Using Simple Marginal Analysis and Density Management Diagrams for Prescribing Density Management

Thomas J. Dean *and* **S. Joseph Chang,** *School of Renewable Natural Resources, Louisiana Agricultural Experiment Station, Louisiana State University Agricultural Center, Baton Rouge, LA, 70803.*

ABSTRACT: *This article presents a procedure to produce management regimes that not only maximize land value but also reflect stand development with simple marginal analyses of the accumulation and control of growing stock. An upper limit of growing stock is determined by translating management objectives into a future desired structure, and with this value as a guide for thinning age, marginal analysis is used to determine the planting density and the residual basal area after thinning. The procedure is demonstrated for a hypothetical loblolly pine plantation growing on land with a site index of 65 ft at 25 yr. The effects of various interest rates for a fixed rotation length and various rotation lengths for a fixed interest rate on initial planting density and residual growing stock after low thinning are analyzed. Optimal planting density decreased with increasing interest rate and rotation length. Thinning ages increased as initial planting density decreased, which caused optimal residual growing stock to increase with increasing interest rate and rotation age. According to this study, maximum land value is achieved when the growing stock limits are set to approximately the lower limit of full-site occupancy and the threshold of self-thinning. In terms of relative density, the ideal limits in growing stock for maximizing land value identified in this study are 35 and 55% of maximum SDI. South. J. Appl. For. 26(2):85–92.*

Key Words: Loblolly pine, interest rates, growing stock, thinning, basal area, stand density index, planting density.

Manipulating and controlling growing stock is often critical in meeting the goals and objectives of forest landowners. However, translating these goals into initial planting density, thinning regimes (if any), and rotation length to meet the requirements of specific management situations of landowners can be a complex process. Density management diagrams have greatly simplified the task of prescribing planting rates and thinning schedules once growing stock objectives and desired stand conditions have been specified because they derive from the density-dependent nature of average tree size (Drew and Flewelling 1977) and embody the natural stages of stand development (Long and Smith 1984). For species that supply a mix of products, however, they do not provide the yield information necessary for economic analysis. Optimization algorithms such as dynamic programming can find economically optimal solutions to density management questions for sets of conditions and constraints, but foresters usually do not have the training or the technical capacity to run these programs. Furthermore, these algorithms must use an extraordinary small time interval between possible management activities to accurately incorporate the chronology of stand development into the analysis; processing time for finding optimal planting rates and thinning schedules increases geometrically with shorter time intervals.

The central task in determining density management prescriptions that optimize land expectation values is finding the ideal combination of planting density, thinning schedules, and rotation length for given stand conditions, financial parameters, silvicultural constraints, and landowner objectives. Prescriptions determined solely from density management diagrams are site specific and can accommodate various constraints, but financial considerations are difficult to incorporate explicitly into the prescriptions. Marginal analysis is a simple means of addressing financial goals but can be extremely inefficient without

NOTE: Thomas J. Dean can be reached at Phone: (225) 578-4216; Fax: (225) 578-4227; E-mail: fwdean@lsu.edu. The authors are indebted to all of the students who have taken our silviculture prescriptions course and endured the development of this technique. Our apologies to the classes of 1992 through 1997 for not finding MERCHLOB sooner. We also thank Brian Roy Lockhart, Scott D. Roberts, and James N. Long for reviewing earlier drafts of this article. Manuscript received August 17, 2000, accepted April 22, 2001.

some guiding parameters, especially with thinning. Density management prescriptions developed with the integration of marginal analysis and density management diagrams necessarily compromise the optimal achievement of biological and economic goals, but contain the necessary components to produce desired stand conditions and nearly optimal profits.

We have developed a relatively simple procedure that not only produces nearly optimal density management regimes for maximizing profit but also reflects the biological development of the stand. The procedure integrates the population behavior of a stand as reflected by the density management diagram and the economic analysis of density management with simple marginal analysis. We briefly describe the components of a density management diagram and summarize its use in setting growing stock guidelines that are consistent with the implicit objective of the marginal analysis, which is maximizing profit through analysis of the variable costs of planting or thinning and variable benefits realized at rotation. We describe the application of marginal analysis to density management and discuss its results within the context of forest size–density relations. A fairly general case for the western Gulf region is used to illustrate the integration of density management diagram and marginal analysis for density management: a loblolly pine plantation growing on a site with a site index of 65 ft at base age 25 yr.

