Forest Sci., Vol. 25, No. 3, 1979, pp. 518–532

Stand Density Management: an Alternative Approach and Its Application to Douglas-fir Plantations

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ABSTRACT. A method of viewing stand density as it relates to volume production and tree size is developed in the form of a simple *density management diagram* applicable to plantations of coastal Douglas-fir, *Pseudotsuga menziesii* (Mirb.) Franco, on all sites. Three prominent points in stand development, crown closure, imminent competition-mortality and the maximum size-density relationship are defined in terms of tree size and stand density, and their implication for stand dynamics is discussed. A relative density index is presented as a basis for quantifying tree growth and stand yield as a function of density. The trade-off between maximizing individual tree size or stand yield is considered; this recurring dilemma for forest managers can be rationalized on the basis of the *density management diagram*. FOREST SCI. 25:518–532.

ADDITIONAL KEY WORDS. *Pseudotsuga menziesii,* tree size, stand yield, management regimes.

IN AN EARLIER MANUSCRIPT (Drew and Flewelling 1977), we discussed the theoretical development of a general principle of plant population biology, the maximum size-density relationship, together with elements of yield-density theory, and introduced a concept of imminent competition-mortality. Here, we develop these concepts into an applied forest management tool, which will take the form of a simple modelling approach to stand development. Our model adds to some of the fundamental concepts of growth, yield, and stand density that have arisen in the fields of forestry, biology, or ecology through the last century.

Growth and development of forest stands can be forecast by a variety of models with differing objectives and degrees of complexity. These models can be incorporated into economic simulations which select optimum management regimes. Most models, with few exceptions, such as Stage's (1973) prognosis model, assume that random effects can be ignored; i.e., only mean trends for a class of stands are forecast. Thus, a consequence of this assumption is a possibility that the management regime chosen as being optimum for the mean stand is not necessarily the optimum regime for any particular stand. However, Adams and Ek (1974) have proposed using stochastic models, which admit to uncertainty, in the evaluation of management regimes. The uncertainty in growth and mortality patterns, particularly at high densities, are too important to be ignored. Stochastic growth models have not been developed to any great extent, possibly because

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foresters are generally more comfortable with the assumption of certainty (Bell 1977), needed error distributions are not known, or because analytical solutions are exceedingly difficult to obtain (Chapman 1967). The error distributions in growth and mortality are quite complex and cannot yet be determined from existing data bases with the precision required for the dynamic programming methods (Hool 1966, Lembersky and Johnson 1975) available in forestry.

We are proposing a simpler model which attempts to delimit stand conditions likely to result in particular patterns of growth and development. The error distribution in mortality trends is recognized, though not specifically defined. The resulting model, not a growth model *per se,* is similar in some respects to earlier hardwood stocking guides (Ginrich 1967, Leak and others 1969). This model should be easily comprehended by nonbiometricians and can, we feel, be extrapolated to untested management regimes with no more "heroic" assumptions than are required by the more complex approaches.

Stand density manipulation has the potential to make a major impact on individual tree size and stand yield. While the literature on spacing, thinning, and stand yield is voluminous, research to date has achieved little more than confirm ". . . what was formerly based on informal observation, namely, that there is an association between the initial spacing and various tree and stand characteristics . . . ." (Evert 1971). In 1971 Evert could state that spacing and thinning studies have contributed little to the assessment of differences in degree in tree size and stand yield that would allow decisionmaking for the achievement of any particular management objective. His statement is essentially true today, and though growth models for some species give good results over limited ranges of age and density, there is still no general framework for relating tree size, stand yield, and stand density.

Central to a discussion of stand density management is an index to quantify the effects of density (trees per unit area) on growth. Stand density indices, functions which are used to estimate the effects of density, have historically been comparisons of stands to reference stands, either in maximum stocking situations or at crown closure. Examples of these are, respectively, Reineke's (1933) stand density index and crown competition factor (Krajicek and others 1961). These and other indices are discussed by Curtis (1970), who regards them as having approximately equal utility. One drawback to many of these indices is that they relate stands in terms of diameter and do not reflect the fact that "the space a . . . tree can utilize is related to both its diameter and height" (Briegleb 1952). Briegleb's (1952) index, based on a standard number of trees per acre by average diameter and average height, is highly regarded by Worthington and Staebler (1961) and Vezina (1964). However, Briegleb's conclusion, that for stands with the same mean diameter the taller stand can support a greater number of trees, may be subject to dispute: his conclusion is based on observed stand structures immediately after thinning and cannot reflect tree space requirements or growth potential.

