth with changes in time. Stocking percent provides this standard. Furthermore, stocking percents for stands of different ages and site qualities are comparable because a constant stocking percent allocates tree area on the basis of tree size.

Literature Cited

BEERS, T. W. 1960. An evaluation of site index for mixed oak in the Central Hardwood Region Diss. Abs. 21(1):11–12.

BICKFORD, C. ALLEN, et al. 1957. Stocking, normality, and measurement of stand density. (Report of S.A.F. Committee on Stocking). J. For. 55:99–104.

CENTRAL STATES FOREST EXPERIMENT STATION and NORTH CENTRAL REGION. 1962. Timber management guide for upland central hardwoods. U.S. Forest Serv., Cent. States Forest Expt. Sta. 33 pp.

CHISMAN, H. H., and F. X. SCHUMACHER. 1940. On the tree-area ratio and certain of its applications. J. For. 38:311–317.

KRAJICEK, J. E., K. A. BRINKMAN, and S. F. GINGRICH. 1961. Crown competition—a measure of density. For. Sci. 7:35–42.

LEXEN, B. 1939. Space requirements of ponderosa pine by tree diameter. U.S. Forest Serv., Southwest. Forest and Range Expt. Sta. Res. Note 63. 4 pp.

LYNCH, D. W. 1958. Effects of stocking on site measurement and yield of second-growth ponderosa pine in the Inland Empire. U.S. Forest Serv., Intermountain Forest and Range Expt. Sta. Res. Paper 56. 36 pp.

MEYER, W. H. 1930. Diameter distribution series in even-aged forest stands. Yale Univ. School of Forestry Bull. 28. 105 pp.

NELSON, T. C. 1964. Diameter distribution and growth of loblolly pine. For. Sci. 10:105–114.

SCHNUR, G. L. 1937. Yield, stand, and volume tables for even-aged upland oak forests. U.S. Dept. Agric. Tech. Bull. 560. 88 pp.

SCHUMACHER, F. X. 1928. Yields, stand and volume tables for red fir in California. Univ. Calif. Col. Agric. Bull. 456.

SMITH, DAVID M. 1962. The practice of silviculture (Ed. 7) John Wiley & Sons, Inc., New York. 578 pp.

TRIMBLE, GEORGE R., JR. 1960. Relative diameter growth rates of five upland oaks in West Virginia. J. For. 58:111–115.

U. S. FOREST SERVICE. 1963. Timber trends of the United States. Forest Resource Rpt. 17. 235 pp.

Volumetric Sampling of Atmospheric Pollen

Grano, Charles X. 1966. An Eight-Day Volumetric Pollen Sampler. U. S. Forest Serv. Res. Note SO-35, 4 pp., illus. Southern Forest Experiment Station, 701 Loyola Avenue, New Orleans, La. 70113.

This paper contains construction details and drawings for a sampler that provides accurate, continuous volumetric estimates of atmospheric pollen load. The sampler is simple to construct.