Methods

Density Management Diagrams

The use of density management diagrams for controlling growing stock, setting initial planting densities, and developing thinning schedules has been described in detail by various authors (Drew and Flewelling 1979, Long 1985, Dean and Baldwin 1993). Growing stock is a combined function of average tree size (usually in terms of quadratic mean diameter) and absolute tree density (in terms of trees per acre) (Jack and Long 1996) and is denoted by the relative distance to the species-specific, uppermost boundary between tree size and absolute density (Curtis 1970). On a density management diagram, constant values of growing stock are represented as series of lines parallel to the 100% relative density line (Figure 1). Lines representing site height (the average height of the dominants and codominants) also are included in the diagram so that age can be determined with the appropriate set of site index curves.

Dean and Baldwin (1993) associate the transition between stages of stand development in loblolly pine plantations with values of relative density. The three key values of relative density for loblolly pine plantations are 25, 35, and 60% of 450, the maximum value of Reineke's stand density index (SDI) [1] for loblolly pine (Reineke 1933). These values correspond to canopy closure, full-site occupancy, and the threshold of self-thinning, respectively. Each of these stages has characteristic structures and properties, and one of the first steps in using the density management diagram is translating management objectives into a specific description of the desired stand conditions and properties at the end and during the course of the rotation. The desired stand condition is achieved by specifying appropriate upper and lower limits of growing stock. Stands managed just below the self-thinning threshold have small average diameters but high growth rates per unit area; stands managed just above canopy closure have large average diameter but low growth rates per unit area. The distance between the limits determines the thinning intensity and the time interval between thinnings, and it must be wide enough to produce a minimum, operational volume. When the stand grows to the upper limit, it is thinned to the lower limit. This is repeated until the final desired stand condition is obtained or the technical specifications for the rotation are achieved. An increase in average stand diameter after thinning is characteristic of low thinning.

For this demonstration, the upper and lower growing stock limits will be set near the self-thinning threshold and the lower limit of full-site occupancy, that is, 60 and 35% of maximum SDI, respectively. These limits will maximize volume growth per acre and minimize density-related mortality. If a cushion is required between the time the plantation is scheduled for thinning and when it reaches the self-thinning threshold, the upper limit could be lowered to 55% of maximum SDI. If this results in insufficient thinning yield, the lower limit may need to be lowered. Rarely should the lower limit be set below canopy closure. The technical rotation for this example is an average stand diameter of 10 in. According to the diagram, this will occur when the site height reaches 71 ft (Figure 1). According to the site index curves developed by Baldwin and Feduccia (1987) and graphed by Dean and Baldwin (1993), it will take 30 yr for site height to attain 71 ft on land with a site index of 65 (base age 25 yr). The plantation will require one thinning to stay within the specified growing stock limits. The plantation will have a site height of 45 ft when it reaches the upper growing stock limit, which will take 13 yr in this example. Without thinning, the plantation would grow into the self-thinning zone to a final relative density of 76%, and by age 30 yr, the average diameter of the plantation would be only 9.4 (Figure 1). Assuming no density-related mortality before the first thinning, the initial planting density is set equal to the number of trees per acre expected before the first thinning, 605 trees/ac.