We define a *relative density index,* ρ_r, as the ratio of actual stand density to the maximum stand density attainable in a stand with the same mean tree volume. Because tree volume varies with both height and diameter, our density index reflects the same factors as does Briegleb's (1952) index; however, the effect of a greater height for a given diameter is viewed differently. The proposed density index is equivalent in concept to a density indexing system proposed by Tadaki (1964), first called a "management base line" and subsequently "relative density." Tadaki's index is proportional to the ratio of a stand's mean tree volume to the maximum mean tree volume attainable at the same density. Though our relative density index and Tadaki's index have identical utility from a computational viewpoint, the relative density which we have defined is directly propor-

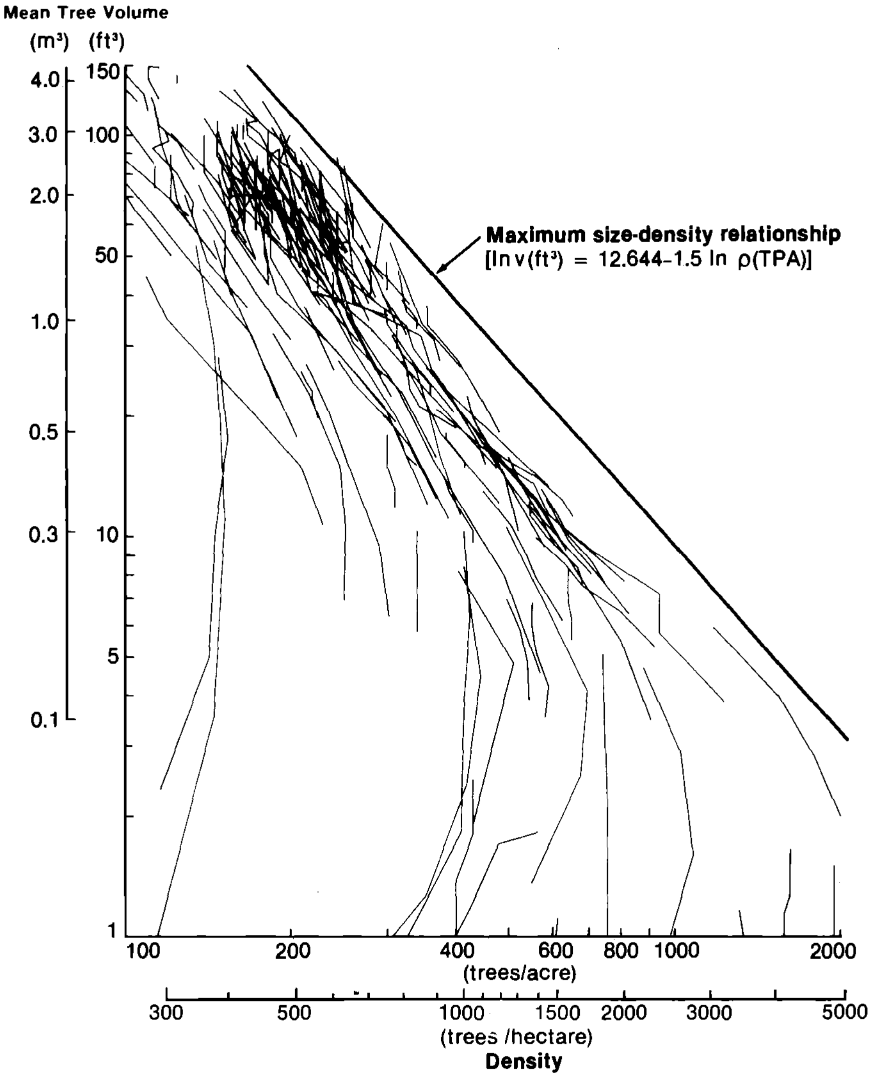

Figure 1. The maximum size-density relationship and the natural stand data used in positioning this relationship.

tional to density. Thus, the new index may be more useful than Tadaki's for understanding the effects of density manipulation.

The model discussed here is partly quantitative and partly conceptual; above all it is simple. The approach was adopted in place of other modelling alternatives in part because of its ease of comprehension, especially by nonbiometricians, and its applicability to a wide range of stand conditions.

THE DENSITY MANAGEMENT DIAGRAM FOR DOUGLAS-FIR

The *density management diagram,* a graphical tool for relating stand density, tree size, and stand yield, is a graph of mean tree volume and stand density on which the following relationships have been superimposed.

Maximum Size-Density Relationship.—We accept the concept of maximum size-density as a general principle of plant population biology: in pure even-aged stands, the maximum mean tree size attainable for any density can be determined by a relationship known as the $-3/2$ power law.

$$v = a\ \rho^{-3/2} \tag{1}$$

where

v = mean tree volume
a = constant
ρ = stand density.

A review of the derivation and an example of the application of this relationship is presented by Drew and Flewelling (1977).

There is no rigorous statistical procedure available for selecting the limiting boundary of a zone, given that some unknown random variation is to be expected. We chose to position the maximum size-density relationship by the following procedure. Volume computations for 1 to 9 repeat measurements of 313 growth and yield plots in natural stands of coastal Douglas-fir, *Pseudotsuga menziesii* (Mirb.) Franco, from Washington and Oregon were graphically summarized on log-log scale (Fig. 1). The data base is essentially that described by King (1970), with some additions. The maximum size-density parameter in Equation 1 was selected for this data set by positioning a line of $-3/2$ slope near the upper limit of the data. The resulting relationship is

$$\ln v = 12.644 - 1.5 \ln \rho \tag{2}$$

where

ln = natural logarithm
v = mean individual tree volume (cubic feet)
ρ = stand density (stems per acre).

For volume in cubic meters and density in trees per hectare the intercept term in this equation becomes 10.437. These coefficients should not be considered significant to five figures. The estimate of a limit, either an upper bound or the division between two regions, is generally less precise than the estimate of a mean. Without prior knowledge of the distribution of random errors near the maximum size-density relationship, there can be no precise unbiased estimator of the upper bound. Parker (1978) proposed using a coefficient obtained from a growth relationship as an estimator of an upper limit; we felt this approach would run counter to our goal of producing as simple a model as possible.