Marginal Analysis

In the procedure demonstrated here, marginal analysis is used to control the variable costs in a density management prescription: planting density and the residual thinning density. Planting density can be determined with marginal analysis independent of the density management diagram; however, a diagram is needed to determine the upper growing stock limit required to control stand development and to efficiently determine when to thin. Optimal planting density is determined by comparing the cost of each additional increment of 50 seedlings with revenue or benefit they produce at the end of the rotation. A thinning age is then set by determining the age when the plantation reaches a predefined upper limit to growing stock. Marginal analysis is used to determine the optimal growing stock to leave after thinning by comparing the cost of leaving additional increments of 5 ft^2/ac of basal area with the benefit it produces at the end of the rotation. All costs are compounded to the end

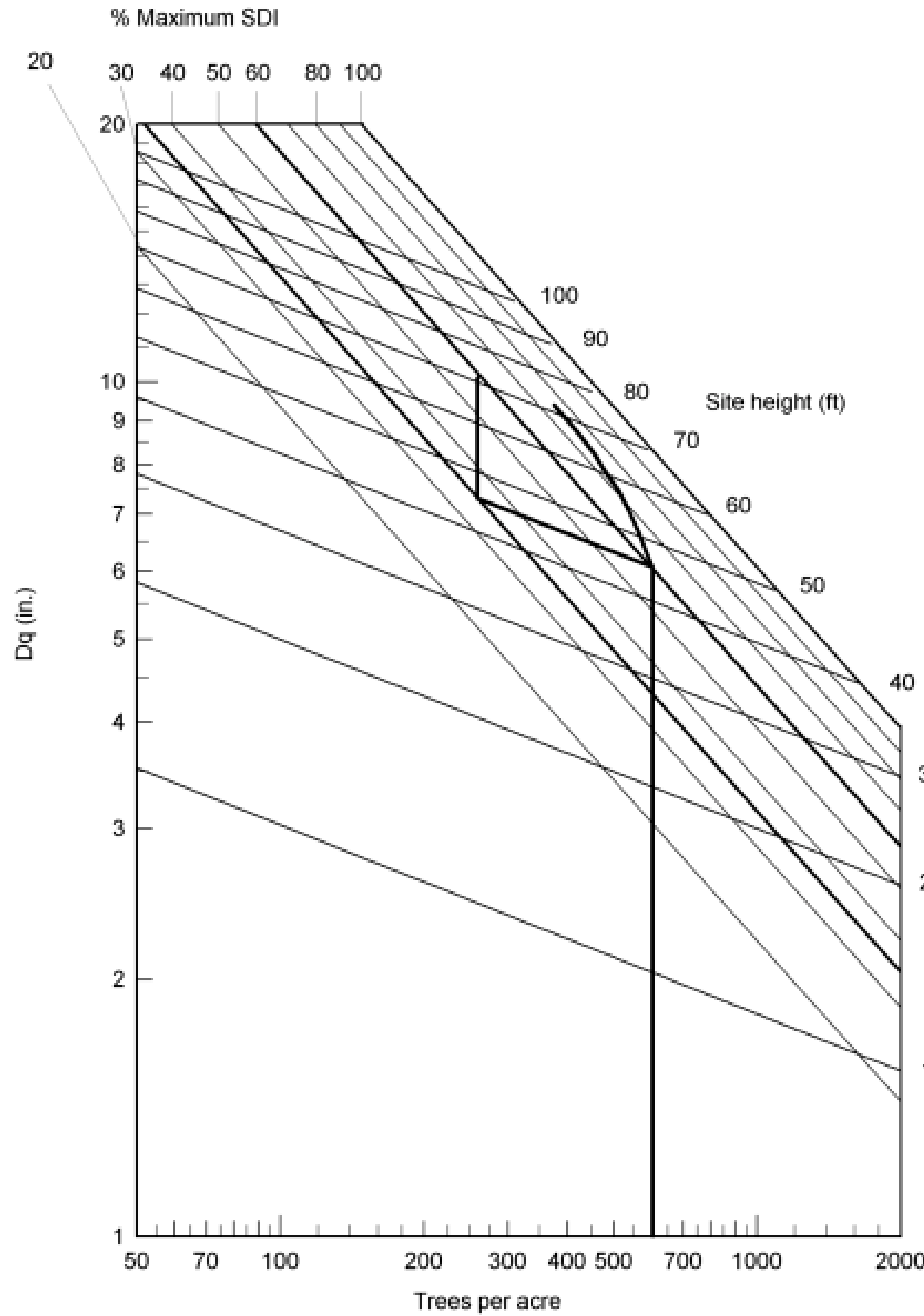

Figure 1. Expected development of thinned and unthinned loblolly pine plantations plotted on a density management diagram. Upper and lower limits of growing stock are indicated by the thicker lines representing percentage maximum SDI.

of the rotation. The optimal planting density or residual basal area is determined by accumulating the 50 seedling or 5 ft^2/ac increments until their marginal costs equal or exceed the marginal benefit they produce.

For this demonstration, each seedling was assumed to cost \$0.12 (\$0.05 per seedling and \$0.07 to hand-plant) (Dubois et al. 1999). Stumpage values at the end of the rotation and stumpage values left after thinning were calculated with a program

Table 1. Product specifications and prices used in COMPUTE_MERCHLOB to calculate stumpage values for even-aged loblolly pine plantations in southeastern Louisiana.

Product category	Price per unit ($)	Minimum diameter	Maximum diameter
		(in.)	
Pulpwood	26/cord	3.5	12.0
Chip-n-saw	90/cord	6.0	12.0
Saw timber	400/mbf*	9.5	18.0

* MBF = 1,000 bd ft Doyle scale.

named COMPUTE_MERCHLOB, or MERCHLOB for short (Busby et al. 1990). This program not only projects growth and yield for specified ages or residual basal areas but also calculates the optimal product mix for maximum, total stumpage value. Stumpage values are calculated with the prevailing prices for pulpwood, chip-and-saw, and saw timber in southeastern Louisiana during 1999 (Table 1). Thinnings were from below and were prescribed when the projected development of the plantation reached the upper growing stock limit of 60% of maximum SDI. The procedure is demonstrated with three interest rates at a fixed rotation age (3, 6, and 9% at 30 yr) and three rotation ages at a fixed interest rate (25, 30, and 35 yr at 6%) to illustrate the sensitivity of density management prescriptions to these variables.

Results

Planting Density

Optimal planting densities decreased with increasing interest rates and with increasing rotation lengths. For the 3, 6, and 9% interest rates, the optimal planting densities for a 30 yr rotation are 750, 700, and 600 seedlings/ac (spa). The optimal planting density for rotation ages of 25, 30, and 35 yr at an interest rate of 6% are 750, 700, and 500 spa. While these results seem to suggest that 5 yr increments in rotation age have the same effect on the optimum planting density as three percentage points of interest rate, rotation age and interest rate do not have interchangeable effects on profit. With no thinning and revenues discounted to year 0, the maximum present net value (PNV) that can be obtained with a 30 yr rotation decreases nearly eightfold from $1772.34/ac at 3% to $229.43/ac at 9% (Table 2). In contrast, the PNV associated with a 6% interest rate increases from $667.94/ac to $710.95/ac when the rotation is increased from 30 to 35 yr. The PNV for a 25 yr rotation and a 6% interest rate is $661.58/ac.

The change in product mix and associated values complicates the comparison between marginal costs and marginal revenues. From 500 to 800 spa, the marginal benefit for a 30 yr rotation from each additional 50 seedlings planted per acre peaked at $95.19/ac between 550 and 600 spa to a low of $2.16/ac between 750 and 800 spa (Table 3). The marginal benefit generally decreased between 500 and 800 spa, but changes in product mixes sometimes created wide changes in direction. Despite the occasional wide swings in marginal benefit, the last increments of seedlings that produced more revenue than they cost were clearly evident. For example, the marginal cost of 50 seedlings compounded 30 yr to the end of the rotation at 9% is $79.61/ac, and although the marginal benefit between 500 and 550 seedlings was $6.52/ac less than the marginal cost, the marginal benefit for the next increment of seedlings was $15.58/ac higher than the marginal cost. After 600 spa, the marginal costs of successive increments of seedlings always exceeded the associated marginal benefits. At 6%, the marginal benefit never again exceeded the compounded marginal cost of $34.46/ac after 700 spa, and at 3%, the marginal benefit dropped to $2.16/ac after 750 spa, $12.30/ac less than the associated marginal cost compounded for 30 yr.