Verification of the approximate positioning of this relationship can be concluded from a comparison with Reineke's (1933) line of maximum number of trees versus quadratic mean diameter. To compare Reineke's relationship for Douglas-fir ($\ln \rho$[TPA] = 10.03 − 1.605 ln $\overline{\text{DBH}}$ [inches]) and our maximum size-density relationship, Equation 2 was modified to predict mean diameter instead of mean volume by incorporating a mean diameter-volume relationship which is discussed later in this paper. The predicted upper limits for mean diameters are in agreement at high densities; at the low density of 200 trees per acre (494 trees per hectare) the maximum size-density relationship predicts a mean diameter 6 percent lower than Reineke's equation. The concept of the maximum size-density relationship and the estimating equation are believed to be approximately correct for coastal Douglas-fir in most even-aged stand conditions.

Imminent Competition-Mortality.—The zone of *imminent competition-mortality,* introduced by Drew and Flewelling (1977), is viewed as that array of stand conditions where competition-related mortality is likely to occur. The zone is bounded by two lines: the maximum size-density relationship and a second line paralleling the first at lower densities for the same mean tree size. In plantations of *Pinus radiata,* the lower line was positioned for stands of a particular mean tree size at a density 54 percent of that indicated by the maximum size-density relationship: a *relative density* of 0.54. Since the lower bound of the zone of imminent competition-mortality had been estimated at relative density 0.54 for *P. radiata,* a relative density of 0.55 is used here as our first estimate of this lower bound for Douglas-fir. This is one of the three parameters in this manuscript which has been rounded. A sufficiently large data base of repeat measurements from Douglas-fir plantations was not available to prove or disprove this hypothesis; however, an examination of several repeat measurements from plantations of Douglas-fir suggests that the hypothesis is reasonable. The basic concept of a zone of imminent competition-mortality for a relatively uniform stand is that while any stand with a mean tree size and density below the lower bound may have mortality, the probability of any tree dying would be unchanged by lowering the stand density. Conversely, for a stand within the zone of imminent competition-mortality, the probability of any tree dying would be reduced if the density were substantially lowered. If a stand is allowed to grow for many years within the zone of imminent competition-mortality, mortality will occur.

Mortality cannot be precisely predicted on the basis of tree size and stand density, because stand density is not a causal agent of mortality. Mortality is due to environmental, pathological, or entomological factors, which may impact on stands at any point in their development, but which are much more likely to occur in stands whose vigor is declining. A decline in vigor can be characterized by a slower than maximum individual tree growth rate. This starts to occur after crown closure, accelerating as stands approach the maximum size-density relationship. Spurr (1962) recognized the association between declining vigor and mortality of Douglas-fir in New Zealand: "Whatever factor or factors administered the final *coup de grace* did so to trees destined to die on the basis of declining vigor." If stands are to be managed without likelihood of significant mortality, the zone of imminent competition-mortality—an indicator of reduced stand vigor and potential mortality—should be considered.

Crown Closure.—Crown closure is often used to approximate the initiation of stand development, *per se,* as opposed to the growth of noncompeting trees. Strub and others (1975) demonstrated the validity of this assumption for loblolly pine. Estimates of the tree size-density relationship at the point of crown closure can be made from observations of crown width for open-grown trees and simple assumptions concerning crown shape and spacing. The crown diameter (CD) of open-grown trees can be described as a linear function of diameter at breast height (DBH) over a limited range. This relationship was fit to data discussed by Dick[1]:

$$\text{CD (feet)} = 3.786 + 1.753\ \text{DBH (inches)}. \tag{3}$$

This equation covers a DBH range of 1 to 17 inches (2 to 43 cm), and has a standard error of 2.3 feet (0.7 m) and a coefficient of determination (R^2) of 0.82; the metric coefficients are 1.154 and 0.2104 respectively for DBH in centimeters and crown diameter in meters.

We are considering crown closure as occurring when the entire area of the stand is first covered by crown; this occurs with the least crown overlap if spacing is triangular (Assmann 1970). Assuming triangular spacing and utilizing the crown

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[1] Dick, J. 1956. Another approach to desirable stocking for Douglas-fir. Unpublished report, Weyerhaeuser Company, Centralia, Washington.