While marginal analyses indicate that the initial investment in growing stock must decrease with either increasing interest rates or longer rotations to maximize profit, the analyses reveal an extreme sensitivity of maximum profit to initial spacing. For a 30 yr rotation, the optimal planting density decreased from 750 to 600 spa as the interest rate increased from 3 to 9%, and it decreased from 800 to 500 spa as the rotation age increased from 25 to 35 yr with a 6% interest rate. A 150 spa difference between the minimum and the maximum interest rates is only a 0.9 ft difference in distance between seedlings. A 300 spa difference between the shortest and longest rotations analyzed in this study is only a 2.0 ft difference in seedling spacing. Such sensitivity between maximum profit and planting density also creates a premium on the survival of every seedling planted, calling into question the practice of factoring seedling mortality into planting density. Planting more seedlings than economically optimal decreases the potential profit obtained from the rotation by increas-

Table 2. Present net value and land expectations of nearly optimum planting densities and residual growing stock determined with marginal analysis and a fixed upper limit to growing stock for even-aged, loblolly pine plantations in southeastern Louisiana.

Rotation (yr)	r* (%)	Planting cost	No thinning PNV	No thinning LEV	With thinning Age[†] yr	With thinning Revenue[††]	With thinning PNV	With thinning LEV	Increase in LEV[§] (%)
		($/ac)				($/ac)			
30	3.00	90.00	1,772.34	2,986.60	14.00	381.72	2,242.60	3,779.04	27.00
30	6.00	84.00	667.94	800.22	15.00	469.89	791.76	948.55	19.00
30	9.00	72.00	229.43	245.96	18.00	608.97	387.80	415.74	69.00
25	6.00	96.00	661.58	851.60	13.00	255.41	740.29	952.81	12.00
35	6.00	60.00	710.95	810.16	22.00	917.85	989.78	1,127.90	39.00

* Interest rate.
[†] Thinning age.
[††] Thinning revenue.
[§] Compared no thinning.

Table 3. Marginal analysis for planting density for an even-aged, loblolly pine plantation in southeastern Louisiana with a site index of 65 ft at base age 25 yr under three interest rates for a rotation age of 30 yr. Seedlings were assumed to cost $0.12; each 50-seedling increment in planting density resulted in a constant marginal cost of $6.00 at year zero of the plantation. Underlined values identify the last 50 seedling increment where the marginal revenue exceeded the marginal cost of the additional seedlings compounded to the end of the rotation.

Planting density (TPA)	Stumpage value at harvest age	Marginal benefit*	Marginal cost compounded to harvest age 3%	6%	9%
	($/ac)				
500	4,316.97				
		73.09	14.56	34.46	79.61
550	4,390.06				
		95.19	14.56	34.46	79.61
600	4,485.25				
		20.93	14.56	34.46	79.61
650	4,506.18				
		42.84	14.56	34.46	79.61
700	4,549.02				
		31.60	14.56	34.46	79.61
750	4,580.62				
		2.16	14.56	34.46	79.61
800	4,582.78				

* The increment in revenue realized from the incremental increase in trees per acre planted.

ing the marginal planting cost at year zero and by increasing the time in which the marginal costs are compounded. This longer time period in which marginal planting costs are compounded is the result of closer spacing between trees. Diameter growth is a function of the size and proximity of neighboring trees. Since seedling mortality usually does not occur uniformly throughout the stand, the closer spacing will slow diameter growth. Given that product specifications are based on diameter, more time will be required to reach commercial minimums. If mortality must be factored into initial planting density, larger marginal costs and longer compounding intervals require *fewer*, not more, trees to be planted. With small differences in optimal planting density, the effort and expense of developing nearly optimal density management prescriptions can be wasted within one step of the tree planter or compensating for seedling mortality.