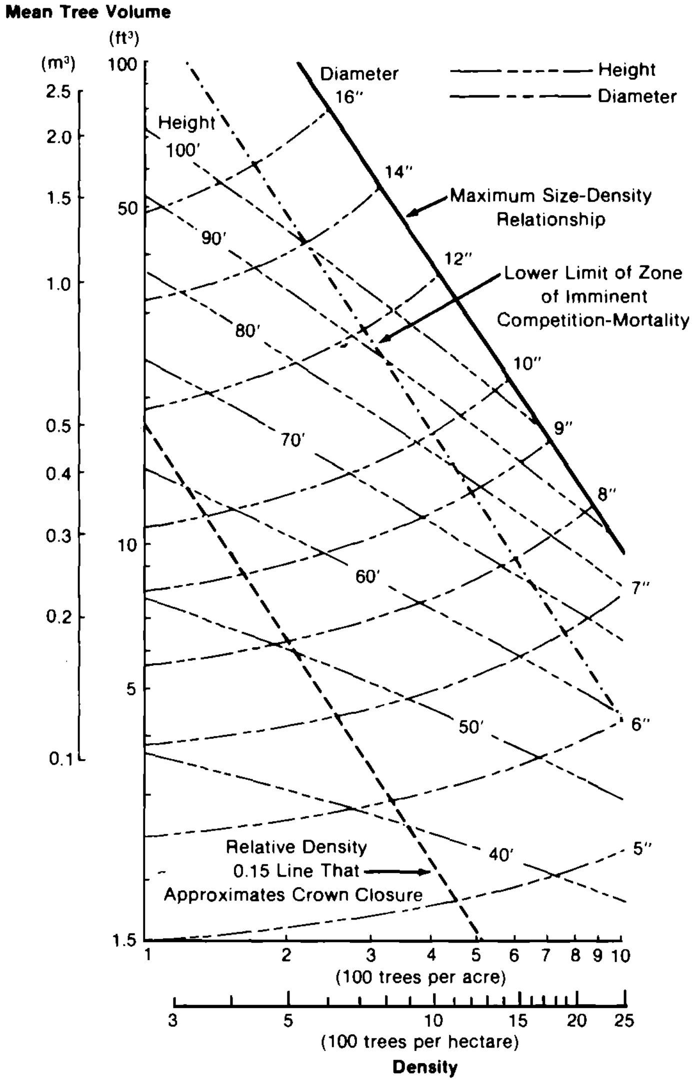

Figure 2. Stand management diagram for Douglas-fir with estimates of diameter and height.

diameter relationship (Equation 3), the maximum number of equally sized trees at crown closure can be estimated for any mean diameter. A relationship between mean diameter and mean volume, which is presented later, allows a crown closure line to be positioned on a graph of mean volume versus density. In the density range of 100 to 600 trees per acre (247 to 1483 trees per hectare), the relative density at crown closure varies from 0.17 to 0.13. We will therefore approximate crown closure, and the onset of competition, as corresponding to a relative density of 0.15.

Estimate of Diameter and Height.—The three relationships—maximum size-density line, lower bound of the zone of imminent competition-mortality, and the line at a relative density of 0.15 (crown closure)—are the basis of our *density management diagram* (Fig. 2). In order to better relate this diagram to actual unthinned stands of Douglas-fir, estimates of mean diameter (DBH) and site height (H) are related to the mean volume-density conditions (Fig. 2).

The diameter estimation equation used,

$$\overline{\mathrm{DBH}} = (68.682\, v - 6.8084)^{0.36716}\,(1 - 0.32375\rho^{0.44709}), \tag{4}$$

was generated as a nonlinear least squares fit to data from 241 measurements on 48 plantation growth plots. The range in mean tree volumes was from 1 to 50 cubic feet (0.02 to 1.4 cubic meters) and in mean diameters from 2.7 to 14.7 inches (7 to 37 cm). In metric units, DBH in centimeters can be predicted from Equation 4 if the first two coefficients are changed to 30719 and 86.23, respectively. The standard error of estimate is 0.3 inches (1 cm). Site height as defined by King (1966) has also been estimated for points on the *density management diagram* using the mean diameter equation (Equation 4) and a mean tree volume equation generated from the same data base:

$$v = (0.008695 + 0.0007764\, \overline{\mathrm{DBH}}^{2.1987})\mathrm{H}^{1.10319}. \tag{5}$$

The standard error of estimate is 0.4 cubic feet (0.01 cubic meters). The metric equivalent of Equation 5 has, for the first two coefficients, 9.131×10^{-4} and 1.045×10^{-5}, respectively. Equation 5 was then solved to find site height.

The observation that the diameter-volume relationship changes with density should be no surprise. While diameter growth decreases with increasing density, height growth is relatively unaffected by changes in density. Utilizing this assumption we would expect a dense stand to be taller than an open-grown stand of the same mean DBH because the dense stand requires more time to reach this DBH: stands with the same mean DBH will have differing heights depending on density. Since mean tree volume is roughly proportional to height for a given mean DBH (Equation 5), the diameter-volume relationship should and does follow the diameter-height relationship discussed above.

Relative density index.—Throughout the past two centuries there have been a number of recurring themes as to the effect of stand density or stand density manipulation on yield. Currently, the thesis that yield is unaffected by density over a broad range of densities is widely accepted. In 1811, Reventlow, a Danish forest owner "asserted that heavy thinning would increase the annual increment, increase the rate of interest on the growing stock, and increase diameter growth" (Heiberg 1954). His conclusions have in general, been accepted except that his predicted increase in annual increment for heavy thinning is now considered to be too optimistic and ". . . it is felt that the degree of thinning, even within wide limits, has no influence upon the average increment over an extended period of time" (Mar: Möller 1947). A conviction similar to Mar: Möller's has continued to be expressed in the forestry literature (Craib 1939, Briegleb 1952, Mar: Möller 1954, Staebler 1955 and 1960, Spurr and others 1957, Gruschow and Evans 1959, Smith 1962, Tadaki 1964, Madgwick 1977). The unresolved question, as pointed out by Staebler (1955), is the range of densities over which full gross production may be achieved.