When the development of these stands is plotted on the density management diagram, the relationship between the ending value of growing stock and either interest rate or rotation length is clearly evident (Figure 2). Left unthinned, the final stand density at age 30 yr with the 3, 6, and 9% interest rates decreases from 74 to 72 to 68% of maximum SDI, respectively (Figure 2a); at the 6% interest rate, the respective final stand densities are 74 and 64 for the 25 yr and 35 yr rotations (Figure 2b). Under a 0% interest rate (where the marginal cost of a 50 seedling increment is $6.00/ac, not compounded) the final stand density reached at the end of an unthinned, 30 yr rotation is projected to be 76% of maximum SDI. Mean annual increment has been shown to culminate at 82% of maximum stand density (Smith and Brand 1988, Smith 1989). The possible correspondence between the final stand density under a 0% scenario and the culmination of mean annual increment is consistent with one of the conclusions reached by Chang (1984) regarding the relationship between optimal rotation age and interest rate. According to his analysis, the optimal rotation age for a 0% interest rate corresponds to the age at which mean annual increment of the stand culminates. This has important implications for stand management: (1) the optimal limit of stand density increases as the prevailing interest rate decreases and (2) stand density must be managed, or at least monitored, to expect a positive return from invested capital.

Thinning

Since increasing interest rate and rotation length reduce the optimum planting density, for a given site index, the time needed to reach to the upper growing stock limit of 60% of maximum SDI increases with either variable. Based on the development of these plantations as projected by MERCHLOB, for the 3, 6, and 9% interest rates and a 30 yr rotation, the respective thinning ages are 14, 15, and 18 yr, and for rotation lengths of 25, 30, and 35 yr with a 6% interest rate, the thinning ages are 13, 15, and 22 yr.

The optimum amount of growing stock to be left after thinning differs little between the 3 and 6% interest rates with a 30 yr rotation (60 vs. 65 ft^2/ac, respectively) and between the 25 and 30 yr rotation lengths with a 6% rate (65 vs. 70 ft^2/ac). The largest differences occurred when interest rate increased to 9% or the rotation length increased to 35 yr. From 6 to 9% the optimal residual basal area increased to 80 ft^2/ac, and from 30 to 35 yr, optimal residual basal area increased to 85 ft^2/ac. While the effect of interest rate and rotation length are confounded in this example by the different thinning ages, the higher residual growing stock after thinning is the combined result of lower initial planting density and later thinning age. The combined effect of the lower initial planting density and later thinning age associated with the 9% interest rate result in larger trees on average that produce larger increments in value at the end of the rotation for each additional 5 ft^2/ac increment in residual basal area than the higher planting densities and earlier thinning ages associated with the 3 and 6% interest rates. The longer 35 yr rotation also has a lower planting density and a later thinning age, thus, larger, average-sized trees after thinning. The increase in optimal residual basal area after thinning specified with this analysis is consistent with the increased