The relationship between density and yield will not be resolved until a general framework relating these variables has been developed and conceptualized in a manner that allows ideas and experimental evidence to be transferred from one experiment to another, from one region to another, and even from one species to another. We propose to use relative density, as defined earlier, for this purpose. Two authors, Briegleb and Tadaki, have drawn general conclusions on how stand growth is related to density indices that are comparable to relative density. Briegleb (1952), in discussing the management of Douglas-fir, stated, "Preferably, density should never fall below 90 percent of the standard (about 0.40 to 0.50 relative density) . . . . Indications are that at least this amount of growing stock will be required to obtain the growth that the site is capable of producing . . . .

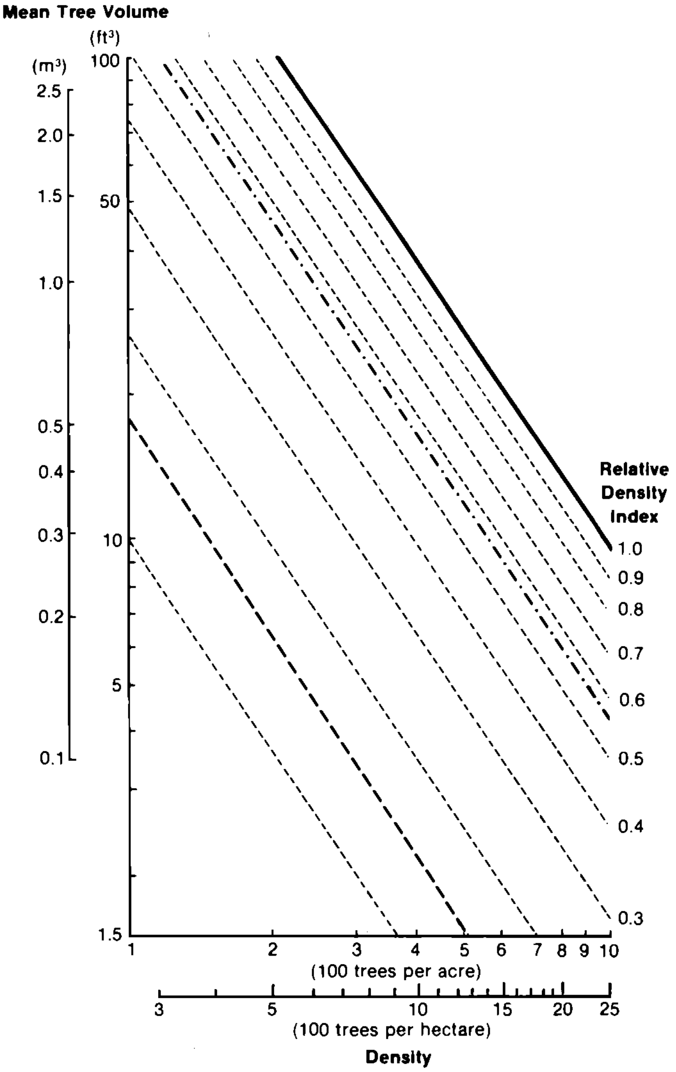

Figure 3. Relative density indices for Douglas-fir.

It is believed that the optimum densities for most combinations of factors will be found to be between 75 and 120 percent of the proposed standard, (relative densities 0.34 and 0.55)." Tadaki (1964), using simulated yield projections, concluded that for stands with thinning regimes not falling below densities equivalent to ρ_r 0.41 the total yields (net plus thinnings) were not particularly affected by the degree or interval of thinning.

A quantitative concept of growth as a function of density can now be stated. At densities below crown closure (less than 0.15 relative density) growth per unit area is proportional to density. At relative densities between 0.15 and 0.40, growth per unit area increases with density, but growth per tree declines. At relative densities between 0.40 and 0.55, growth per unit area is unaffected by density. For relative densities greater than 0.55, gross growth is the same as in the 0.40 to 0.55 region, but net growth may be considerably less than this if substantial mortality has occurred (Fig. 3). These conclusions are very similar to a theory

Table 1. Yields from a Douglas-fir thinning experiment at Golden Downs State Forest, New Zealand.

	Treatment					
	Control		Light Thinning		Heavy Thinning	
Final Density, TPA (TPH)	643	(1588)	250	(617)	150	(370)
Final Mean Tree Volume, ft^3 (m^3)	23	(0.64)	37	(1.04)	48	(1.34)
Final Total Volume, cunits/acre (m^3/ha)	150	(1050)	92	(643.7)	72	(503.8)
First Thinning, cunits/acre (m^3/ha)			3	(21.0)	3	(21.0)
Second Thinning, cunits/acre (m^3/ha)			14	(98.0)	22	(153.9)
Third Thinning, cunits/acre (m^3/ha)			26	(181.9)	25	(174.9)
Final Volume + Thinning, cunits acre (m^3/ha)	150	(1050)	135	(944.6)	122	(853.6)
Percent yield relative to unthinned net yield			90			82
Percent yield relative to unthinned gross yield	94		85			77

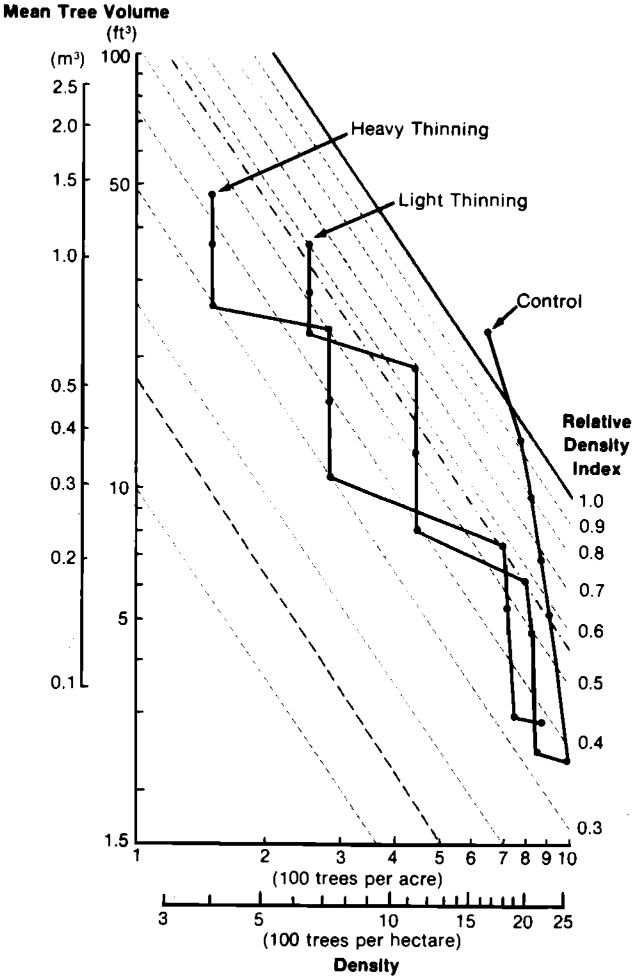

Figure 4. Size-density trends for three management regimes in a Douglas-fir plantation at Golden Downs, New Zealand.

proposed by Langsaeter (1941 cited by Smith 1962). The major difference is that we have quantified and positioned the bounds of the density types described by Langsaeter.

Assumptions.—The construction and interpretation of the *density management diagram* require certain assumptions:

1. The location of the maximum size-density relationship is correct for all sites. Though the exact position of this line is difficult to determine, earlier Japanese work indicates a single line applies to all sites.
2. The lower limit of the zone of imminent competition-mortality is correct for all sites. This assumption is untested.
3. Individual tree growth is not related to stand density prior to crown closure which we approximated by the ρ_r 0.15 line.
4. Stand growth is at a maximum in the ρ_r 0.40 to 0.55 region. Both bounds are approximate; the region of maximum stand growth may be wider.
5. The height and diameter trends shown in Figure 2 are approximately correct for unthinned plantations on all sites. The data indicate that any biases are small.
6. Following thinning, a stand's growth potential temporarily falls below that indicated by its relative density. This falldown is assumed to be short-lived.
7. Within reasonable limits, the uniformity of tree distributions does not affect stand volume growth. This assumption is supported by a recent study of thinning in red pine (Stiell 1978).

MANAGEMENT APPLICATIONS OF THE *DENSITY MANAGEMENT DIAGRAM*

Douglas-fir Plantations in New Zealand.—Density manipulation in a Douglas-fir plantation at the Golden Downs State Forest,[2] New Zealand (Table 1) is characterized by size-density trends for three 0.4-acre plots: a control and two thinning treatments that are displayed on the *density management diagram* (Fig. 4). The densities at which the stands were managed are negatively correlated with final mean tree size. The least dense plot produced trees which, at harvest, were over twice the size of trees on the dense plot. Net yields including thinning were decreased by the thinning operation; the heavily thinned plot yielded 82 percent of the control volume. This falldown is larger than experience suggests and could be due to random plot to plot variation in growth rate. The fact that the control plot is above the maximum size-density line is also unexpected; we assume that this plot is in an unstable condition and predict that significant mortality will occur in the near future.

Yields from a Douglas-fir thinning experiment at Compartment 1153 of the Kaingaroa State Forest (Spurr 1963) are summarized (Table 2) and are overlayed on the *density management diagram* for Douglas-fir (Fig. 5). The plot with heavy thinning drops after thinning to a relative density index of 0.30; yield from this plot is 97 percent of the gross yield in the unthinned plot. Only the unthinned plot had any significant mortality, for this plot had been allowed to proceed unchecked into the zone of imminent competition-mortality. A theoretical regime can be designed to capture total gross yield, as shown in Figure 5. This experimental evidence is in accord with the biological relationships discussed earlier.

Basic Tenets.—We propose four tenets to be used in conjunction with our *density management diagram* as a basis for the manipulation of stocking in plantations:

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[2] Data provided by Bryan Johnson, New Zealand Forest Service.

Table 2. Yields from a Douglas-fir thinning experiment at Compartment 1153, Kaingaroa State Forest, New Zealand (Spurr 1963) and a theoretical regime for optimum yield.