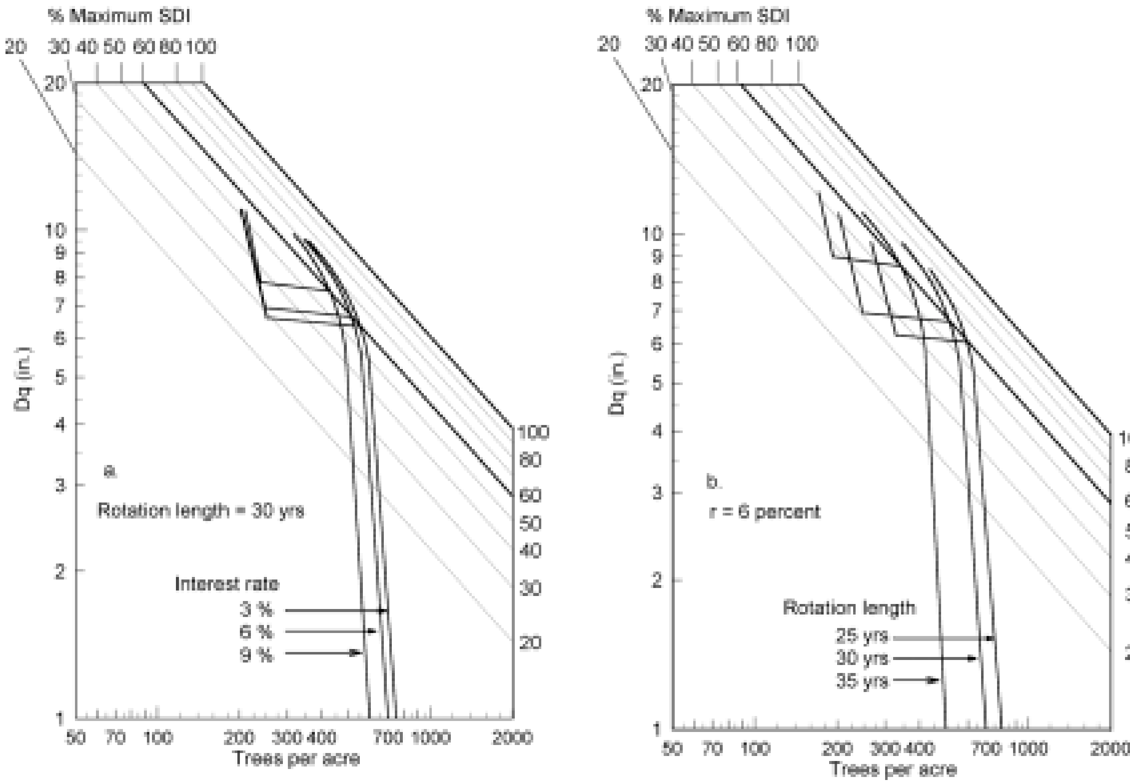

Figure 2. Expected development of thinned and unthinned loblolly pine plantations planted or thinned to optimal densities according to marginal analysis using three interest rates with a fixed rotation of 30 yr (a) and three rotation lengths with 6% interest rate (b). Upper growing stock limit indicated by thicker line representing percentage of maximum SDI.

residual basal area that would be prescribed for older stands when specified upper and lower limits of relative density are used to control growing stock (Reukema and Bruce 1978, Drew and Flewelling 1979).

Product mixture affects both the marginal benefit and the marginal costs in a thinning analysis. Whereas in the marginal analysis of planting density the marginal cost is constant in this example, the marginal cost of the residual growing stock within the range of 50 to 70 ft^2/ac for the 6% and 30 yr rotation scenario varies between $94.45 to $124.21/ac for each 5 ft^2/ac increase in residual basal area (Table 4). Overall, the marginal benefit decreased with each 5 ft^2/ac increment in residual basal area, but it spiked at $581.31/ac when basal increased from 60 to 65 ft^2/ac.

Table 4. The marginal analysis for residual basal area after thinning for a 15-yr-old loblolly pine plantation in southeastern Louisiana with a site index of 65 ft (base age 25 yr) for a rotation age of 30 yr and a 6% interest rate. Underlined values identify the last increment in residual basal area where the marginal revenue at harvest exceeds the marginal cost of the additional basal area compounded to the end of the rotation.

Residual BA* (ft^2/ac)	Revenue†	Marginal benefit††	Cost§	Marginal cost‖
	($/ac)			
50	3,143.43		485.89	
		230.78		94.45
55	3,374.21		525.30	
		186.75		122.11
60	3,560.96		576.25	
		<u>581.31</u>		<u>100.37</u>
65	4,142.27		618.13	
		37.80		122.82
70	4,180.07		669.38	
		50.45		124.21
75	4,230.52		721.21	
		66.08		84.36
80	4,296.60		756.41	

* After thinning (ft^2/ac).
† Received at harvest age.
†† Increment in revenue realized from the incremental increase in residual basal area.
§ Stumpage value of residual stand.
‖ Stumpage value of residual stand from the incremental increase in residual basal area ft^2/ac compounded by the respective interest rate for 15 yr, the difference between the thinning age and the harvest age.