	Treatment							
	Unthinned Control		Light Thinning		Heavy Thinning		Theoretical Regime for Optimum Yield	
Final Density, TPA (TPH)	444	(1096)	156	(385)	112	(276)	40	(346)
Final Mean Tree Volume, ft³ (m³)	29.9	(0.84)	53.1	(1.49)	61.6	(1.72)	60	(1.68)
Final Total Volume, cunits/acre (m³/ha)	133	(930.6)	83	(580.7)	69	(482.8)	83	(580.7)
First Thinning, cunits/acre (m³/ha)	–		18	(125.9)	29	(202.9)	15	(104.9)
Second Thinning, cunits/acre (m³/ha)	–		30	(209.9)	33	(230.9)	17	(118.9)
Third Thinning, cunits/acre (m³/ha)	—						19	(132.9)
Final Volume + Thinning, cunits/acre (m³/ha)	133	(930.6)	132	(923.6)	131	(916.6)	134	(937.6)
Percent yield relative to unthinned net yield			99		99		101	
Percent yield relative to unthinned gross yield	99		98		97			

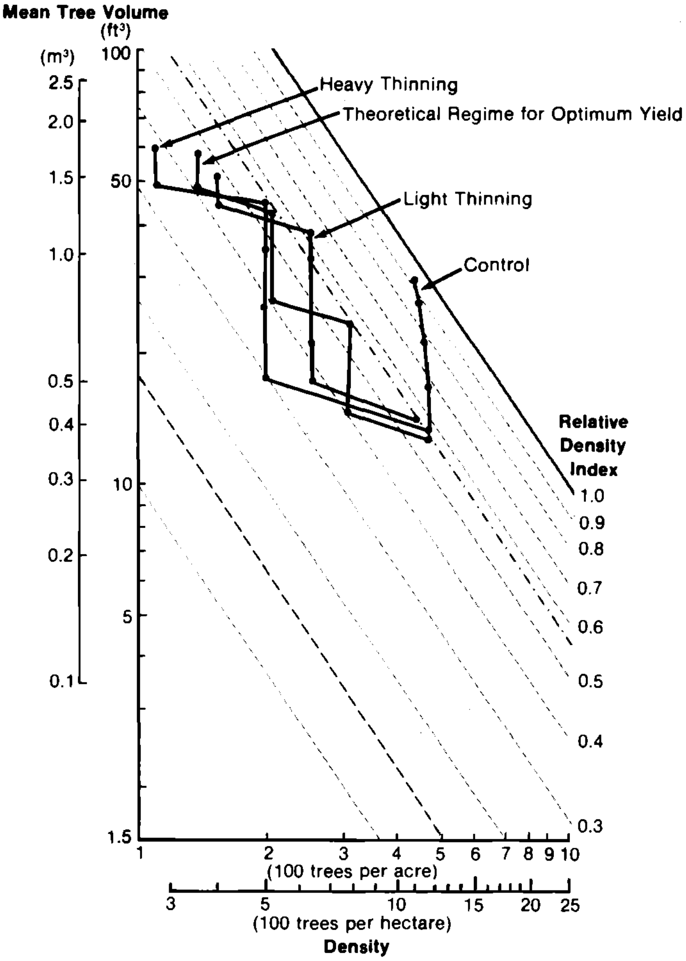

Figure 5. Size-density trends for three management regimes in a Douglas-fir plantation at Kaingaroa, New Zealand (Spurr 1963) and a theoretical regime for optimum stand yield.

1. Stands growing below the crown closure line (ρ_r = 0.15) are not fully utilizing the site, and density could be increased without decreasing mean tree growth.
2. Maximum tree size can be obtained by managing stands near or below the crown closure line (ρ_r = 0.15).
3. Stands managed near the lower bound of the zone of imminent competition-mortality down to a relative density of 0.40 will have somewhat greater total stand growth but considerably smaller individual tree sizes than stands managed at lesser densities.
4. Stands should not be allowed to enter the zone of imminent competition-mortality (ρ_r = 0.55) until several years before the final harvest in order to avoid a severe reduction in vigor and potential damage to the crop trees.

Managing stands in the range of 0.15 to 0.40 relative density will probably become more prevalent even though some growth per unit area will be sacrificed. No one has yet solved the problem of how to remove modest numbers of small trees at a reasonable cost. The practical alternative, namely reducing the frequency and increasing the intensity of thinning, will necessitate that the forest manager resolve the dilemma of having a profitable intermediary source of raw material and yet, by doing so, causing a reduction in total yield. For Douglas-fir growing in Compartment 1153 of the Kaingaroa State Forest (Spurr 1963) and displayed in Table 2 and Figure 5, if stand size-density conditions are not reduced in thinning to below a ρ_r of 0.30, the loss in total yield is only one percent of the final net yield of the unthinned control.

Planning Research Experiments.—The Blue Mountain thinning experiment is a recent study of the effect of thinning and fertilizer on the growth of a Douglas-fir plantation in western Washington. The study, established in 1962, was located in a 16-year-old well-stocked plantation growing on high site land. Five-year results are presented by Steinbrenner (1967). The *density management diagram* (Fig. 6) is used to aid in interpreting this experiment, and to show that the response to thinning is similar to what might have been predicted. The basic results are computed from the 1973 remeasurement of the plots (Table 3). It is apparent that the heavily thinned plots yielded substantially less than the unthinned plots. Thinning treatments were applied with an objective of not altering the diameter distribution and did not result in maximum mean tree sizes. If the control plots were now thinned from below to the same volume as the heavily thinned plots, a mean tree size could be obtained which exceeds that of the heavily thinned plots. Further, the thinning volume would be two and one-half times that obtained on the heavily thinned plots—a clear indication that all thinning treatments were too early or too severe to maximize yield. The utility of the *density management diagram* in selecting reasonable thinning treatments and planting densities should thus be apparent.

Further Development.—The *density management diagram* that we have developed is not a growth model *per se* for, at this stage of refinement, it lacks a time dependent treatment of stand yield. When utilized with the four basic tenets of forest management which are discussed in this paper, the *density management diagram* can be used as a basis for designing stand density regimes with emphasis on particular tree size or stand yield objectives. The approach is not time dependent and therefore cannot be used in forest management as a basis for making ordinal, economic decisions.

Consideration is now being given to the derivation of a growth and yield model that is compatible with the *density management diagram* approach. The basic hypothesis of the model now under development incorporates the concepts of Langsaeter (1941) and the *density management diagram.*

Table 3. Unfertilized yields for the Blue Mountain Douglas-fir thinning experiment (three plots per treatment).

	Treatments							
	Control		Light Thinning		Medium Thinning		Heavy Thinning	
Final Density TPA (TPH)	440	(1086)	300	(741)	253	(625)	127	(314)
Final Mean Tree Volume ft^3 (m^3)	9.58	(0.27)	11.31	(0.32)	10.12	(0.28)	13.70	(0.38)
Final Total Volume, cunits/acre (m^3/ha)	42	(291)	34	(235)	26	(180)	17	(118)
Thinnings, cunits/acre (m^3/ha)	0		4	(28)	5	(35)	10	(69)
Final Yields + Thinning, cunits/acre (m^3/ha)	42	(291)	38	(263)	31	(214)	27	(187)

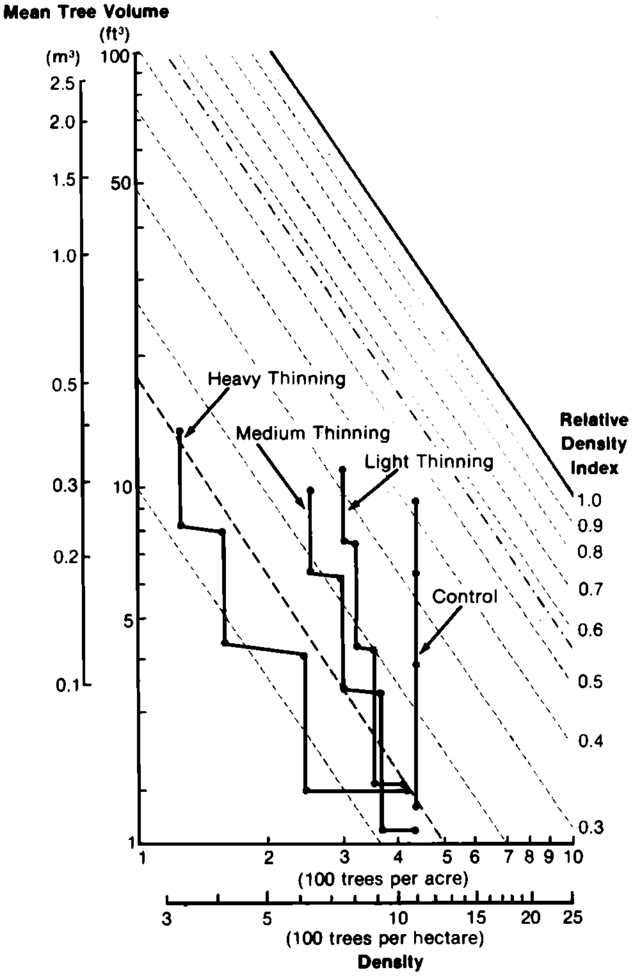

Figure 6. Size-density trends for four thinning regimes in a Douglas-fir plantation at Blue Mountain (western Washington).

SUMMARY

A graphical tool for relating stand density to tree size and stand yield is developed in the form of a simple *density management diagram* applicable to coastal Douglas-fir plantations. A *relative density index* is defined as the ratio of stand density to the maximum attainable density for a stand of the same mean tree volume; this maximum density is estimated from the $-3/2$ power law. Crown closure occurs at relative densities of close to 0.15, and the zone of imminent competition-mortality is bounded by relative densities of 0.55 and 1.00. Stands should be managed, for most of their postestablishment development, in the relative density range between 0.15 to 0.55; little mortality will occur in this region. Maximum gross production is obtained at relative densities greater than 0.40. At lower densities in the relative density range of between 0.15 and 0.40, less growth per unit area is obtained, but this will be offset by greater growth per tree. Management regimes tailored to different management objectives ranging from maximum fiber production to saw-log production in the shortest possible time, can be described and implemented from this single *density management diagram*.

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ERRATUM

A Method for Visual Estimation of Leaf Area, by B. A. Carbon, G. A. Bartle, and A. M. Murray, Forest Science 25(1):53–58 (March 1979).

The printer omitted measurements for Figure 1 which appeared on pages 56–57. The measurements, which should be written under the proper photographs, are as follows:

Page 56, Figure 1:

upper left	4.4 m^2
upper right	3.2 m^2
lower left	2.3 m^2
lower right	1.4 m^2

Page 57, Figure 1, continued:

left	0.5 m^2
right	0.08 m^2