In all cases, thinning increases the land expectation value (LEV) over the LEV obtained with no thinning during the rotation (Table 2). Land expectation value (LEV) is a better means of comparing the various density management prescriptions than present net value because it reflects the present net worth of a series of rotations, each receiving uniform management (Davis 1966). Present net value simply reflects the profit discounted to year zero for a single rotation and does not account for the effects of long rotations and high interest rates on the capital value of the land. The average increase in LEV produced by including thinning in the prescription was 33%. The largest change, 69%, was observed with a 30 yr rotation and the 9% interest rate, and smallest change, 12%, was observed with the 25 yr rotation and 6% interest rate. While the largest increase in LEV due to thinning occurred with the rotation length and interest combination resulting in the lowest, absolute LEV, the smallest change in LEV due to thinning was in the midrange of land expectation values of the five scenarios analyzed in this study. The increase in LEV is a result of thinning increasing average stand diameter at the end of the rotation and using trees that would otherwise be lost in self-thinning.

For all five combinations of interest rates and rotation lengths, the optimal relative density to leave after thinning ranged from 29 to 36% of maximum SDI and averaged 33%. The minimum value of the optimum relative density to leave after thinning, which occurred for the 3% interest rate with a 30 yr rotation length, was slightly greater than the relative density corresponding with canopy closure. The maximum value of the optimum residual density, which was observed for two combinations of interest rate and rotation length (9% interest with a 30 yr rotation and 6% interest with a 35 yr rotation), corresponded with the lower limit of full site occupancy.

For all five scenarios, the final stand densities (at rotation age) after thinning exhibited extremely little variation: the range in final stand densities is only 3 percentage points and averaged 53% of maximum SDI. This result indicates that for the rotation ages and interest rates considered in these examples, nearly optimum density management regimes result in approximately equivalent growing stock at harvest age. While the optimum growing stock to leave after thinning seems to correspond with lower limit of full-site occupancy, the amount of growing stock projected for the end of the rotation with thinning is close to the self-thinning threshold. Thus, the results for these examples suggest that maximum profit is obtained by maintaining growing stock at roughly the lower and upper edge of full-site occupancy without self-thinning, that is, between relative densities of 35 and 55% of maximum SDI.

Conclusions

As evidenced by the changes in planting density, marginal analysis indicates that initial investments in plantation establishment must decrease with increasing interest rate and increasing rotation length or profit will be sacrificed. Increasing interest rate reduces the value of the land as reflected by decreases in LEV, even with the additional value gained with thinning. While the effects of initial investment rate and rotation length on LEV are complicated by changing product mix with age, overall, if the product stumpage prices remain constant, LEV will decrease with rotation length.

While planting densities and residual densities to leave after thinning for nearly optimal profit are readily identified with marginal analyses, actually establishing these densities in the field requires extraordinary precision in spacing. The largest difference in optimal spacing is only 2 ft between seedlings and 1.5 ft between trees after thinning: most spacings identified as optimal for each scenario in this analysis differ by less than 1 ft. While the precision in initial spacing indicated with marginal analysis may be greater than the inherent precision of the growth-and-yield simulator used in this analysis, one important conclusion may be drawn from these results: maximizing profit depends on the precision and quality of the field work to establish a plantation and to manage its density throughout the rotation.

The results of this study indicate that growing stock must be controlled, or at least monitored, to obtain any profit from the land. To maximize profit and land value, this study suggests that growing stock be managed at the thresholds of full-site occupancy and self-thinning. That profit coincides with stage of stand development is logical since stage of development, stand structure, and growth are interrelated (Dean and Baldwin 1996). Below full-site occupancy, growing stock is inadequate for rapid, overall stand growth, and above the self-thinning threshold, individual tree growth is inadequate for sufficient growth in stumpage value. Consequently, 55 and 35% of maximum SDI represent good upper and lower boundaries for growing stock when landowner objectives include maximizing profit. Individual circumstances will determine the actual LEV that can be attained through their influence on initial planting density, thinning age, and thinning intensity.

Endnote

[1] *SDI* = TPA × (*Dq*/10)1.6, where *TPA*= trees per acre and *Dq* = average stand diameter.

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